SYK模型における多体局在：フォック空間での局在としての定量的解析

arXiv:2005.12809

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Fock space localization in the Sachdev-Ye-Kitaev model

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We study the physics of many body localization in the Majorana Sachdev-Ye-Kitaev (SYK) model perturbed by a one-body Hamiltonian. Specifically, we consider the statistics of many body wave functions and spectra as the strength of the one-body term is ramped up from an ergodic phase via a regime of non-ergodic yet extended states into a (Fock space) Anderson localized phase. Our results are obtained from an effective low energy theory, derived from the microscopic model by matrix integral techniques standard in the theory of disordered electronic systems. Applicable to systems of arbitrarily large particle number, the analytical results produced by this formalism are compared to exact diagonalization for systems containing up to 30 Majorana fermions. The statistics of many body spectra and wave functions, and the indications of the localization transition are in quantitative agreement with numerics. We believe that this is the first many body system where a localization transition is observed in parameter free agreement with first principle analytical calculations.

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Localization phenomena in quantum systems: what is Fock space localization?

Anderson localization of (uncorrelated) single particle wave functions: Analytical techniques, good agreements with numerics

Many-body localization (MBL) in 1D & higher dimensions with correlation: Few analytical results, numerics exponentially hard, many open issues

Fock space localization

$$|\Psi\rangle \to |\phi_1\rangle|\varphi_2\rangle|\zeta_3\rangle \cdots |\xi_N\rangle$$

- Spatially confined quantum systems (e.g. quantum dots)
- Many-body eigenstates localizing in the Fock space

[Altshuler-Gefen-Kamenev-Levitov PRL 1997, Silvestrov PRL 1997, PRB 1998, …]

ergodic, uniform states

Nonergodic, extended (NEE) states

nonergodic, localized states

parameter

The Sachdev-Ye-Kitaev model

N Majorana- or Dirac- fermions with all-to-all random couplings

[Majorana version]

$$\hat{H} = \frac{\sqrt{3!}}{N^{3/2}} \sum_{1 \le a < b < c < d \le N} J_{abcd} \hat{\chi}_a \hat{\chi}_b \hat{\chi}_c \hat{\chi}_d$$

J_{abcd} independently chosen from Gaussian random ensemble

[A. Kitaev: talks at KITP (2015)]

[Dirac version]

$$\hat{H} = \frac{1}{(2N)^{3/2}} \sum_{ij;kl} J_{ij;kl} \hat{c}_i^\dagger \hat{c}_j^\dagger \hat{c}_k \hat{c}_l$$

[Kitaev's talks]
[S. Sachdev: PRX **5**, 041025 (2015)]
"Two-body random ensemble" since 1970s

cf. S. Sachdev and J. Ye, PRL 70, 3339 (1993)

最近の研究から

Sachdev–Ye–Kitaev 模型，ブラックホール，冷却気体系

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超弦理論は重力の量子論の有力な候補として長年研究され続けている．究極の目標は自然界のあらゆる相互作用を統一する万物の理論の構築だが，副産物として数学や物理学の様々な分野との繋がりが見出されてきた．特に，この2，3年で物性理論，量子情報理論と超弦理論の意外な関係が明らかになってきた．

超弦理論を非摂動的にどう定義したらよいかというのは長年の問題だが，重力を含まないある種の量子場の理論が超弦理論，あるいはより一般的な量子重力理論の定義になっているのではないかというホログラフィー原理という考えがここ20年ほど有力視されている．対応が最もよく理解されているのは極大超対称ゲージ理論と呼ばれる一見特殊な理論の場合だが，最近，SYK模型という物性分野から出てきた理論が重力理論の少なくともある種の特徴をとらえていることがわかってきた．

SYK模型は，N個のフェルミオンが非局所的にランダムに相互作用している模型である．元々は1990年代初頭にサチデフ（Sachdev）と叶（Ye）が銅酸化物高温超伝導体の関連物質の実験に関係して非フェルミ液体状態を記述するために提案したSY模型というものがあったが，SYK模型はこれを簡単化してキタエフ（Kitaev）が2015年に提案した模型である．サチデフはもともと物性理論への応用という立場からホログラフィー原理に興味を持っていたようだが，途中から，SY模型を使って量子重力理論を定義するという方向性も追求し始めた．2015年にキタエフがSYK模型が「カオスの上限」を実現することを示し，量子重力の観点からの研究に火が付いた．

現在では，SYK模型と対応する重力理論が何かはまだわかっていないものの，量子重力や量子カオスの研究の舞台として積極的に研究され，また，関連する模型も多々提案されている．著者らは，光格子中の冷却気体を用いてSYK模型を実現する方法を提案した．この方法では，深い光格子の1サイトに複数のフェルミ原子を捕獲し，光会合レーザーにより，任意の2準位から分子状態への遷移を可能とする．形成された分子が別の2準位の原子へと速やかに光解離する状況で，分子の内部自由度を活用することにより，必要な相互作用のランダムさを実現できるという提案である．

SYK模型や超対称ゲージ理論のような量子重力理論の定義となると目されている理論を実験的に実現することができたとすると，量子重力系の様々な性質，たとえばブラックホールの生成や蒸発などを実験的に調べることができると期待される．そのような意味で，物性理論や冷却気体実験の専門家が，量子重力の研究に貢献できる可能性が拓かれつつある．

Keywords

SYK模型：
多数のフェルミオンが非局所的にランダムに相互作用する模型．量子重力，量子カオスなどの文脈で注目を集めている．詳しくは本文を参照．

ホログラフィー原理：
一見重力を含まない量子論が非自明な形で量子重力理論の定義になっているという予想．非重力理論と重力理論の次元は一般に異なる．1993年にトフーフト（'t Hooft）とサスキンド（Susskind）によって提案された．最も有名な具体例として1997年にマルダセナ（Maldacena）が提案したゲージ/重力対応がある．

