Characterization of quantum many-body chaos with quantum Lyapunov exponents and by two-point correlations: application to a generalized Sachdev-Ye-Kitaev model

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Collaborators in this work

arXiv:1809.01671
arXiv:1902.11086

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Characterization of quantum many-body chaos

- Random-matrix like energy level correlation

- Exponential Lyapunov growth of out-of-time-order correlators (OTOC)

$$\langle \hat{W}^\dagger(t)\hat{V}^\dagger(0)\hat{W}(t)\hat{V}(0)\rangle \sim C + \#\, e^{2\lambda_\mathrm{L} t}$$

Example: the Sachdev-Ye-Kitaev model

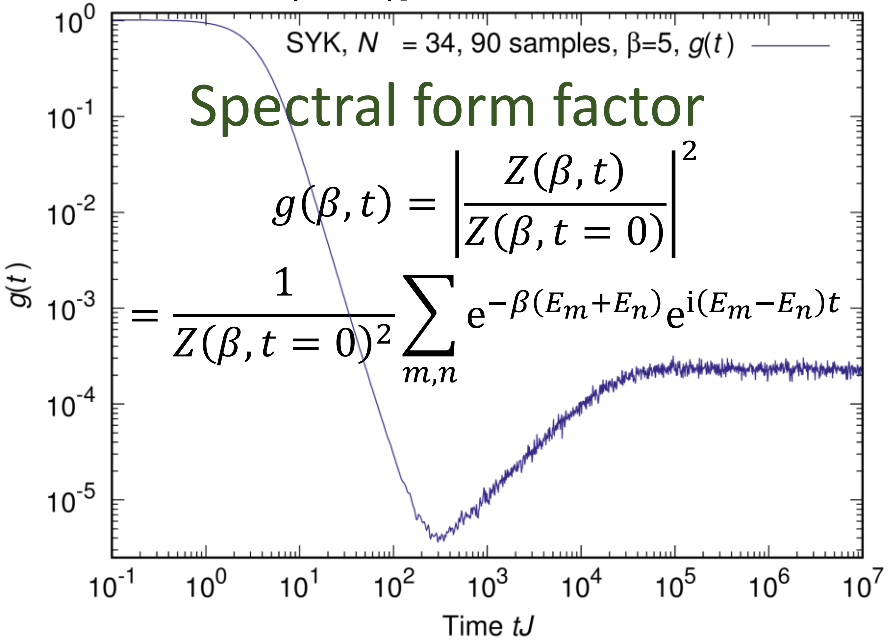
$$\hat{H} = \frac{\sqrt{3!}}{N^{3/2}} \sum_{1\le a<b<c<d\le N} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d$$

$\{\hat{\chi}_a, \hat{\chi}_b\} = \delta_{ab}$

J_{abcd} : Gaussian random ($\langle {J_{abcd}}^2\rangle = J^2 = 1$)

[Kitaev 2015]

[Cotler, MT et al., JHEP **1705**, 118 (2017)]



Spectral form factor

$$g(\beta,t) = \left|\frac{Z(\beta,t)}{Z(\beta,t=0)}\right|^2 = \frac{1}{Z(\beta,t=0)^2}\sum_{m,n} \mathrm{e}^{-\beta(E_m+E_n)}\mathrm{e}^{\mathrm{i}(E_m-E_n)t}$$

N mod 8	RMT
0	GOE
2	GUE
4	GSE
6	GUE

Lyapunov exponent

$$\lambda_\mathrm{L} = \frac{2\pi k_\mathrm{B} T}{\hbar} \text{ in low } T \text{ limit}$$

(Maldacena-Shenker-Stanford chaos bound)

We propose two new characterizations

- Quantum Lyapunov spectrum: Quantum version of finite-time Lyapunov spectrum

$\hat{M}_{ab}(t)$: (anti)commutator of $\hat{O}_a(t)$ and $\hat{O}_b(0)$

$$\hat{L}_{ab}(t) = \sum_{j=1}^{N} \hat{M}_{ja}(t)^\dagger \hat{M}_{jb}(t)$$

$\left\{\lambda_k(t) = \frac{\log s_k(t)}{2t}\right\}$ for singular values $\{s_k(t)\}_{k=1}^{N}$ of $N \times N$ matrix $\langle\phi|\hat{L}_{ab}(t)|\phi\rangle$.

arXiv:1809.01671

- Two-point correlations:

$G_{ab}^{(\phi)} = \langle\phi|\hat{O}_a(t)\hat{O}_b(0)|\phi\rangle$ as matrix, log (singular values)

