N33.00007

arXiv:2111.03857

Ferromagnetism in tilted fermionic Mott insulators

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Introduction: tilted potential in periodic systems

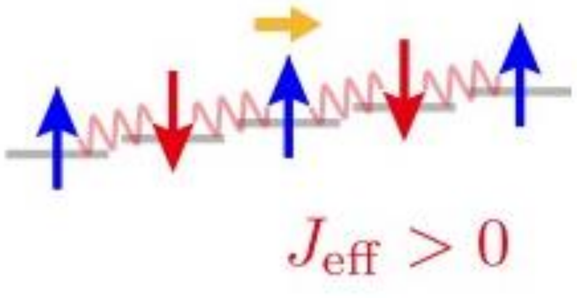
Long history of research:
Response of electrons in solids to electric field

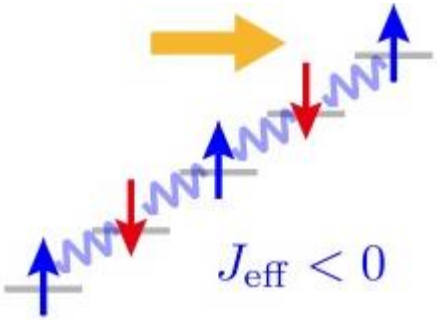
e.g. Bloch oscillation, Zener tunneling

Renewed interest in the context of localization of quantum many-body system

e.g. Hilbert space fragmentation,
Hilbert space shuttering,
quantum many-body scars

Introduction: Magnetism of tilted Mott insulators





Static tilt enhances $J_{\rm eff}$
[K. Takasan and M. Sato,
PRB **100**, 060408 (2019)]

Localized spin system?
→ Wannier-Stark
localization occurs!

What happens for
larger E?

Main result: Effective interaction
becomes ferromagnetic

cf. Mentink, Balzer and Eckstein, Nat. Commun. 6, 6708 (2015)
Reversible control of exchange interaction **by periodic modulation**

Setup

One-dimensional Hubbard model with a linear potential

$$\hat{H} = -t_h \sum_{j=1}^{L-1} \sum_{\sigma=\uparrow,\downarrow} \left(\hat{c}_{j+1,\sigma}^{\dagger}\hat{c}_{j,\sigma} + \mathrm{h.c.}\right) + U \sum_{j=1}^{L} \hat{n}_{j,\uparrow}\hat{n}_{j,\downarrow} + \sum_{j=1}^{L} \sum_{\sigma=\uparrow,\downarrow} jE\hat{n}_{j,\sigma}$$

[length gauge]

Time-dependent gauge transformation

$$U(t) = \exp\left[-iEt\sum_{j,\sigma} jn_{j,\sigma}\right], \tilde{H}(t) = U^{\dagger}HU - iU^{\dagger}\partial_t U$$

$$\tilde{H}(t) = -t_h \sum_{j=1}^{L-1} \sum_{\sigma=\uparrow,\downarrow} \left(e^{iEt}\hat{c}_{j+1,\sigma}^{\dagger}\hat{c}_{j,\sigma} + \mathrm{h.c.}\right) + U \sum_{j=1}^{L} \hat{n}_{j,\uparrow}\hat{n}_{j,\downarrow}$$

[velocity gauge]

Initial state: antiferromagnetic

$$|\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\cdots\uparrow\downarrow\rangle$$