「カオスの上限」(Chaos bound)：
カオス系の典型的性質として初期条件鋭敏性があるが，これはリアプノフ（Lyapunov）指数λで定量的に特徴づけられる（初期条件の微小摂動が時間tとともに$\exp(\lambda t)$で増大する）．量子多体系の場合，ある種の自然な物理的仮定のもとに，このλには温度Tに比例した上限値$2\pi k_B T/\hbar$（「カオスの上限」）があることがマルダセナ，シェンカー（Shenker），スタンフォード（Stanford）によって2015年に示された．超弦理論と対応する量子系はこの上限を満たすと信じられている．

最近の研究から Sachdev–Ye–Kitaev 模型，ブラックホール，冷却気体系 569
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<u>Solvable in the large-N limit</u>

(after sample average $\langle \cdots \rangle_{\{J\}}$)

<u>➔ Maximally chaotic ($\lambda_{\mathrm{L}} = 2\pi k_{\mathrm{B}} T/\hbar$)</u>

See e.g. [J. Polchinski and V. Rosenhaus, JHEP 1604 (2016) 001]
[J. Maldacena and D. Stanford, PRD **94**, 106002 (2016)]
[段下一平, 手塚真樹, 花田政範: 日本物理学会誌 73(8), 569 (2018)]

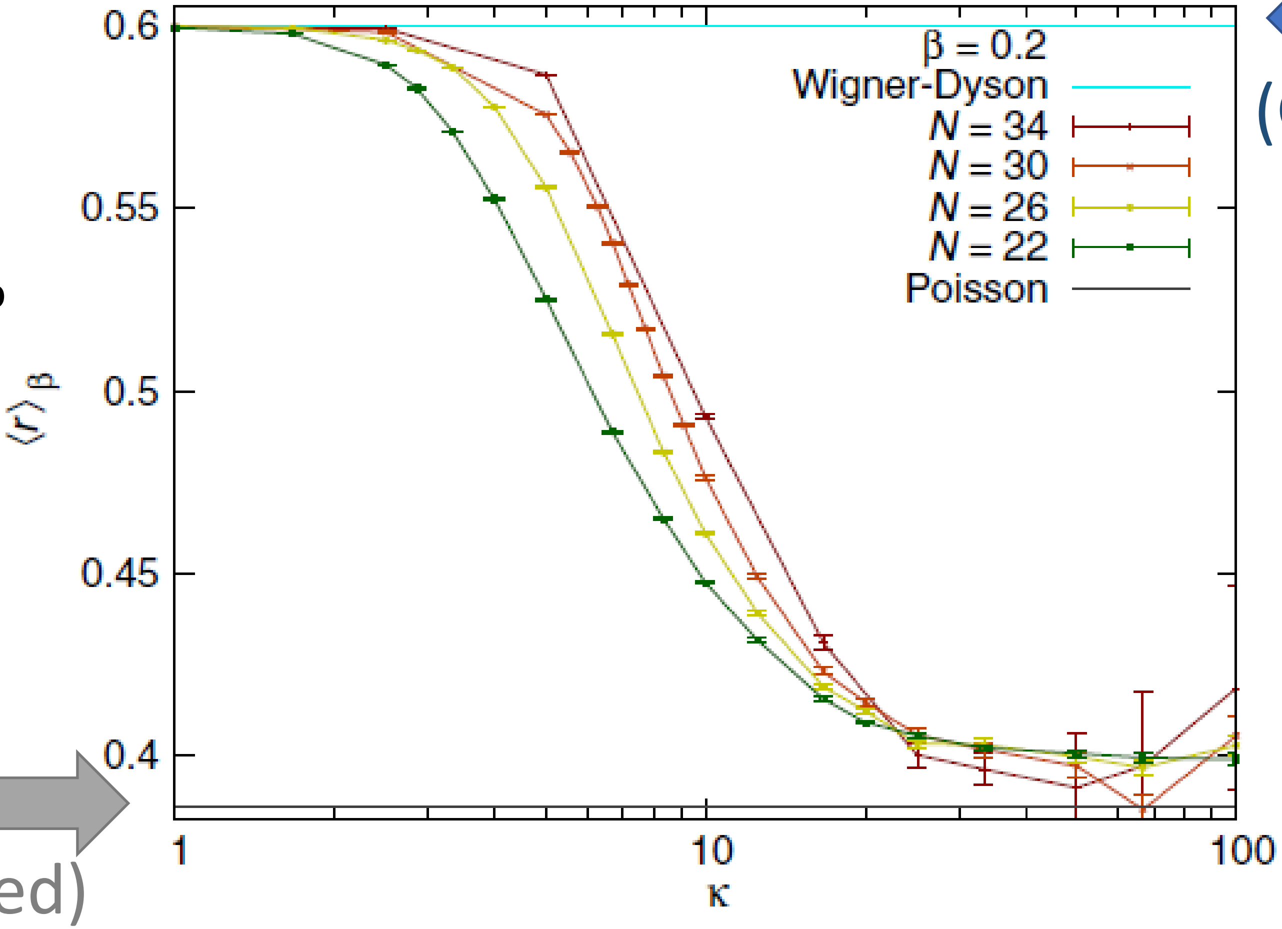
Candidate of quantum system holographically corresponding to black holes (experimental study of quantum gravity)

$\text{SYK}_{q\geq 4} + \text{SYK}_2$: breakdown of chaos

$$\hat{H} = \overbrace{\sum_{1\leq a<b<c<d}^{N} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d}^{\text{SYK}_4} + \overbrace{i\sum_{1\leq a<b}^{N} K_{ab}\hat{\chi}_a\hat{\chi}_b}^{\text{SYK}_2}$$

K_{ab}: standard deviation $= \kappa/\sqrt{N}$

Averaged ratio between neighboring energy level separations



GUE (Gaussian Unitary Ensemble)

Poisson (uncorrelated)

A. M. Garcia-Garcia, B. Loureiro, A. Romero-Bermudez, and MT, PRL **120**, 241603 (2018)

(cf. T. Nosaka, D. Rosa, and J. Yoon, JHEP09(2018)041 for GOE, GSE; Yu Yi-Xiang, Fadi Sun, Jinwu Ye, W.M. Liu, arXiv:1809.07577 for odd N cases)