arXiv:1902.11086

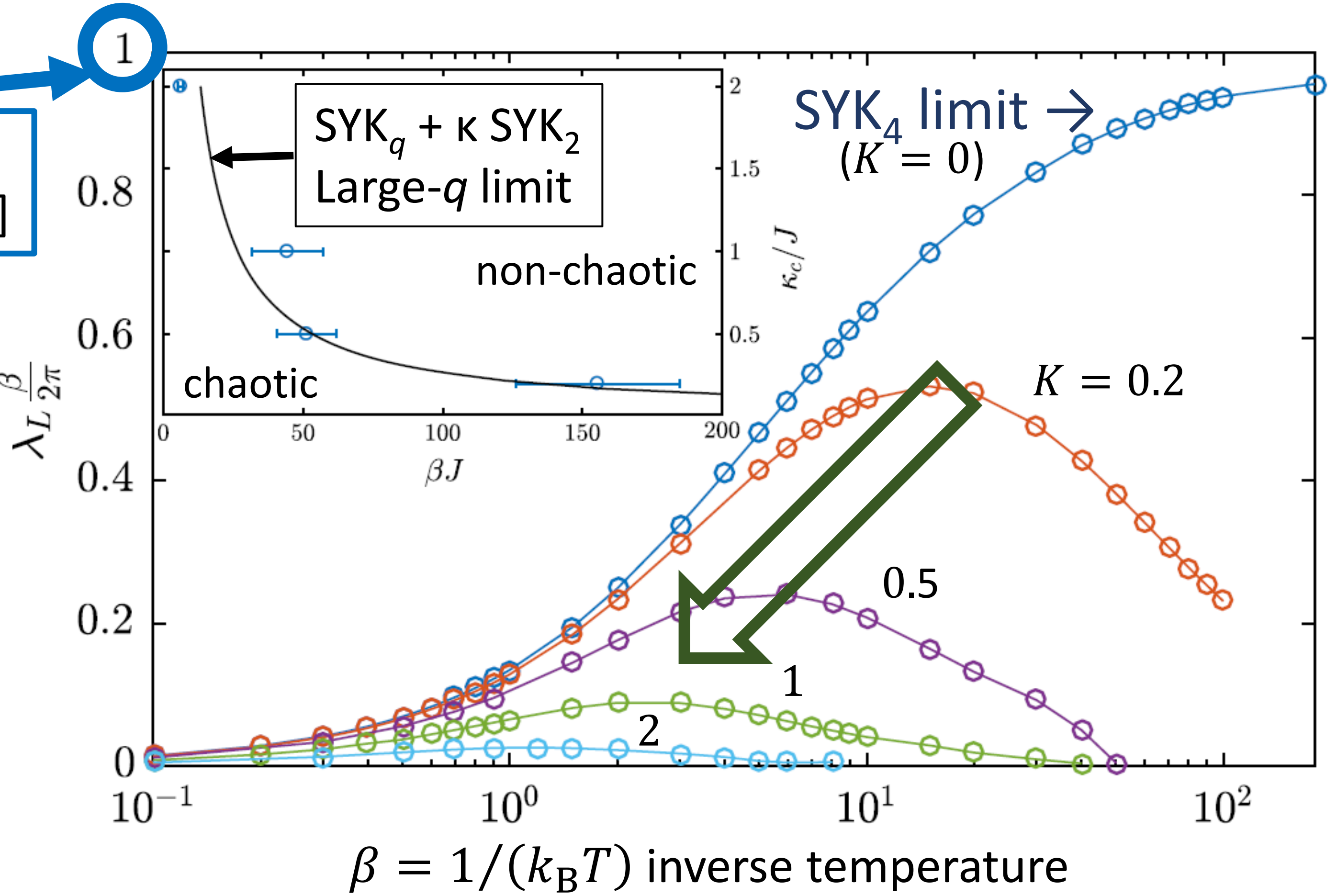
Modified SYK model: Large-N calculation for OTOC

$$\hat{H} = \overbrace{\sum_{1\le a<b<c<d}^{N} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d}^{\text{SYK}_4} + i\overbrace{\sum_{1\le a<b}^{N} K_{ab}\hat{\chi}_a\hat{\chi}_b}^{\text{SYK}_2}$$

K_{ab}: standard deviation $\frac{K}{\sqrt{N}}$

Chaos bound [Maldacena, Shenker, and Stanford 2016]

normalized Lyapunov exponent



A. M. Garcia-Garcia, B. Loureiro, A. Romero-Bermudez, and MT, PRL **120**, 241603 (2018)

Deviation from the chaos bound as SYK$_2$ component is introduced

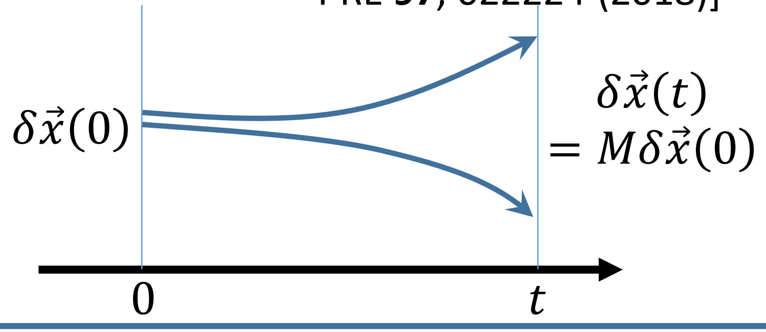
1. Quantum Lyapunov spectrum

Gharibyan, Hanada, Swingle, and MT, submitted (arXiv:**1809.01671**)

Finite-time **classical Lyapunov spectrum**: obeys RMT statistics for chaos

[Hanada, Shimada, and MT: PRE **97**, 022224 (2018)]

Singular values of $M_{ij} = \left(\frac{\partial x_i(t)}{\partial x_j(0)}\right)$ **at finite *t***: $\{s_k(t)\} = \{e^{\lambda_k t}\}$



$L = \{x_i(t), p_j(0)\}_{\text{PB}}{}^2 = \left(\frac{\partial x_i(t)}{\partial x_j(0)}\right)^2 \to e^{2\lambda_{\text{L}} t}$ for large t

OTOC: $C_T(t) = \left\langle \left|[\hat{W}(t), \hat{V}(t=0)]\right|^2 \right\rangle = \langle \hat{W}^\dagger(t)\hat{V}^\dagger(0)\hat{W}(t)\hat{V}(0)\rangle + \cdots$

Quantum Lyapunov spectrum: Define $\hat{M}_{ab}(t)$ as (anti)commutator of $\hat{O}_a(t)$ and $\hat{O}_b(0)$

$$\hat{L}_{ab}(t) = \sum_{j=1}^{N} \hat{M}_{ja}(t)^\dagger \hat{M}_{jb}(t)$$

For $N \times N$ matrix $\langle \phi | \hat{L}_{ab}(t) | \phi \rangle$, obtain singular values $\{s_k(t)\}_{k=1}^{N}$.

The Lyapunov spectrum is defined as $\left\{\lambda_k(t) = \frac{\log s_k(t)}{2t}\right\}$.

