Chaotic-integrable transition and many-body localization in generalized SYK models

17:10 – 17:40, 27 Sep 2018

Anderson Localization and Interactions, MPI-PKS, Dresden

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Contents and collaborators

- Chaotic-integrable transition in a generalized SYK model

With Antonio M. García-García, Bruno Loureiro, and Aurelio Romero-Bermúdez

PRL **120**, 241603 (2018) (arXiv:1707.02197)

- Many-body localization in a short-range SYK model

With Antonio M. García-García

arXiv:1801.03204

- Quantum Lyapunov spectrum in SYK and XXZ models

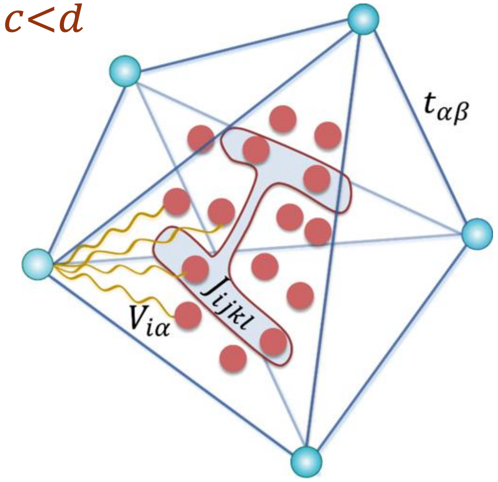
With Hrant Gharibyan, Masanori Hanada, and Brian Swingle

arXiv:1809.01671

Part 1: Stability of chaos in the SYK model

Antonio M. García-García, Bruno Loureiro, Aurelio Romero-Bermúdez, and Masaki Tezuka, Phys. Rev. Lett. **120**, 241603 (2018) (arXiv:1707.02197)

$$\hat{H}_{\mathrm{SYK}(q=4)} = \sum_{1\leq a<b<c<d}^{N} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d$$



[S. Banerjee and E. Altman 2017]

Various modifications of SYK have been studied

e.g.

$$Q = i \sum_{1\leqslant i<j<k\leqslant N} C_{ijk}\psi^i\psi^j\psi^k$$

- Supersymmetric SYK [Fu, Gaiotto, Maldacena, and Sachdev 2016]
- Non-random couplings [Witten, 1610.09758]
- Higher-dimensional generalizations [Gu, Qi, and Stanford 2017]
  [Davison, Fu, Georges, Gu, Jensen, and Sachdev 2017]

Addition of new Fermi species can induce a transition to a Fermi liquid [Banerjee and Altman PRB **95**, 134302 (2017)] or MBL transition [S.-K. Jian and H. Yao PRL **119**, 206602 (2017)], additional interaction can induce a metal-insulator transition [C.-M. Jian, Bi, and Xu PRB **96**, 115122 (2017)], …

Phys. Rev. Lett. **120**, 241603 (2018) (arXiv:1707.02197)

Our motivation and model

Q.: Minimum requirements for chaotic behavior? (→ gravity interpretation?)
Here we study a simple model with analytical + numerical methods

$$\hat{H} = \overbrace{\sum_{1\le a<b<c<d}^{N} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d}^{\text{SYK}_4} + i \overbrace{\sum_{1\le a<b}^{N} K_{ab}\hat{\chi}_a\hat{\chi}_b}^{\text{SYK}_2}$$

Gaussian random couplings
J_{abcd}: average 0, standard deviation $\frac{\sqrt{6}J}{N^{3/2}}$
K_{ab}: average 0, standard deviation $\frac{K}{\sqrt{N}}$