no double occupancy

$t_{\mathrm{h}} = 1$: unit of energy, $E = |e|a\mathcal{E}/\hbar$

$L = 10$ in numerical simulations

e: elementary charge

a: lattice constant

$\mathcal{E}$: electric field

<u>Time-periodic system</u> Period: $\frac{2\pi}{E}$

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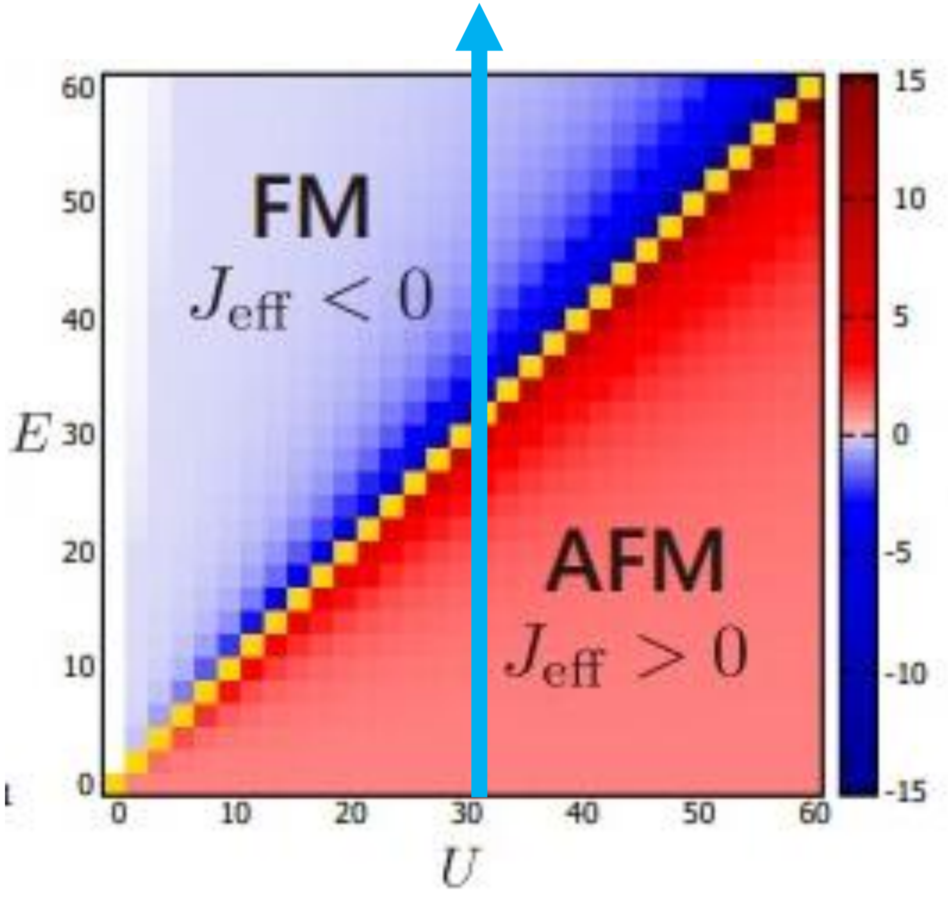
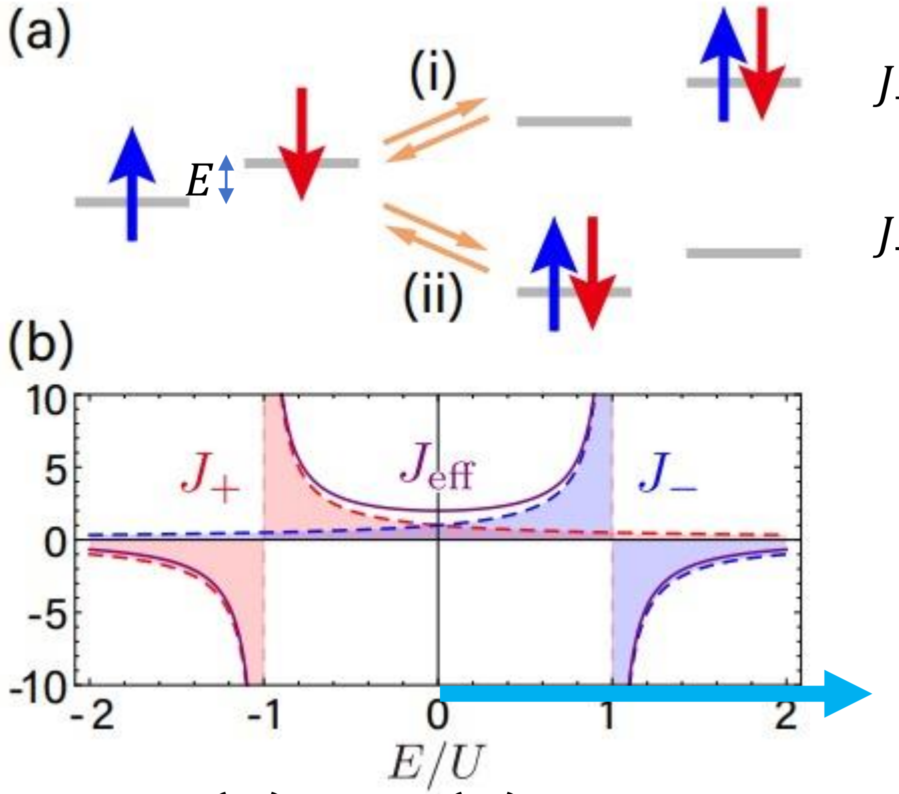
Effective spin Hamiltonian