Quantum Lyapunov spectrum for SYK model + modification

$$\hat{H} = \sum_{1\le a<b<c<d}^{N} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d + i\sum_{1\le a<b}^{N} K_{ab}\hat{\chi}_a\hat{\chi}_b$$

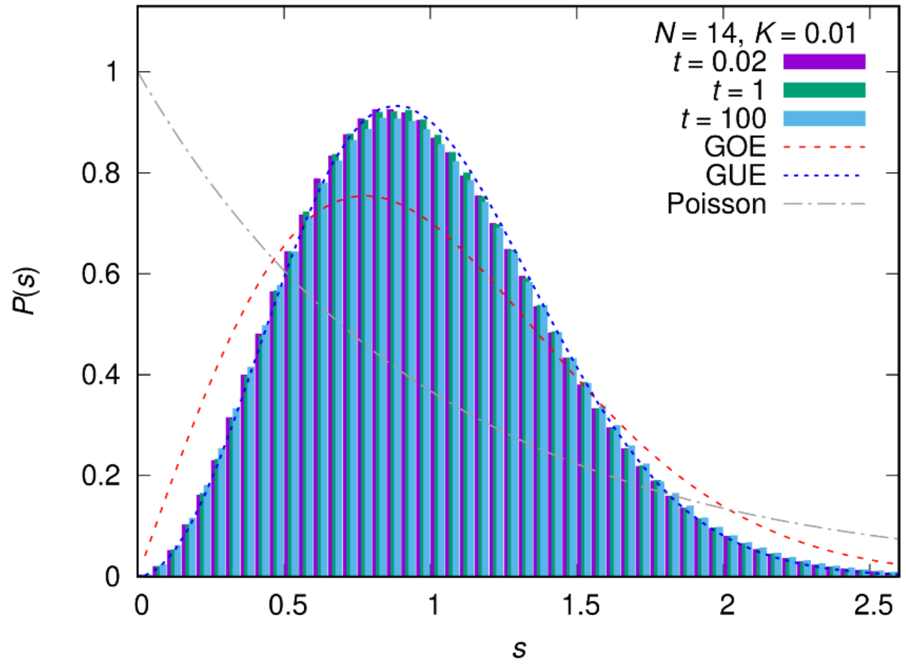
J_{abcd}: s. d. $= \frac{\sqrt{6}}{N^{3/2}}$

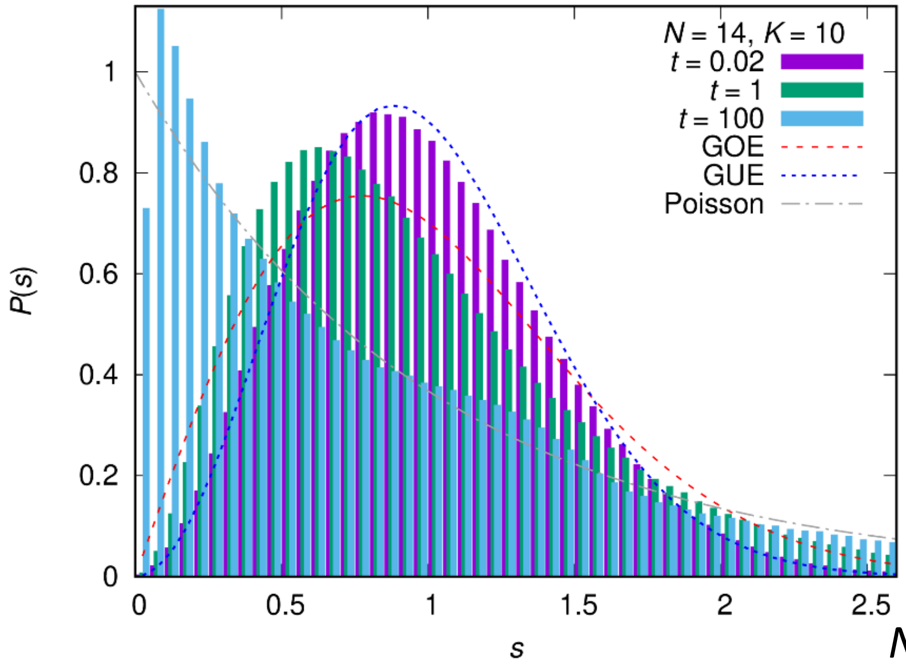
K_{ab}: s. d. $= \frac{K}{\sqrt{N}}$

- Define $\hat{L}_{ab}(t) = \sum_{j=1}^{N} \hat{M}_{ja}(t)\hat{M}_{jb}(t)$ for time-dependent anticommutator $\hat{M}_{ab}(t) = \{\hat{\chi}_a(t), \hat{\chi}_b(0)\}$.
- Obtain the singular values $\{a_k(t)\}_{k=1}^{K}$ of $\langle\phi|\hat{L}_{ab}(t)|\phi\rangle$
- Quantum Lyapunov spectrum: $\left\{\lambda_k(t) = \frac{\log a_k(t)}{2t}\right\}_{k=1,2,\ldots,K}$ (also dependent on state ϕ)

Other possibilities: see Rozenbaum-Ganeshan-Galitski, 1801.10591; Hallam-Morley-Green: 1806.05204