$J = 1$: unit of energy

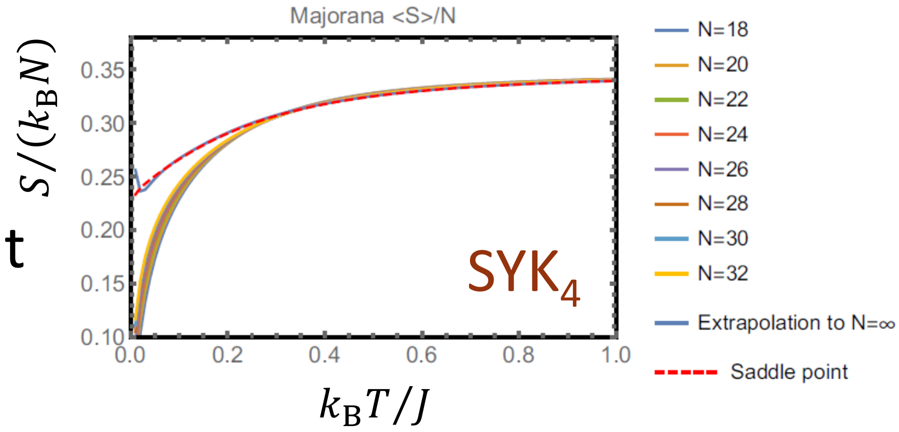
SYK$_4$ as unperturbed Hamiltonian,
K controls the strength of SYK$_2$ (one-body random term, solvable)

In this work:
$N \equiv 2 \pmod 4$
cf. Nosaka, Rosa, Yoon
Lunkin, Tikhonov, Feigel'man
Yu-Xiang, Ye, Liu

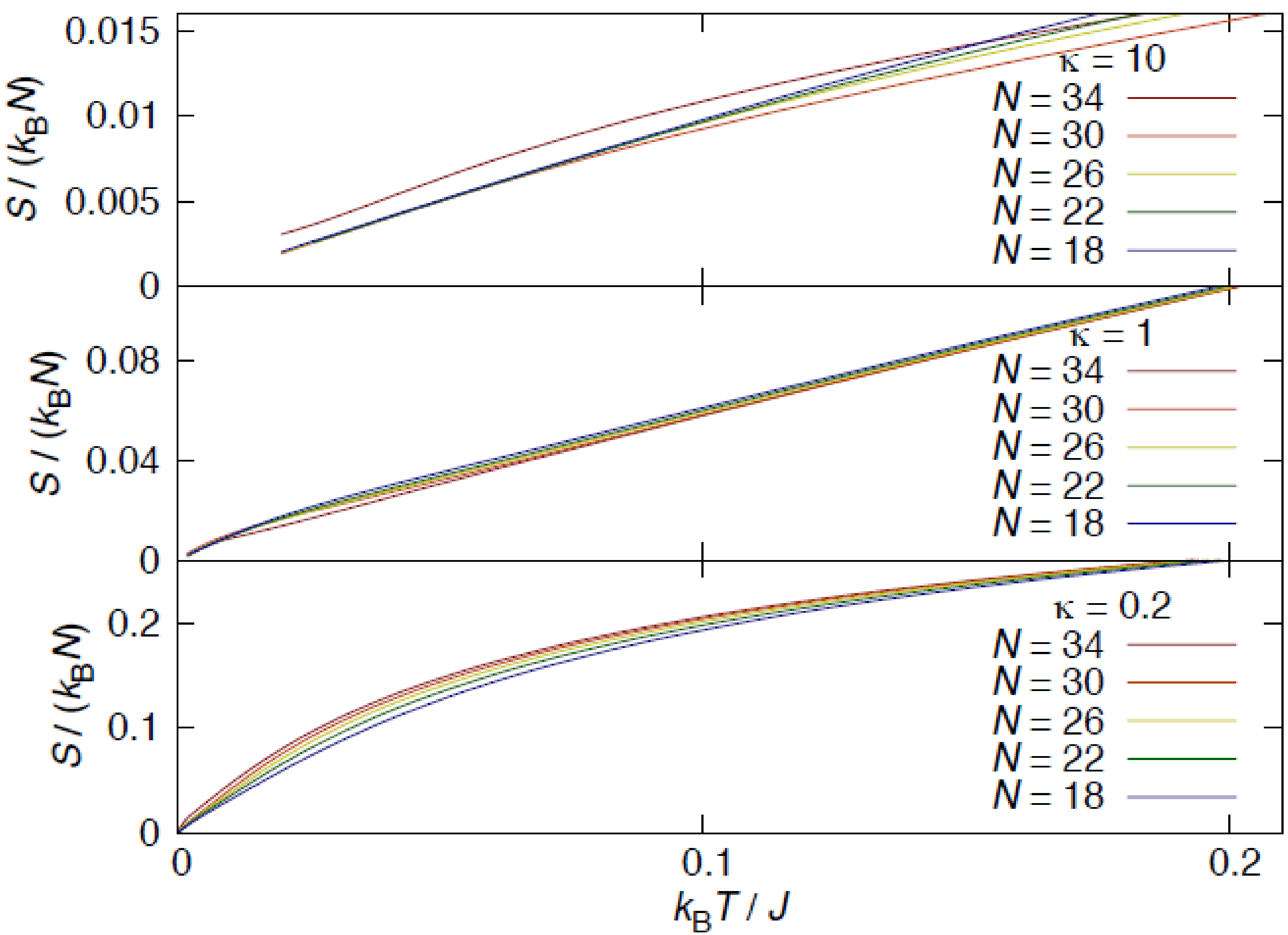
Both terms respect charge parity in complex fermion description
➔ Full numerical exact diagonalization (ED) of $2^{N/2-1}$-dimensional matrix, $N \lesssim 34$ possible