$$\hat{H}_{\text{eff}} = \sum_j J_{\text{eff}}(E)\boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1}, J_{\text{eff}}(E) = \frac{J_0}{1-\left(\frac{E}{U}\right)^2}$$

For $E = 0$,

$$\hat{H}_{\text{eff}} = \sum_j J_0 \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1}, J_0 = \frac{4t_h^2}{U}.$$





$$J_+(E) = \frac{2t_h^2}{U+E}$$

$$J_-(E) = \frac{2t_h^2}{U-E}$$

$$J_{\text{eff}}(E) = J_+(E) + J_-(E)$$

See also [K. Takasan and M. Sato, PRB **100**, 060408 (2019)]

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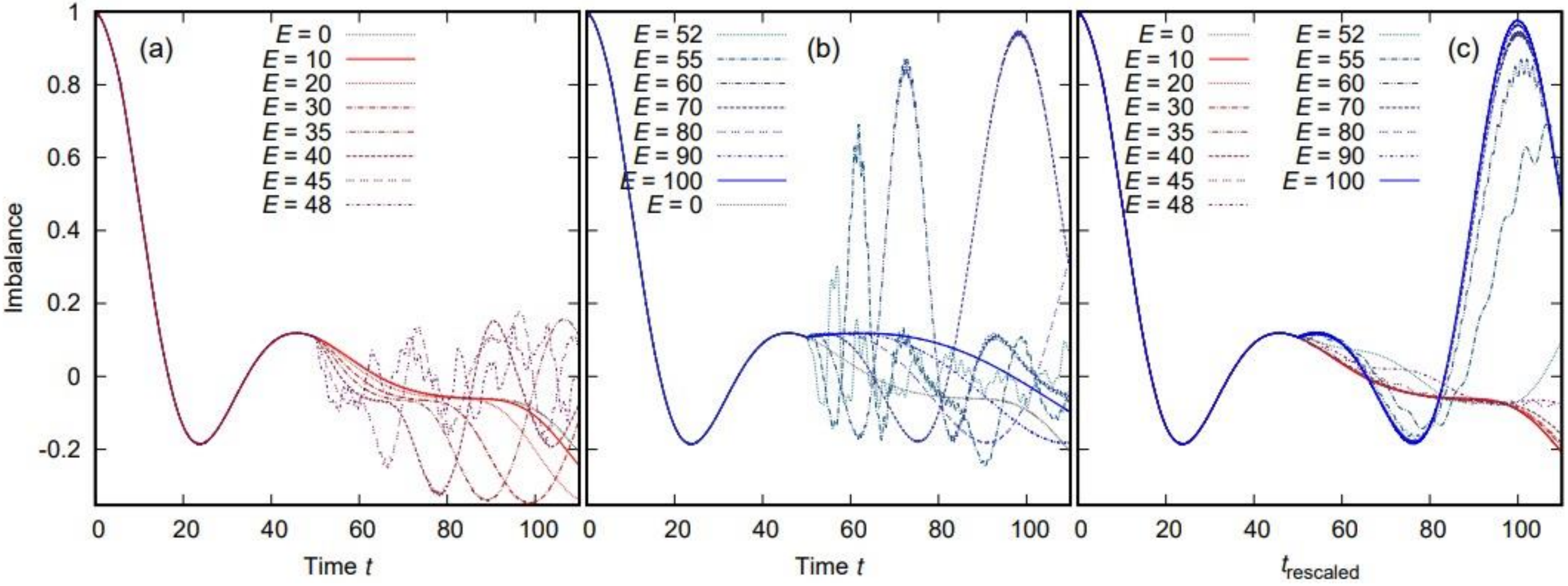
Time reversal by the strong electric field