Spectral statistics of quantum Lyapunov spectrum: SYK





arXiv:**1809.01671**

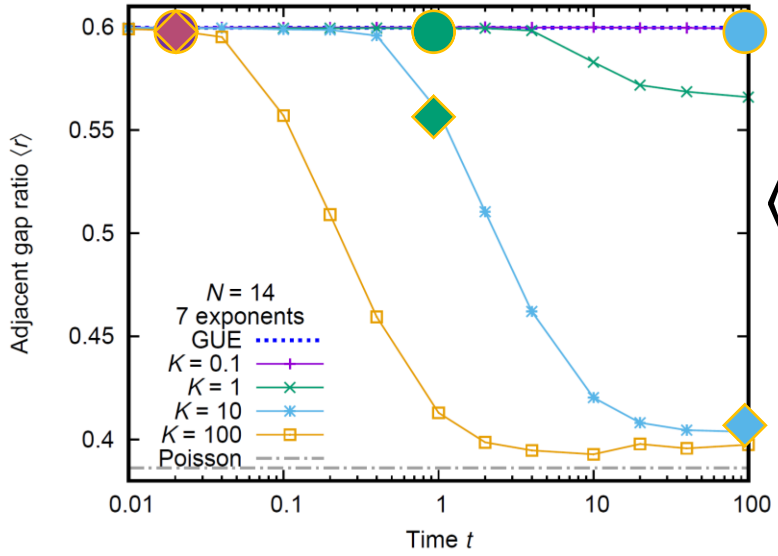
Energy eigenstates

$N/2$ larger exponents

$K = 0.01$ (●):

Remains GUE for long time

Exponents are nearly constant until the singular values of $\langle\phi|\hat{L}_{ab}(t)|\phi\rangle$ saturate: Lyapunov growth



$K = 10$ (◆):

Approaches Poisson

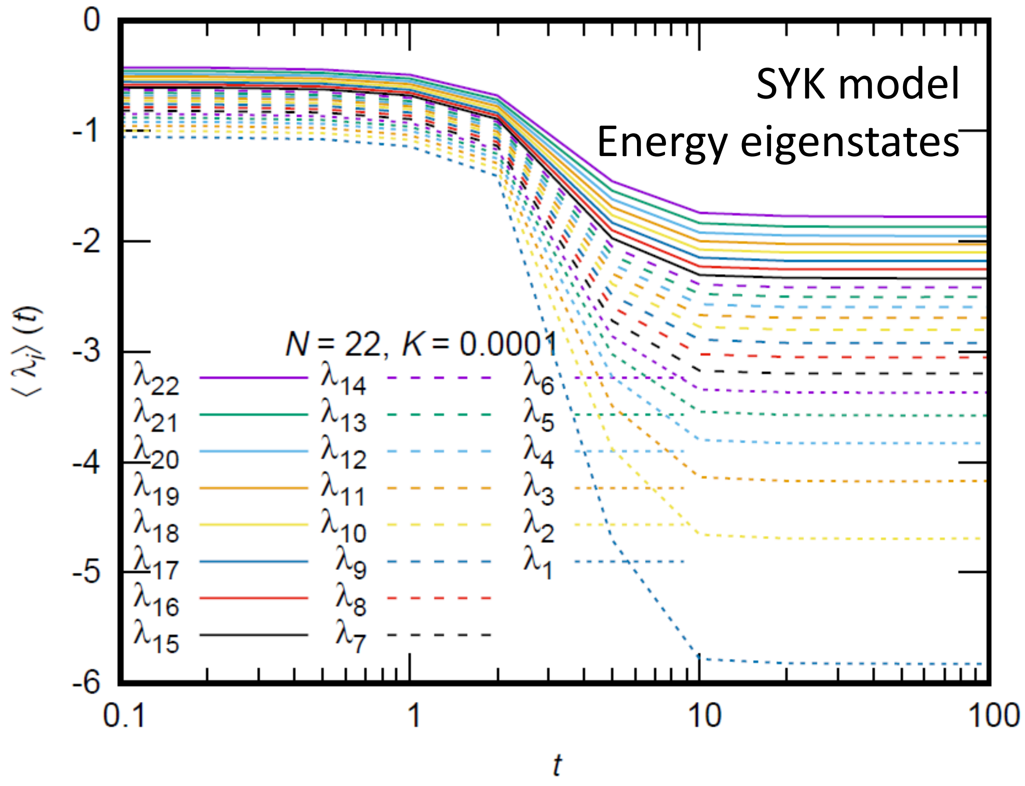
$\langle r \rangle$: average of $\frac{\min(\epsilon_{i+1}-\epsilon_i\,,\epsilon_{i+2}-\epsilon_{i+1})}{\max(\epsilon_{i+1}-\epsilon_i\,,\epsilon_{i+2}-\epsilon_{i+1})}$

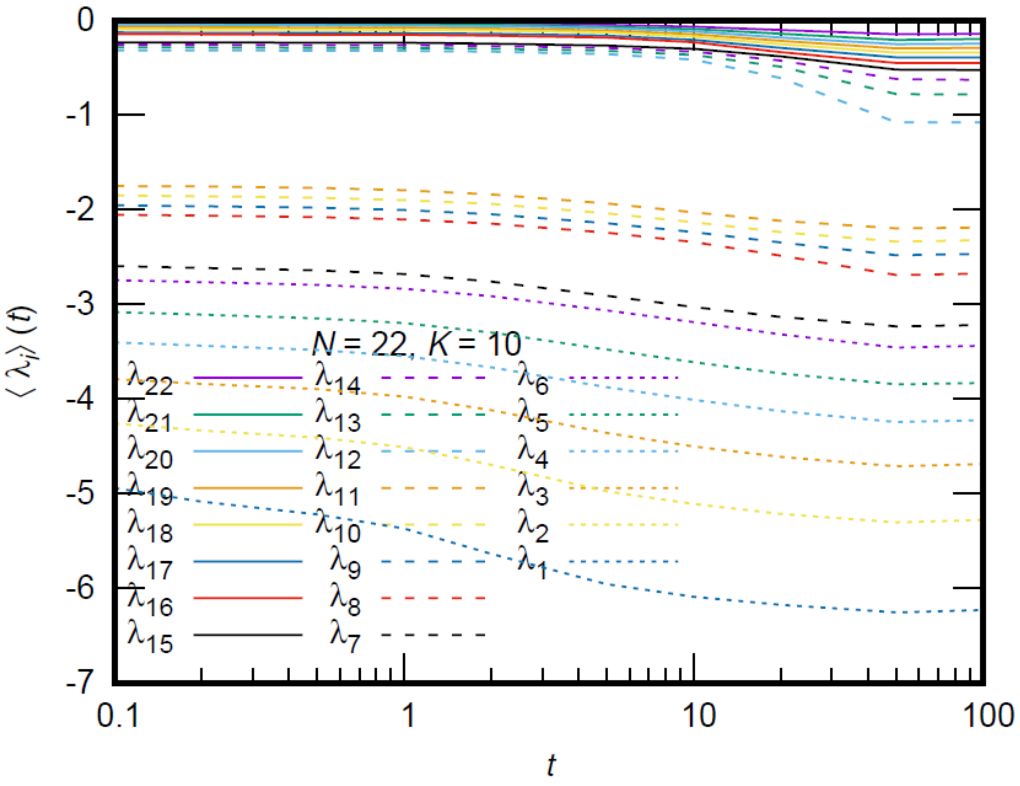
(fixed-i unfolding: unfold each gap $\lambda_{i+1} - \lambda_i$ using its average)