Entropy per fermion

In SYK$_4$, entropy S per fermion at low $T \rightarrow$ 0.2324 in large-N limit



[Cotler, …, and MT, JHEP **1705**, 118 (2017)]



Large K (weak SYK$_4$):
S / N almost independent of N, and vanishing as $T \rightarrow 0$

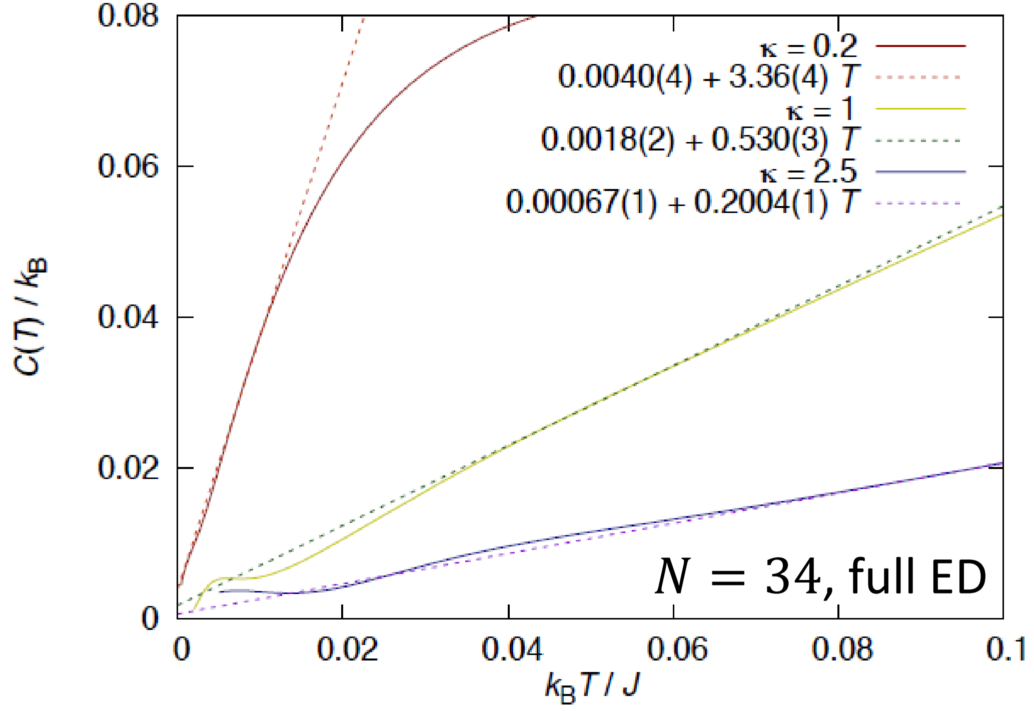
$$\hat{H} = \sum_{1 \leq a<b<c<d}^{N} J_{abcd} \hat{\chi}_a \hat{\chi}_b \hat{\chi}_c \hat{\chi}_d + i \sum_{1 \leq a<b}^{N} K_{ab} \hat{\chi}_a \hat{\chi}_b$$