Imbalance $I = \frac{1}{L}\sum_{j=1}^{L}(-1)^j \langle\psi(t)|(\hat{n}_{j\downarrow} - \hat{n}_{j\uparrow})|\psi(t)\rangle$

$|\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\cdots\uparrow\downarrow\rangle$

$$J_{\mathrm{eff}}(E) = \frac{J_0}{1-\left(\frac{E}{U}\right)^2}$$



$L = 10$
$U = 50$
$t_{\mathrm{on}} = 50$

Fast-forward dynamics

$J_{\mathrm{eff}}(E) > J_0$ for $0 < E < U$

Reversed dynamics

$J_{\mathrm{eff}}(E) < 0$ for $E > U$

$$t_{\mathrm{rescaled}} = t_0 + \frac{J_0}{|J_{\mathrm{eff}}(E)|}(t - t_0)$$

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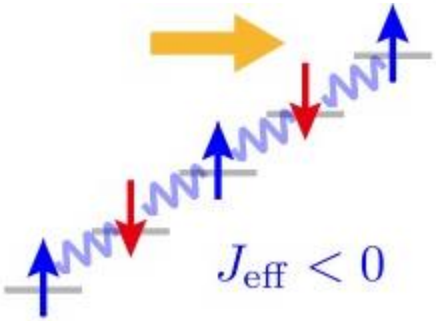
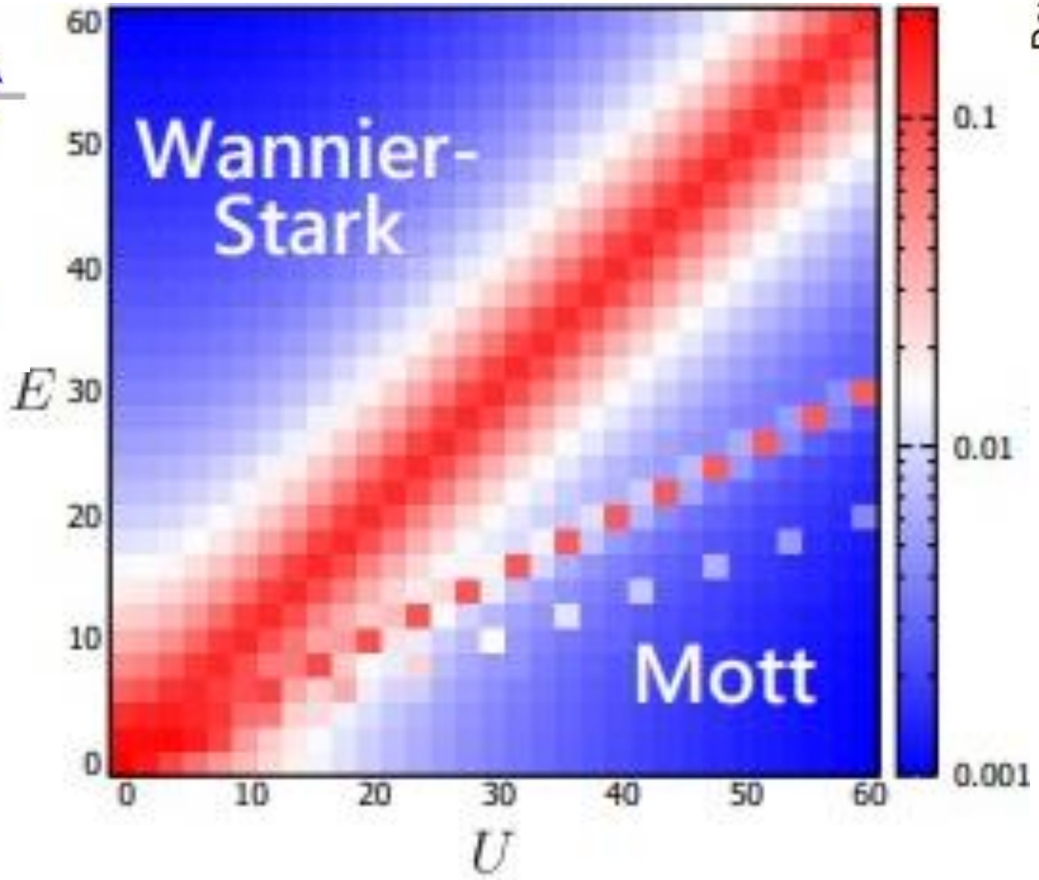
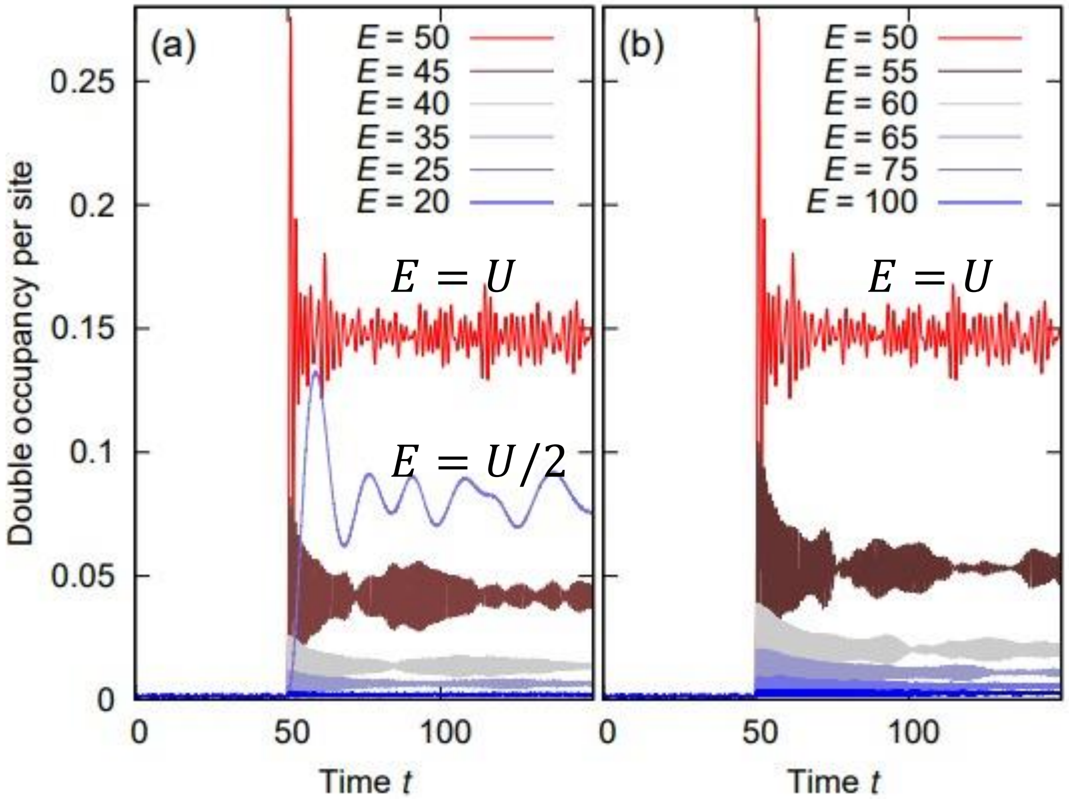
Double occupancy?

$$\overline{\mathcal{N}_{\mathrm{double}}} = \frac{\int_{t=t_0}^{t_1} \frac{1}{L}\sum_{j=1}^{L} \langle \Psi(t) | \hat{n}_{j\uparrow}\hat{n}_{j\downarrow} | \Psi(t) \rangle}{t_1 - t_0}$$