2. Singular value statistics of two-point functions

$$G_{ab}^{(\phi)} = \langle \phi | \hat{\chi}_a(t) \hat{\chi}_b(0) | \phi \rangle$$

$$\lambda_j = \log\left[\text{singular values of}\left(G_{ab}^{(\phi)}\right)\right]$$





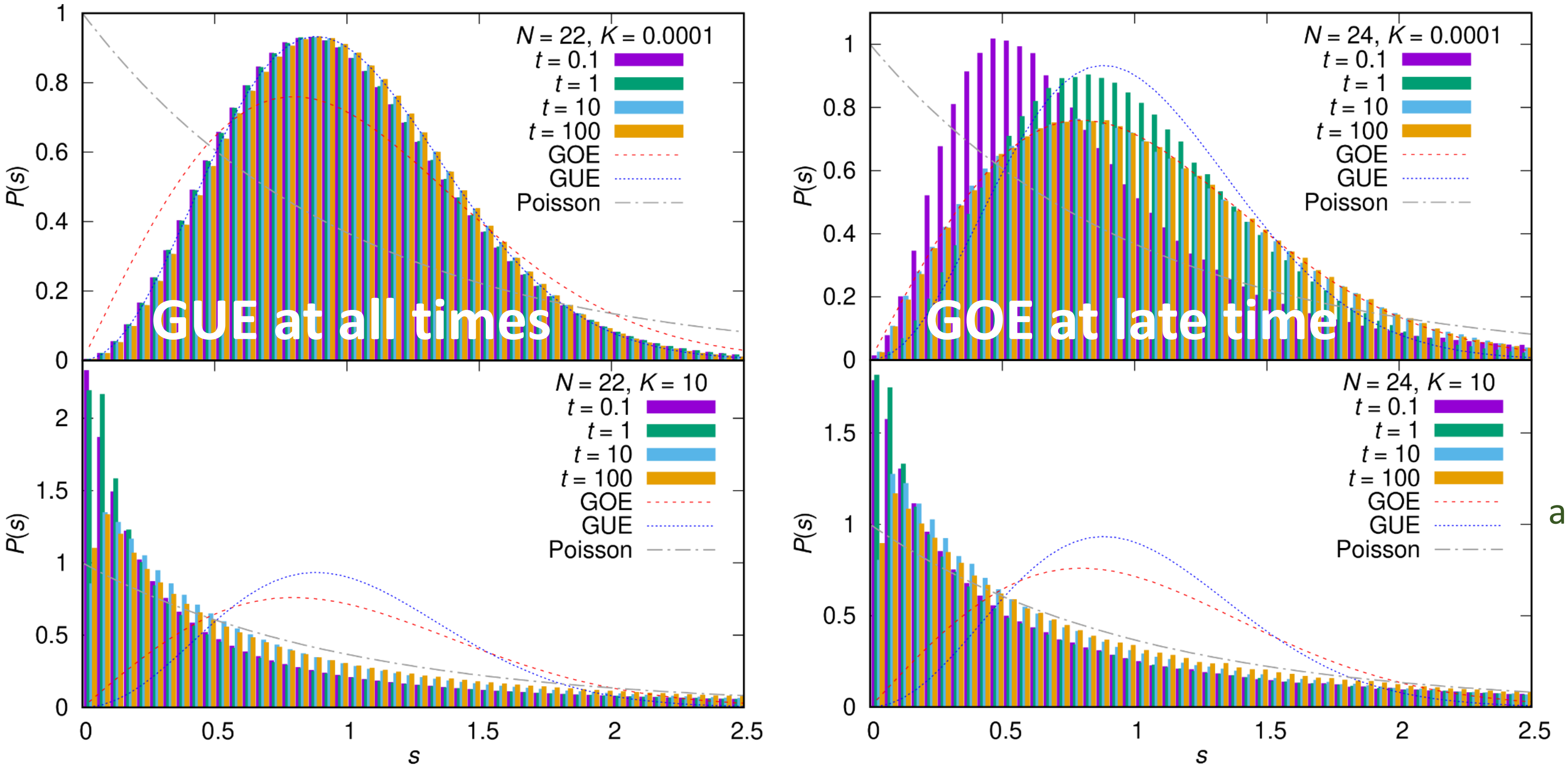
H. Gharibyan, M. Hanada, B. Swingle, and MT, arXiv:1902.11086

2. Singular value statistics of two-point functions

$$G_{ab}^{(\phi)} = \langle \phi | \hat{\chi}_a(t) \hat{\chi}_b(0) | \phi \rangle$$