We consider N Majorana fermions with normalization $\{\hat{\chi}_a, \hat{\chi}_b\} = \delta_{ab}$ here

Deviation from Gaussian random matrix as SYK_2 component is introduced

arXiv:2005.12809

Our model and choice of basis

$$\mathrm{SYK}_4 + \delta\, \mathrm{SYK}_2$$

$$\hat{H} = -\sum_{1\le a<b<c<d}^{2N} J'_{abcd}\hat{\psi}_a\hat{\psi}_b\hat{\psi}_c\hat{\psi}_d + i\sum_{1\le a<b}^{2N} K_{ab}\hat{\psi}_a\hat{\psi}_b$$

Block-diagonalize the SYK_2 part
(the skew-symmetric matrix (K_{ab}) has eigenvalues $\pm v_j$)

$$\hat{H} = -\sum_{1\le a<b<c<d}^{2N} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d + i\sum_{1\le j\le N}^{N} v_j\hat{\chi}_{2j-1}\hat{\chi}_{2j}$$

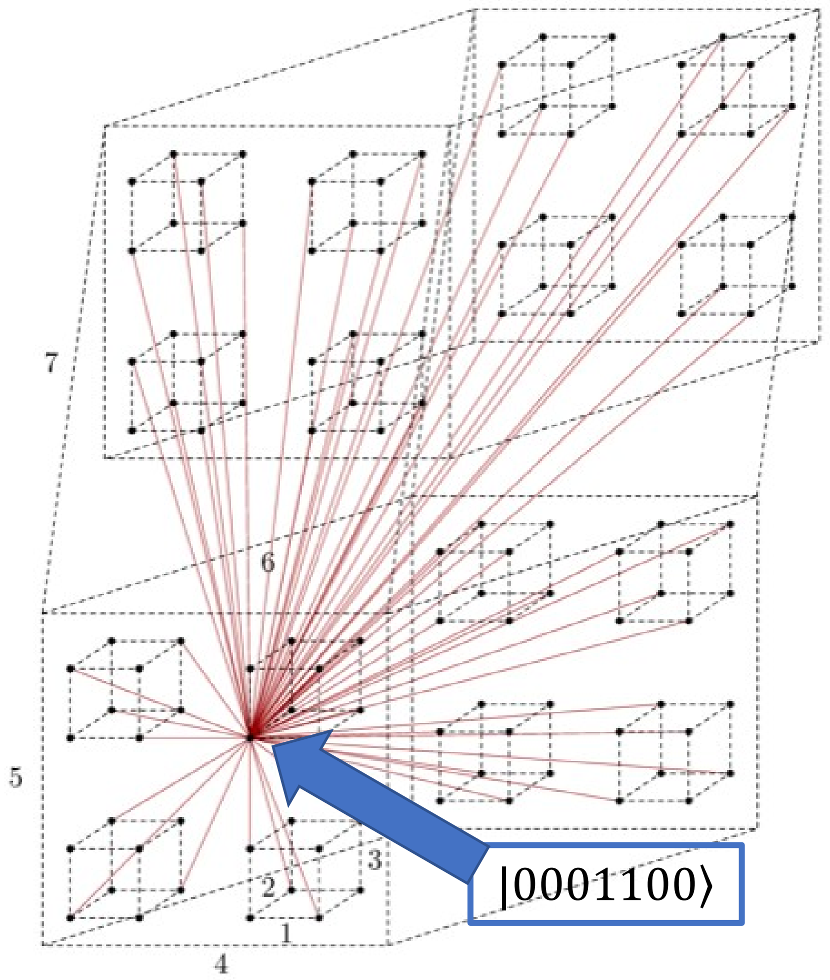
Normalization of J_{abcd}, v_j :
SYK_4 bandwidth = 1,
Width of v_j distribution = δ

We choose $\{\hat{\psi}_a, \hat{\psi}_b\} = \{\hat{\chi}_a, \hat{\chi}_b\} = 2\delta_{ab}$ as the normalization for the $2N$ Majorana fermions.
For $\hat{c}_j = \frac{1}{2}\left(\hat{\chi}_{2j-1} + \mathrm{i}\hat{\chi}_{2\mathrm{j}}\right)$ we have $\{\hat{c}_i, \hat{c}_j^\dagger\} = \delta_{ij}$.

arXiv:2005.12809

Our model and choice of basis

$N = 7$: 2^7 = 128 states



Basis diagonalizing the complex fermion number operators

$\hat{n}_j = \hat{c}_j^\dagger \hat{c}_j$ → Sites: the 2^N vertices of an N-dim. hypercube.

$$\hat{c}_j = \frac{1}{2}\left(\hat{\chi}_{2j-1} + \mathrm{i}\hat{\chi}_{2j}\right)$$

$$\hat{H} = -\sum_{1\le a<b<c<d}^{2N} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d + i\sum_{1\le j\le N}^{N} v_j\hat{\chi}_{2j-1}\hat{\chi}_{2j}$$

$$= -\sum_{1\le a<b<c<d}^{2N} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d + \sum_{1\le j\le N}^{N} v_j\left(2\hat{n}_j - 1\right)$$