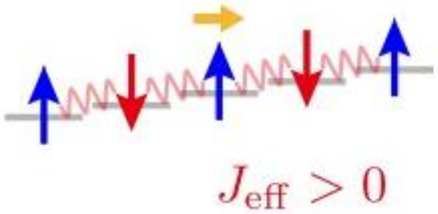




$U = 50$



$E = U, \frac{U}{2}, \frac{U}{3}, \ldots$: resonances



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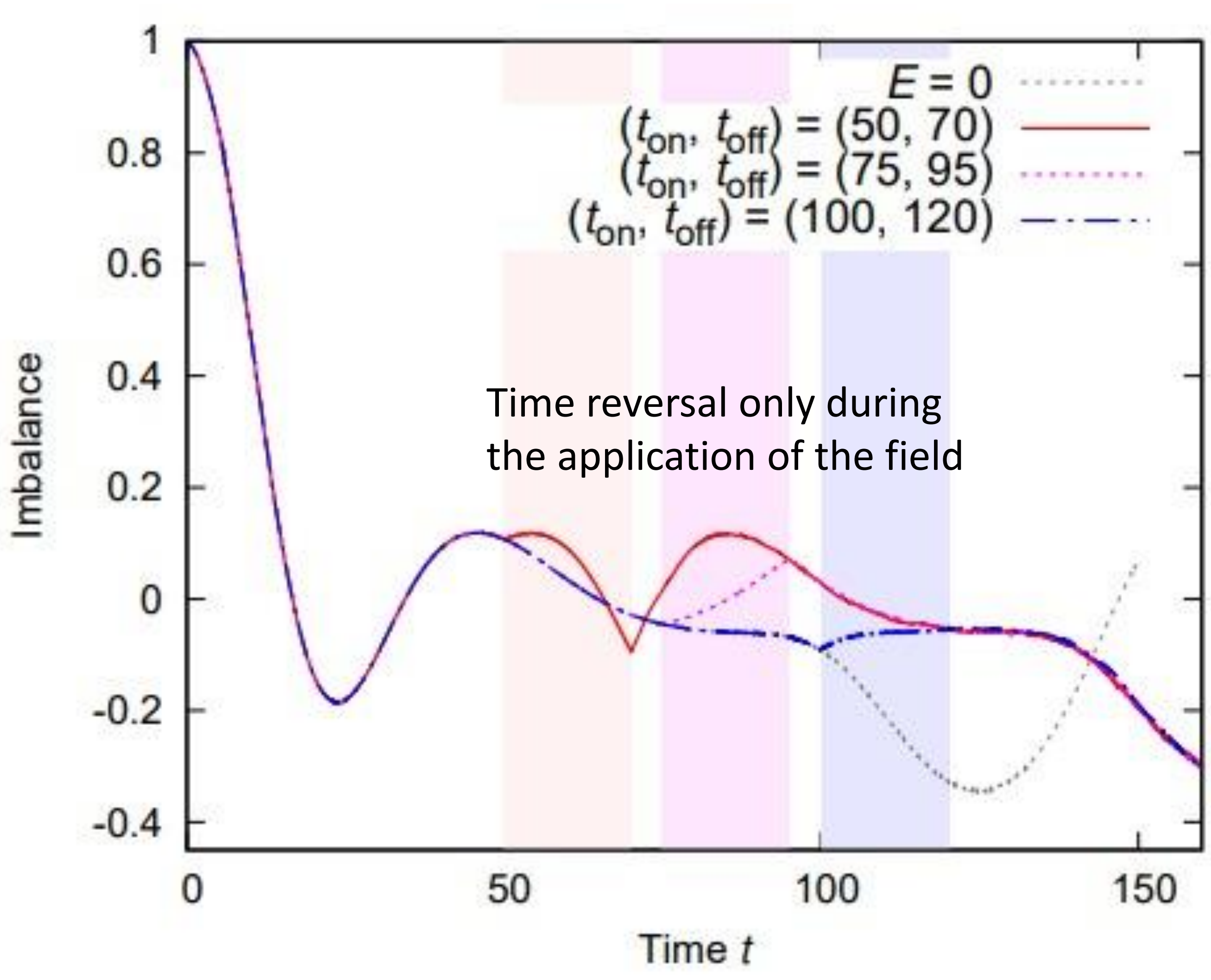
Dynamics after limited-time field application