SYK, larger $N/2$ exponents

$$\lambda_j = \log \left[\text{singular values of} \left(G_{ab}^{(\phi)} \right) \right]$$



arXiv:**1902.11086**

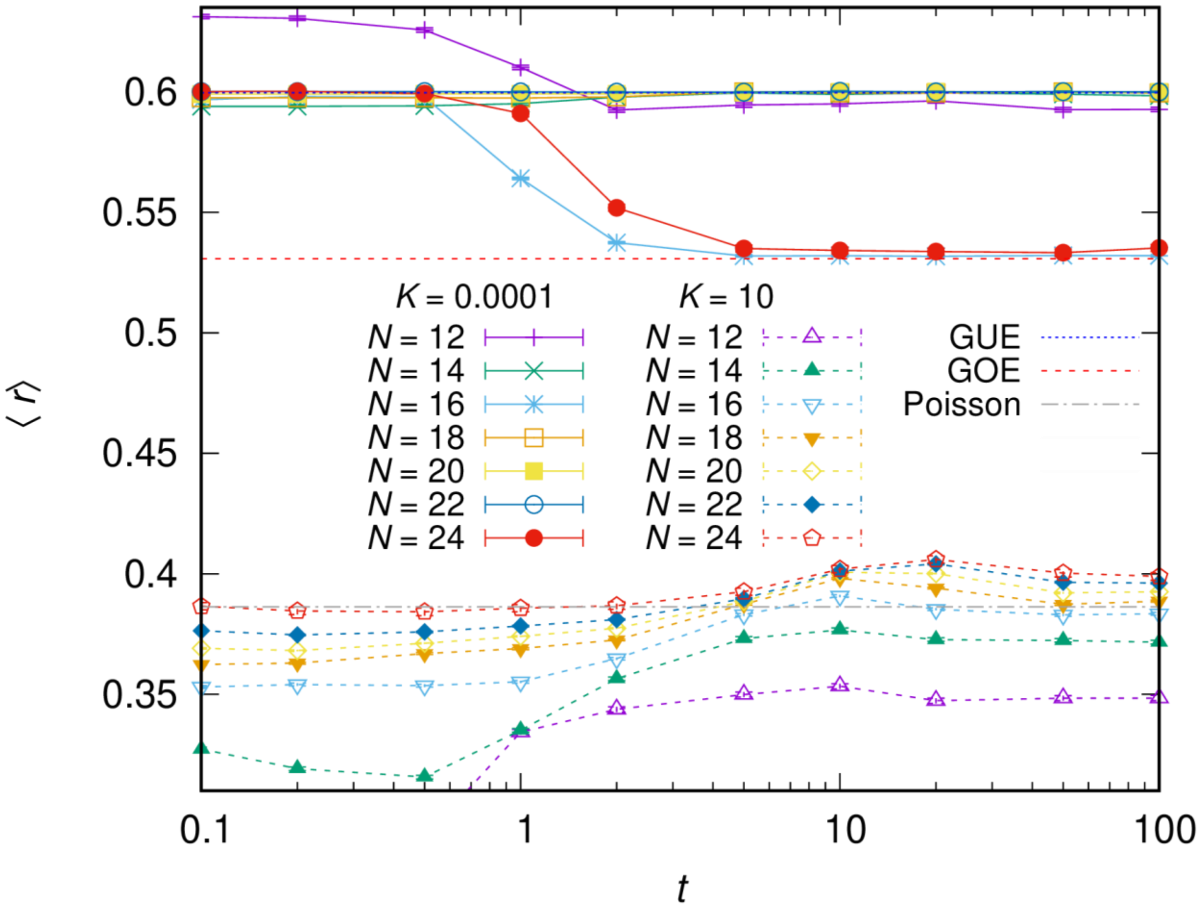
$\langle r \rangle$: average of the adjacent gap ratio $\frac{\min(\lambda_{i+1}-\lambda_i\,,\lambda_{i+2}-\lambda_{i+1})}{\max(\lambda_{i+1}-\lambda_i\,,\lambda_{i+2}-\lambda_{i+1})}$

Uncorrelated (Poisson): $2\log 2 - 1 \approx 0.386$

Correlated: larger (GOE: 0.5307, GUE: 0.5996 etc.) [Atas *et al.*, PRL 2013]

SYK, larger $N/2$ exponents

ϕ: energy eigenstates



N mod 8 = 2, 4, 6: GUE

N mod 8 = 0: GOE

(the matrix is symmetric)

$$G_{ab}^{(\phi)} = \langle \phi | \hat{\chi}_a(t) \hat{\chi}_b(0) | \phi \rangle$$

$$\lambda_j = \log\left[\text{singular values of} \left(G_{ab}^{(\phi)}\right)\right]$$

fixed-i unfolded

arXiv:**1902.11086**

At late time,

Random matrix behavior ⇔ chaotic (also for XXZ model + random field)

Summary

- Quantum Lyapunov spectrum defined from local operators: characterizes quantum chaos [1809.01671]
  - Lyapunov growth
  - Fastest entropy production in the SYK model?
  - Random matrix behavior in chaotic systems

$\hat{L}_{ab}(t) = \sum_{j=1}^{N} \hat{M}_{ja}(t)\hat{M}_{jb}(t)$ for
$\hat{M}_{ab}(t) = \{\hat{\chi}_a(t), \hat{\chi}_b(0)\}$
QLS: log(singular values of $\langle\phi|\hat{L}_{ab}(t)|\phi\rangle$)/(2t)