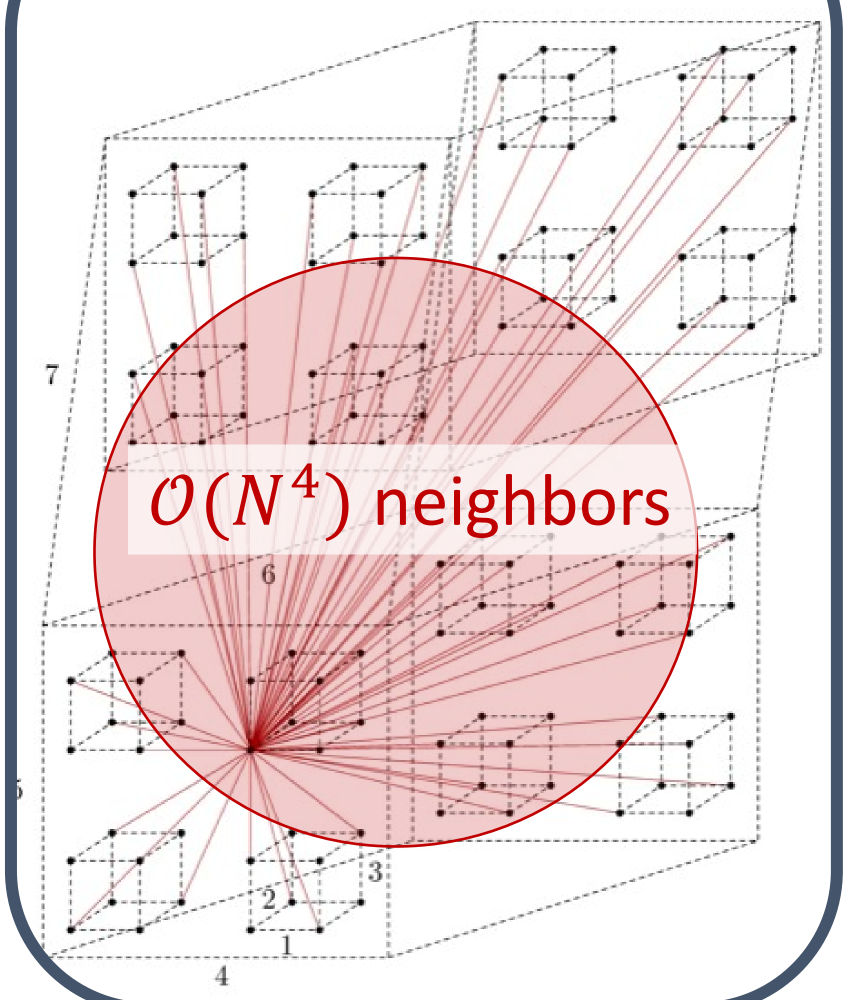
Each term of SYK_4 connects vertices with distance = 0, 2, 4.

For $N = 7$, each vertex is directly connected with 1 (distance=0, itself) + 21 (distance=2) + 35 (distance=4) vertices out of the possible $2^N = 128$ (64 per parity).

arXiv:2005.12809

Our model and choice of basis

2^N Fock states



Basis diagonalizing the complex fermion number operators $\hat{n}_j = \hat{c}_j^\dagger \hat{c}_j$ → Sites: the 2^N vertices of an N-dim. hypercube.

$$\mathrm{SYK}_4 + \delta\, \mathrm{SYK}_2$$

$$\hat{H} = -\sum_{1\le a<b<c<d}^{2N} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d + \sum_{1\le j\le N}^{N} v_j\left(2\hat{n}_j - 1\right)$$

Each term of SYK_4 connects vertices with distance = 0, 2, 4.

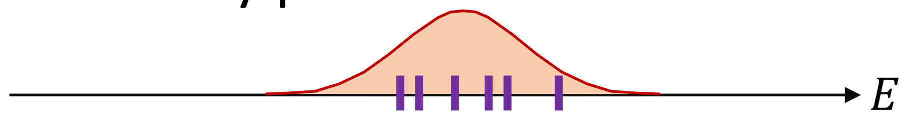
For $N = 17$, each vertex is directly connected with 1 (distance=0, itself) + 136 (distance=2) + 2380 (distance=4) vertices out of the possible $2^N = 131072$ (65536 per parity).

Four regimes of disorder strengths

arXiv:2005.12809

I

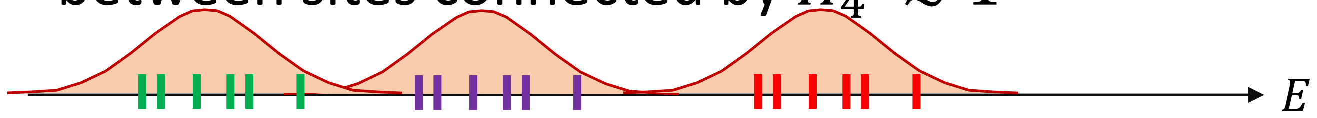
- Typical energy difference between arbitrary pair of sites $\lesssim 1$



$N^{-1/2}$

II

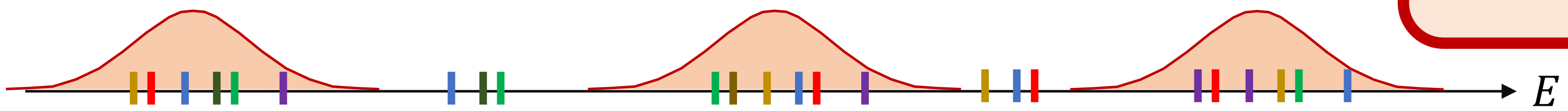
- Typical energy difference > 1, but difference between sites connected by $\widehat{H}_4 \ \lesssim 1$



1

III

- Difference between sites connected by $\widehat{H}_4 > 1$



δ_c

IV

- Fock space localization ($\delta_c \sim N^2 \log N$ for Bethe lattice)

[Altshuler, Gefen, Kamenev, and Levitov, PRL **78**, 2803 (1997)]

δ

$$\widehat{H} = \widehat{H}_4 + \widehat{H}_2 \qquad \widehat{H}_2 = \sum_{1\le j\le N}^{N} v_j(2\hat{n}_j - 1)$$

width of v_j dist. = δ

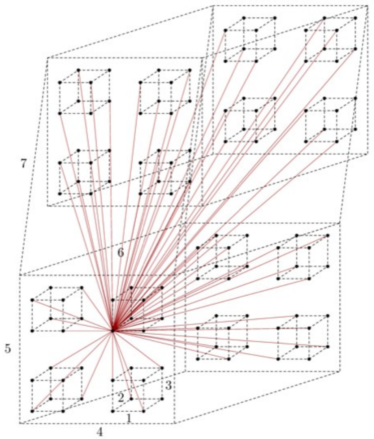
Site energy of site #m:

$$\epsilon_{(m=\sum_{1\le j\le N}^{N} 2^{j-1} n_j)} = \sum_{1\le j\le N}^{N} (-1)^{n_j - 1} v_j$$