$$\frac{E}{\sqrt{2}} = U = 50,$$

$$J_{\mathrm{eff}} = \frac{J_0}{1 - \left(\frac{E}{U}\right)^2} = -J_0$$

Same $\Delta t = t_{\mathrm{off}} - t_{\mathrm{on}}$

→ Same dynamics after time reversal

Potential application

Out-of-time ordered correlations:
characterize operator scrambling

[Wiener 1938][Larkin and Ovchinnikov, JETP 1969]

Non-commutating

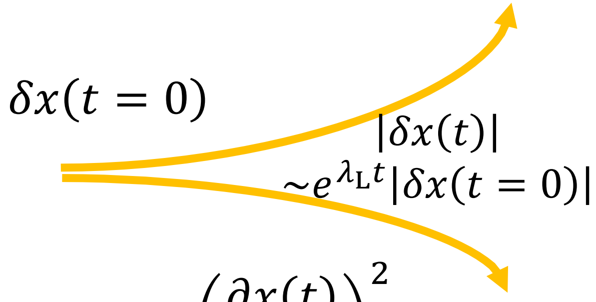
$VW(t) \neq W(t)V$

scrambling

Commuting

$VW(0) = W(0)V$

cf. chaos in classical dynamics



$$\left(\frac{\partial x(t)}{\partial x(0)}\right)^2 = \{x(t), p(0)\}_{PB}{}^2 \sim e^{2\lambda_L t}$$

λ_L: Lyapunov exponent

$$C(t) = \langle |[W(t), V(t=0)]|^2 \rangle = \langle W^\dagger(t) V^\dagger(0) W(t) V(0) \rangle + \cdots$$

OTOC $\sim e^{2\lambda_L t}$ at long times with $\lambda_L > 0$: chaotic

$\lambda_L \leq 2\pi k_B T/\hbar$ (chaos bound)

[Maldacena, Shenker, and Stanford, JHEP08(2016)106]

$$W(t) = \boxed{e^{iHt}}\, W\, e^{-iHt}$$

Time evolution by $-H$ is key to the measurement

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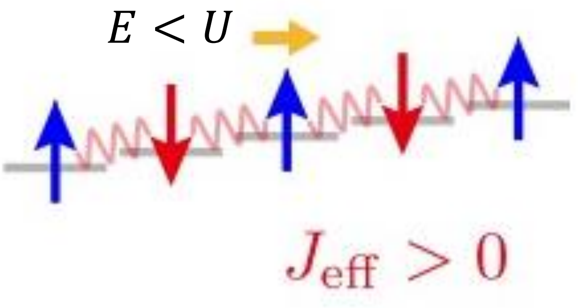
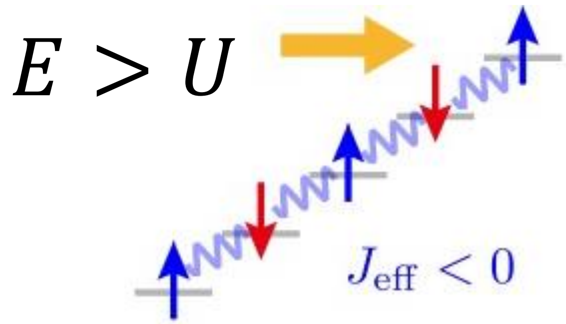
Kazuaki Takasan and Masaki Tezuka, arXiv:2111.03857

Summary

- One-dimensional Mott insulator (half-filled Hubbard model) + tilt
- Effective spin interaction: $J_{\text{eff}}(E) = J_0 / \left(1 - \left(\frac{E}{U}\right)^2\right)$





- Double occupancy suppressed for $U \gg t_{\text{h}}$ away from $E = U, \frac{U}{2}, \frac{U}{3}, \ldots$
- Sign reversal may be useful for measuring out-of-time-order correlators