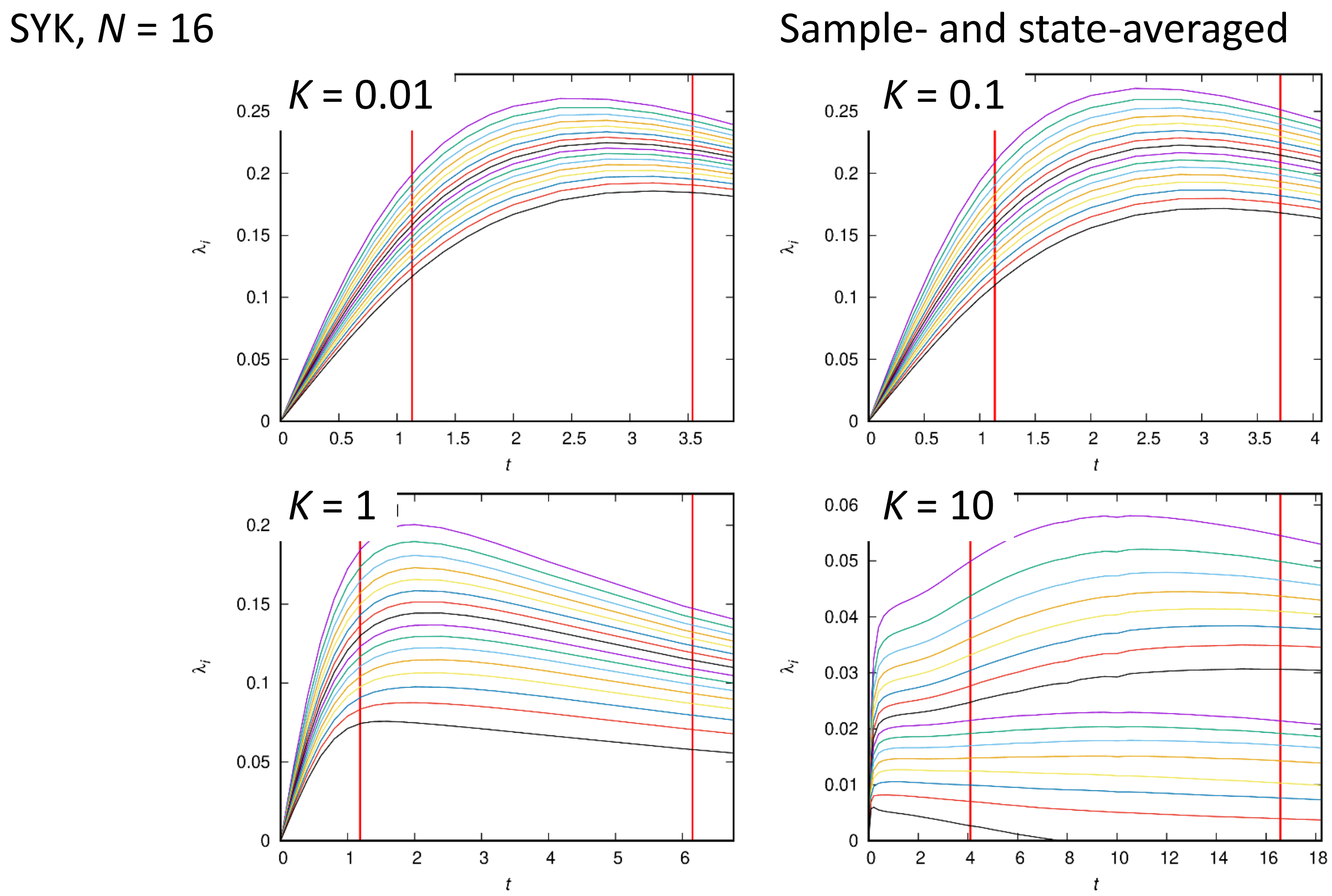
- Two-point correlation function: singular values exhibit random matrix behavior in chaotic cases [1902.11086]
  - Experiments should be possible with phase-sensitive measurements

$G_{ab}^{(\phi)} = \langle\phi|\hat{\chi}_a(t)\hat{\chi}_b(0)|\phi\rangle$

- Both characterizations of chaos demonstrated for the SYK model
  - Also for XXZ spin chain + random field (see our papers)

Gharibyan, Hanada, Swingle, and MT, submitted (arXiv:**1809.01671**)

Full Lyapunov spectrum



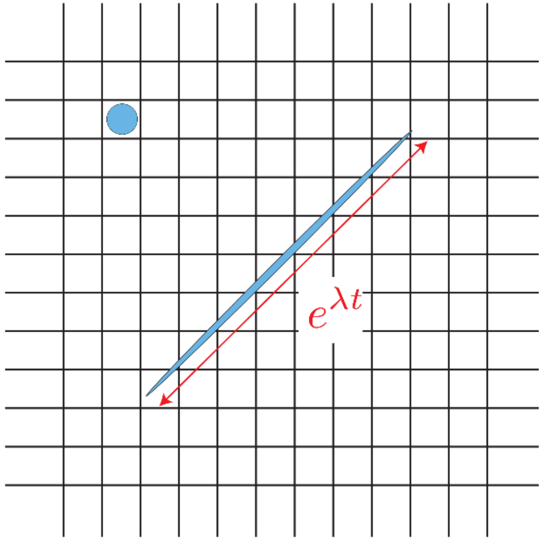
$\lambda_i(t)$: Close to constant between red lines

(20 % and 80 % of the saturated value of $\lambda_N t$)

arXiv:**1809.01671**

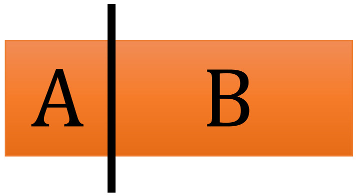
Kolmogorov-Sinai entropy vs entanglement entropy production

Coarse-grained entropy
= log(# of cells covering the region)
~ (sum of positive λ) t