Width of ϵ_m dist. $= \sqrt{N}\delta$

$$\widehat{H}_4 = -\sum_{1\le a<b<c<d}^{2N} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d$$

SYK_4 bandwidth = 1



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Diagnostic quantities: Moments of wave functions and spectral two-point correlation function

- Moments of eigenstate wave functions

$$I_q = \nu^{-1} \sum_{n,\psi} \left\langle |\langle \psi | n \rangle|^{2q} \delta(E_\psi) \right\rangle_J$$

with average density of states at band center

$$\nu = \nu(E \simeq 0), \nu(E) = \sum_{\psi} \left\langle \delta(E - E_\psi) \right\rangle_J$$

➔ Parametrizes localization, allows comparison with numerics

$I_2 = \nu^{-1} \sum_{n,\psi} \left\langle |\langle \psi | n \rangle|^4 \delta(E_\psi) \right\rangle_J$:

inverse participation ratio (IPR), $\frac{1}{D} \leq I_2 \leq 1$

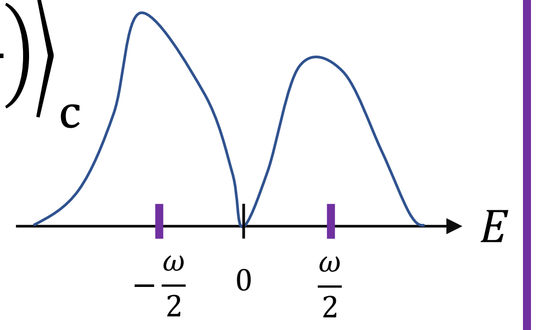
Equal weights

Single non-zero element

D: dimension of $\{|n\rangle\} = 2^{N-1}$

- Spectral two-point correlation function

$$K(\omega) = \nu^{-2} \left\langle \nu\left(\frac{\omega}{2}\right) \nu\left(-\frac{\omega}{2}\right) \right\rangle_c$$



c: connected part

$\langle AB \rangle_c = \langle AB \rangle_J - \langle A \rangle_J \langle B \rangle_J$

➔ Reflects level repulsion if the spectrum is random matrix-like

➔ We calculate these quantities for large N and compare against numerical results

Analytical results

Method: Exact matrix integral representation of I_q and $K(\omega)$;
mapping to a supersymmetric sigma model;
saddle point equations; effective medium approximation

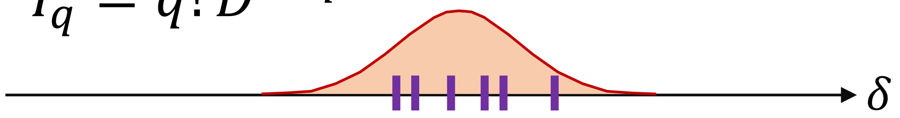
arXiv:2005.12809

$(c = O(1), D = 2^{N-1})$

I

- I: Average density of states (ADoS) at band center $\nu = cD$
- $I_q = q!\, D^{1-q}$

ergodic

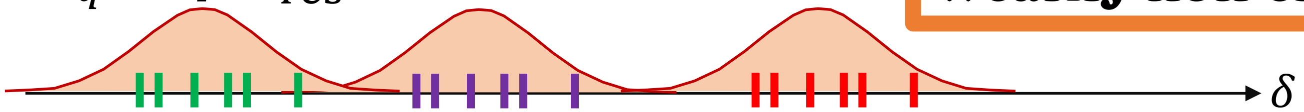


$N^{-1/2}$

II

- II: ADoS $\nu = \frac{cD}{\sqrt{N}\delta}$, spread of wave functions $D_{\text{res}} \simeq \frac{D}{\sqrt{N}\delta}$
- $I_q = q!\, D_{\text{res}}^{1-q}$

weakly non-ergodic

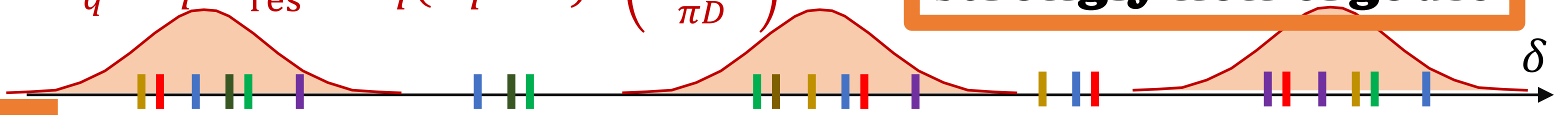


1

III

- III: ADoS $\nu = \frac{cD}{\sqrt{N}\delta}$, spread of wave functions $D_{\text{res}} \simeq \frac{D}{\sqrt{N}\delta^2}$
- $I_q = q!\, D_{\text{res}}^{1-q} = q(2q-3)!!\left(\frac{4\sqrt{N}\delta^2}{\pi D}\right)^{q-1}$

strongly non-ergodic



δ_{c}

IV

$$\delta_{\text{c}} = \frac{N^2}{4\sqrt{3}} \log N \text{ for large } N$$

- IV: All eigenstates localized to $\mathcal{O}(1)$ sites

δ

Eigenenergy spectral statistics (for odd N case for simplicity)

$$\widetilde{K}(s) = 1 - \frac{\sin^2 s}{s^2} + \delta\left(\frac{s}{\pi}\right),$$

$s = \pi\omega\nu$ in I, II, III :
agrees with Gaussian Unitary Ensemble (GUE)

IV: **Poisson statistics**

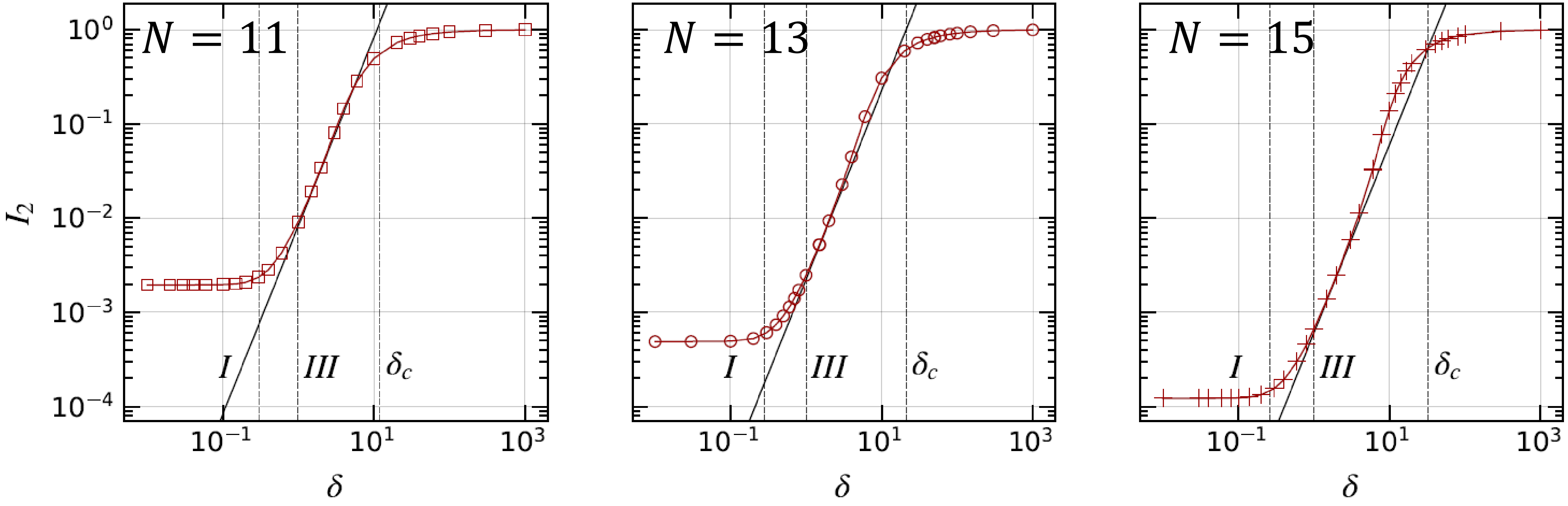
arXiv:2005.12809

Inverse participation ratio vs prediction for III

IPR I_2 = average of $\sum_n |\langle\psi|n\rangle|^4$ for normalized ψ, $\frac{1}{D} \leq I_2 \leq 1$

Equal weights

Single non-zero element



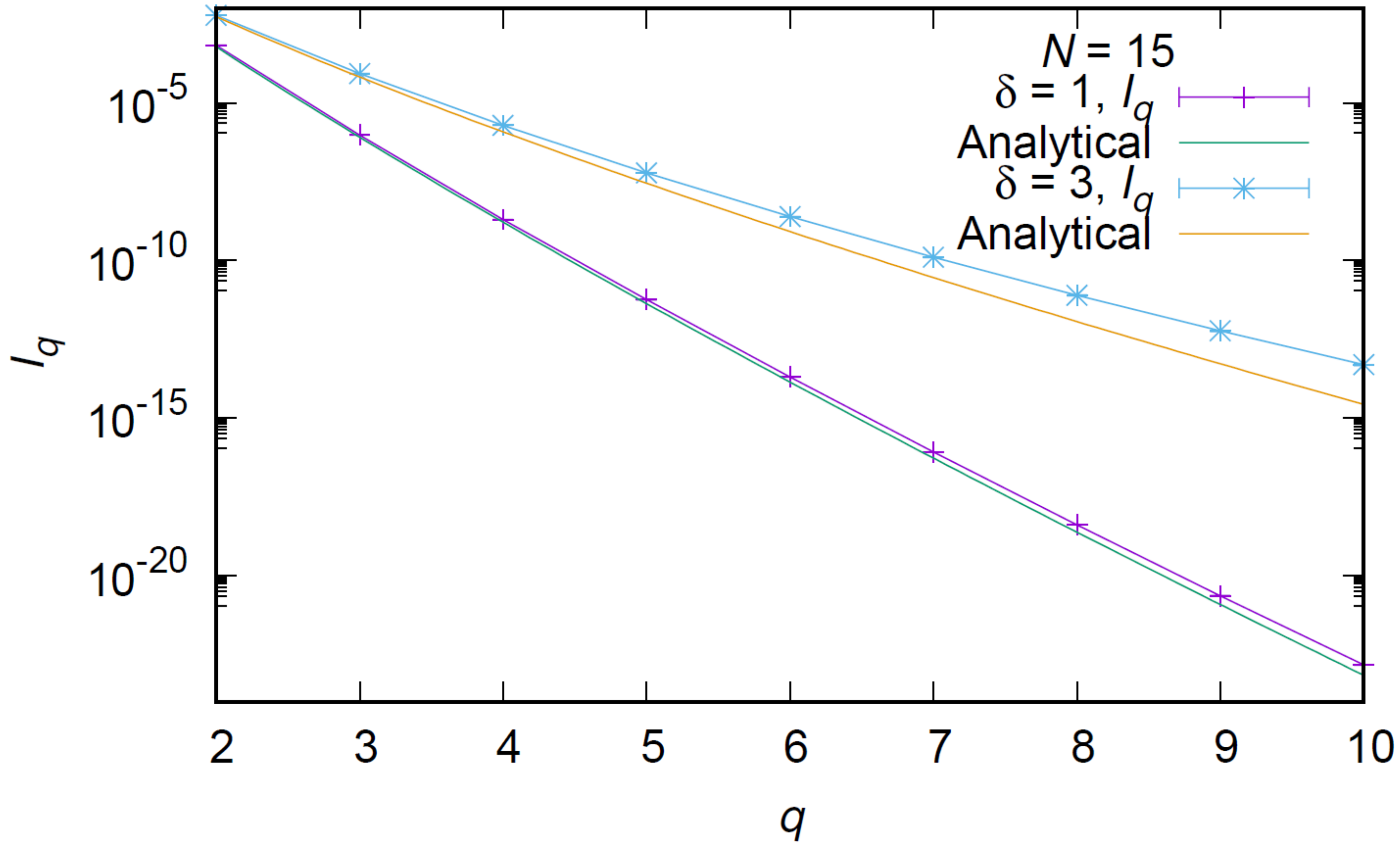
$$I_q = \frac{q(2q-3)!!}{\delta^{2(1-q)}}\left(\frac{\pi D}{4\sqrt{N}}\right)^{1-q} = q(2q-3)!!\left(\frac{4\sqrt{N}\delta^2}{2^{N-1}\pi}\right)^{q-1} \text{ in III}$$