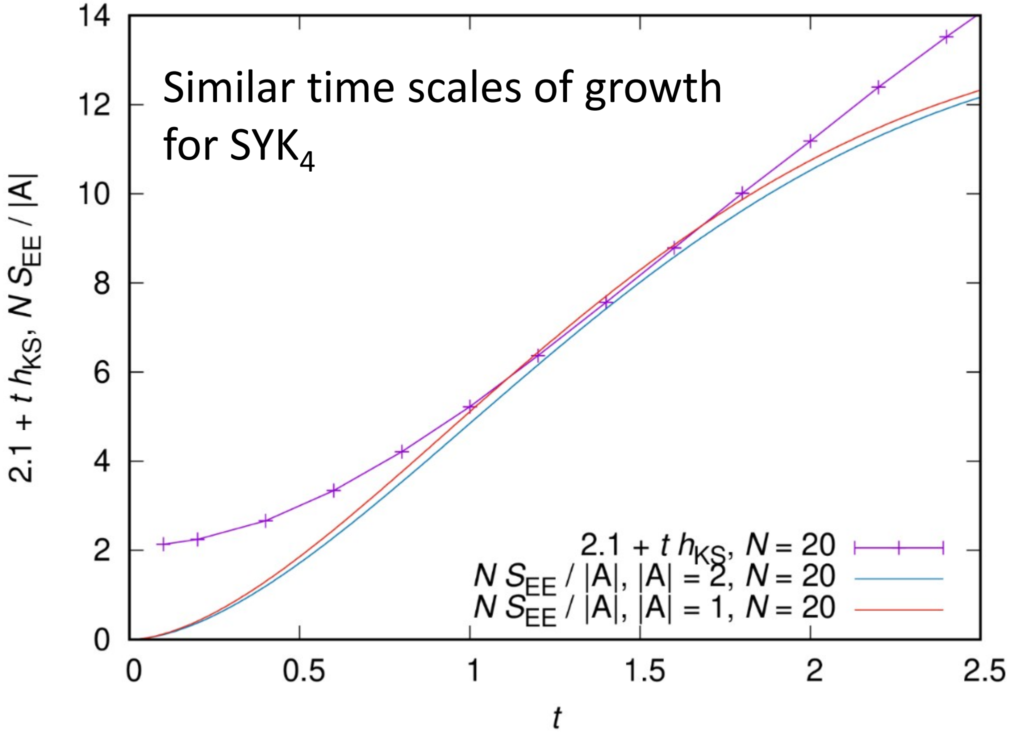
Kolmogorov-Sinai entropy h_{KS}
= (sum of positive λ)
= entropy production rate

Time evolution by $\widehat{H}_{\mathrm{SYK4}}$ from an initial state with $S_{\mathrm{EE}} = 0$:
$|\psi(t=0)\rangle = |\underbrace{000\ldots000}_{N/2}\rangle$ in the complex fermion basis $\hat{c}_j = \frac{(\chi_{2j-1} + i\chi_{2j})}{\sqrt{2}}$



$$\rho_{\mathrm{A}}(t) = \mathrm{Tr}_{\mathrm{B}}\, \rho(t),\ \rho(t) = |\psi(t)\rangle\langle\psi(t)|$$

$$S_{\mathrm{EE}}(t) = -\mathrm{Tr}\, \rho_{\mathrm{A}}(t) \log\big(\rho_{\mathrm{A}}(t)\big)$$



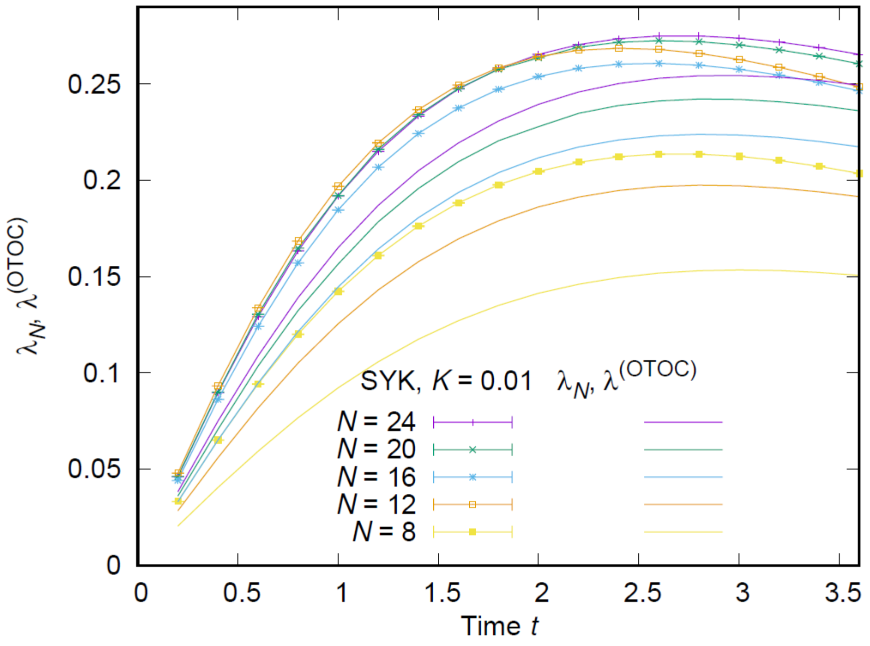
arXiv:**1809.01671**

Fastest entropy production?

SYK$_4$ limit

- λ_N and $\lambda_{\text{OTOC}} = \frac{1}{2t}\log\left(\frac{1}{N}\sum_{i=1}^{N} e^{2\lambda_i t}\right)$ approach each other; difference decreases as $1/N$
- Same for λ_N and λ_1:

  all exponent → single peak
- All saturate the MSS bound at strong coupling (low *T*) limit
- Growth rate of entanglement entropy $\sim h_{\text{KS}} =$ sum of positive (all) λ_i



➔ [conjecture] SYK model (and black holes): not only the fastest scramblers, but also fastest entropy generators