Central 1/7 of the energy spectrum

Moments of eigenvectors

Analytical prediction:

$$I_q = \frac{q(2q-3)!!}{\delta^{2(1-q)}}\left(\frac{\pi D}{4\sqrt{N}}\right)^{1-q} = q(2q-3)!!\left(\frac{4\sqrt{N}\delta^2}{2^{N-1}\pi}\right)^{q-1} \text{ in III}$$



Good agreement up to large q for $\delta \sim 1$

Central 1/7 of the energy spectrum

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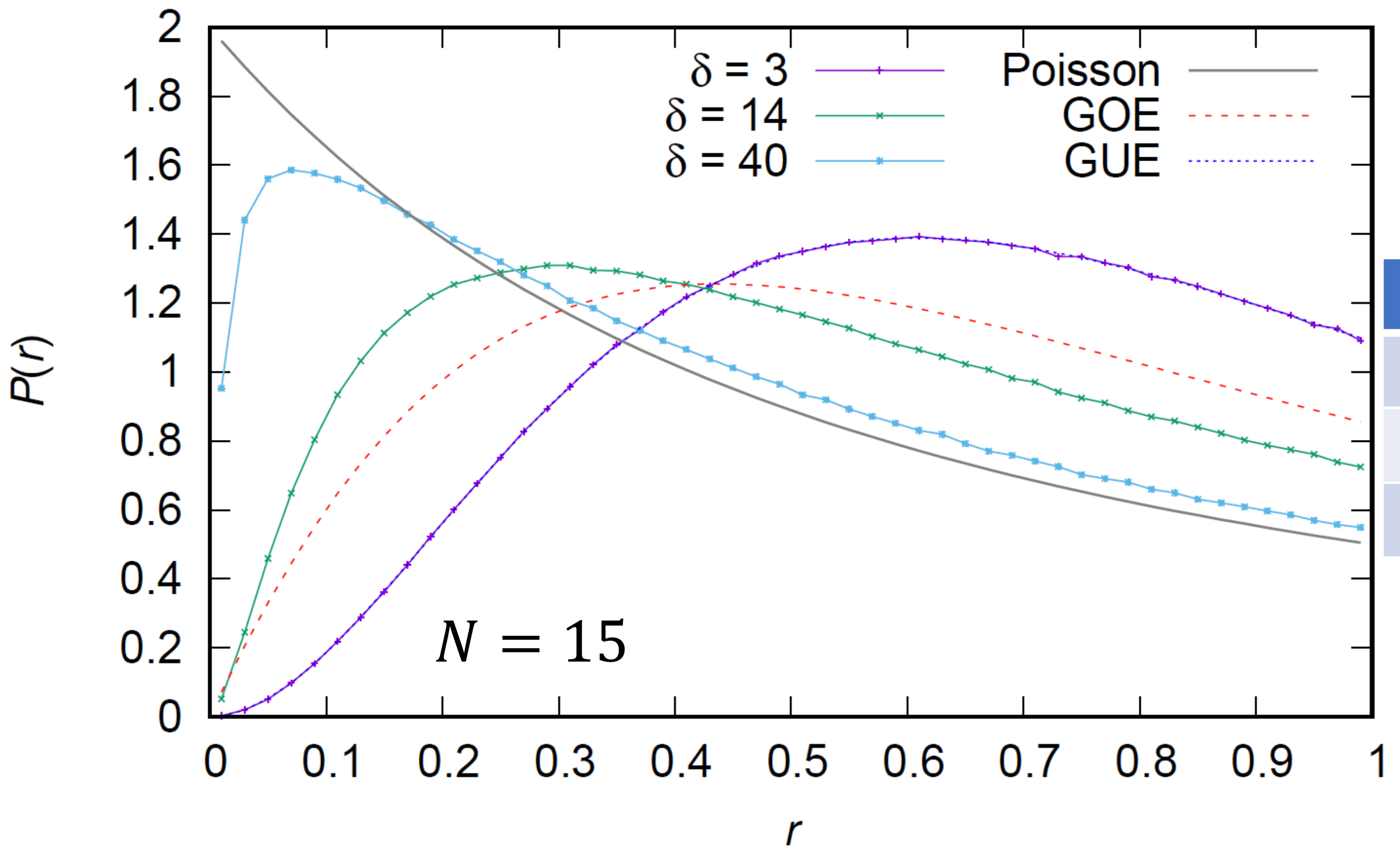
Spectral statistics: gap ratio distribution



$(\delta_c = \frac{Z}{\sqrt{2\rho}} W(2Z\sqrt{\pi}) = 38.47)$

Measure difference by Kullback-Leibler (KL) divergence: $D_{\mathrm{KL}}(P||Q) = \sum_x P(x) \log \frac{P(x)}{Q(x)}$.

$\boldsymbol{\delta}$	$D_{\mathrm{KL}}(P(\delta, r)\|\|P_{\mathrm{Poisson}}(r))$	$D_{\mathrm{KL}}(P(\delta, r)\|\|P_{\mathrm{GUE}}(r))$
3	0.3608	5×10^{-6}
14	0.1234	0.1463
40	0.0096	0.5705

$$r = \frac{\min(E_{i+1} - E_i\,,\ E_{i+2} - E_{i+1})}{\max(E_{i+1} - E_i\,,\ E_{i+2} - E_{i+1})}$$

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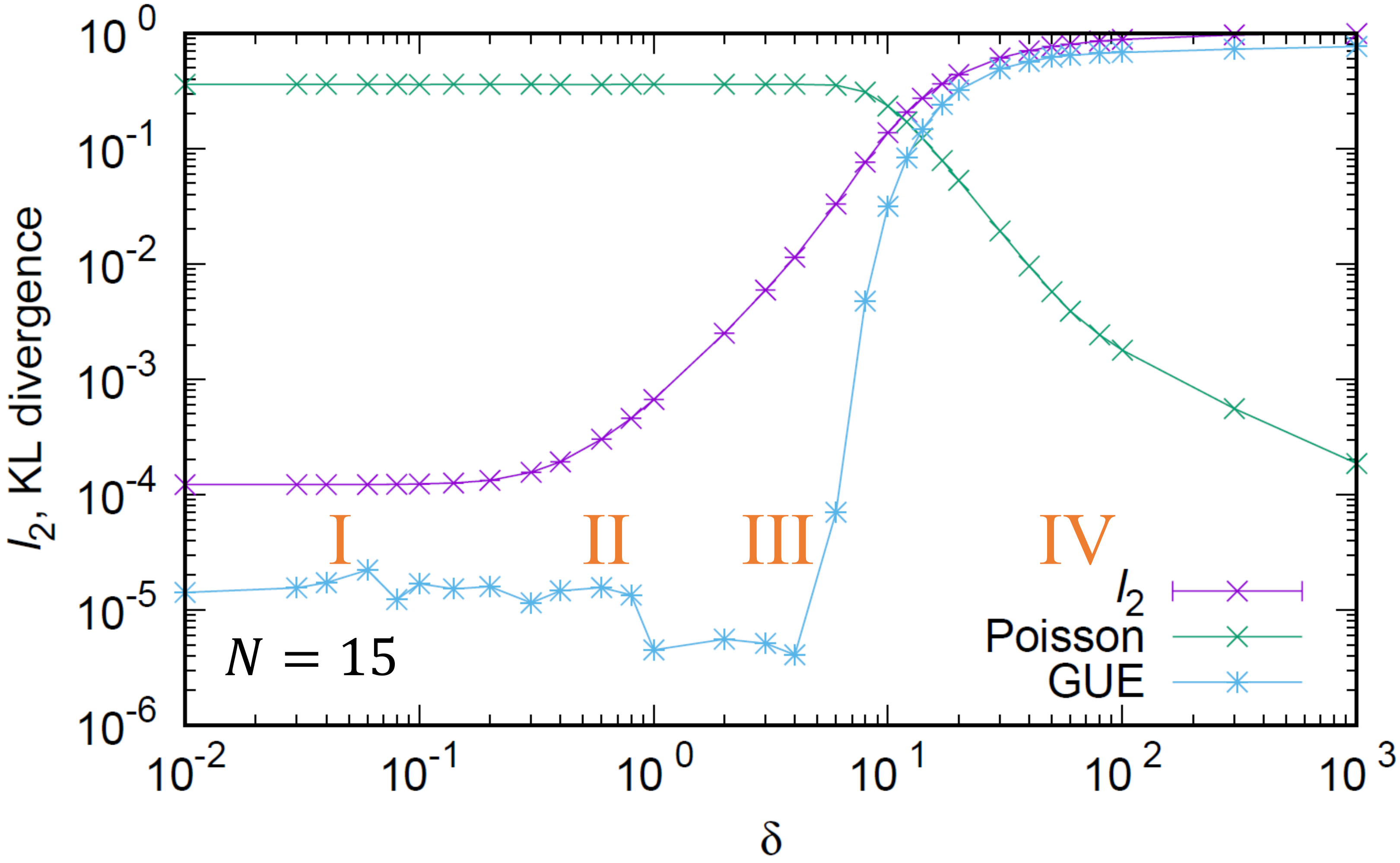
Departure from random matrix $P(r)$ occurs after I_2 has grown significantly



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Felipe Monteiro, Tobias Micklitz, Masaki Tezuka, and Alexander Altland,
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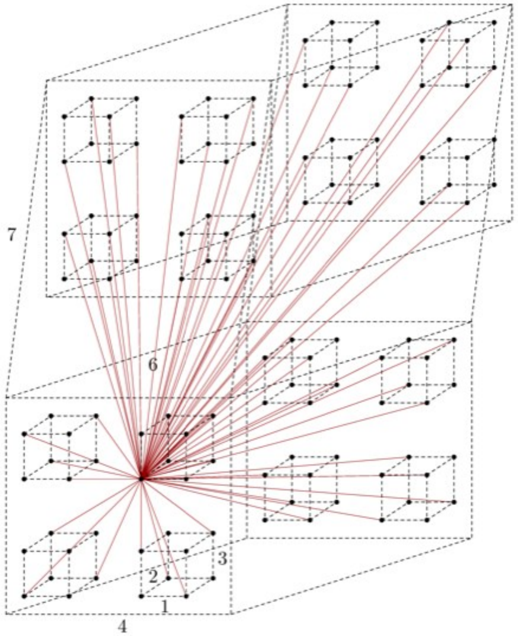
Summary

Fock space localization in many-body quantum systems

Sachdev-Ye-Kitaev model as tractable system



Analytical estimate of inverse participation ratio, spectral statistics

Numerical calculation of inverse participation ratio, energy spectrum correlation

Four regimes (I: ergodic, II: localization starts, III: localization rapidly progresses, IV: MBL) found in $SYK_4 + \delta\, SYK_2$ system (in SYK_2-diagonal basis);
I, II, III are chaotic while IV is not

Prediction for momenta of eigenstate wavefunctions I_q is verified by **parameter free comparison**, and **energy spectrum statistics** is consistent with GUE/Poisson transition well after entering regime III