J_{abcd}: average 0, std. dev. $\frac{\sqrt{6}}{N^{3/2}}$

K_{ab}: average 0, std. dev. $\frac{K}{\sqrt{N}}$

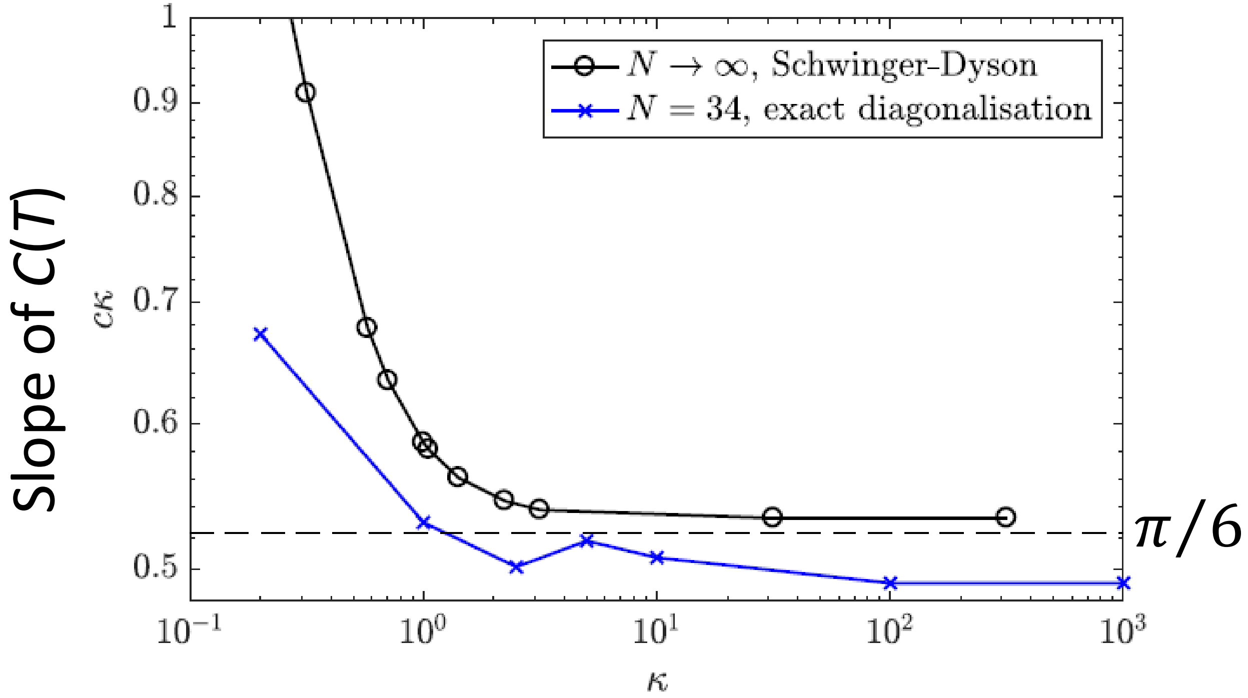
Specific heat and large-N analysis

Low T: specific heat $C(T)$ → linear in T



$$C(T) = \left\langle \frac{1}{NZ} \sum_k \frac{(E_k - \bar{E})^2}{T^2} e^{-\beta E_k} \right\rangle$$

Large N: replica fields + Hubbard-Stratonovich transf.
[cf. Maldacena and Stanford 2016, Sachdev 2015]



← SYK_4 limit

SYK_2 limit →

Out-of-time order correlator (OTOC)

$$F(t_1,t_2) \equiv \frac{1}{N^2}\sum_{i,j}^{N} Tr\left[\rho(\beta)^{1/4}\chi_i(t_1)\rho(\beta)^{1/4}\right.$$
$$\left.\times\chi_j(0)\rho(\beta)^{1/4}\chi_i(t_2)\rho(\beta)^{1/4}\chi_j(0)\right]$$
$$\simeq G_R(t_1)G_R(t_2) + \frac{1}{N}\mathcal{F}(t_1,t_2) + \mathcal{O}(N^{-2})$$

$$\rho^{1/4}(\beta) = \left(\frac{e^{-\beta H}}{Z}\right)^{1/4}$$

$$\mathcal{F}(t_1,t_2) = e^{\lambda_L (t_1+t_2)/2} f(t_1-t_2)$$

$$\mathcal{F}(t_1,t_2) = \int dt_3 dt_4 K_R(t_1,t_2,t_3,t_4)\mathcal{F}(t_3,t_4),$$

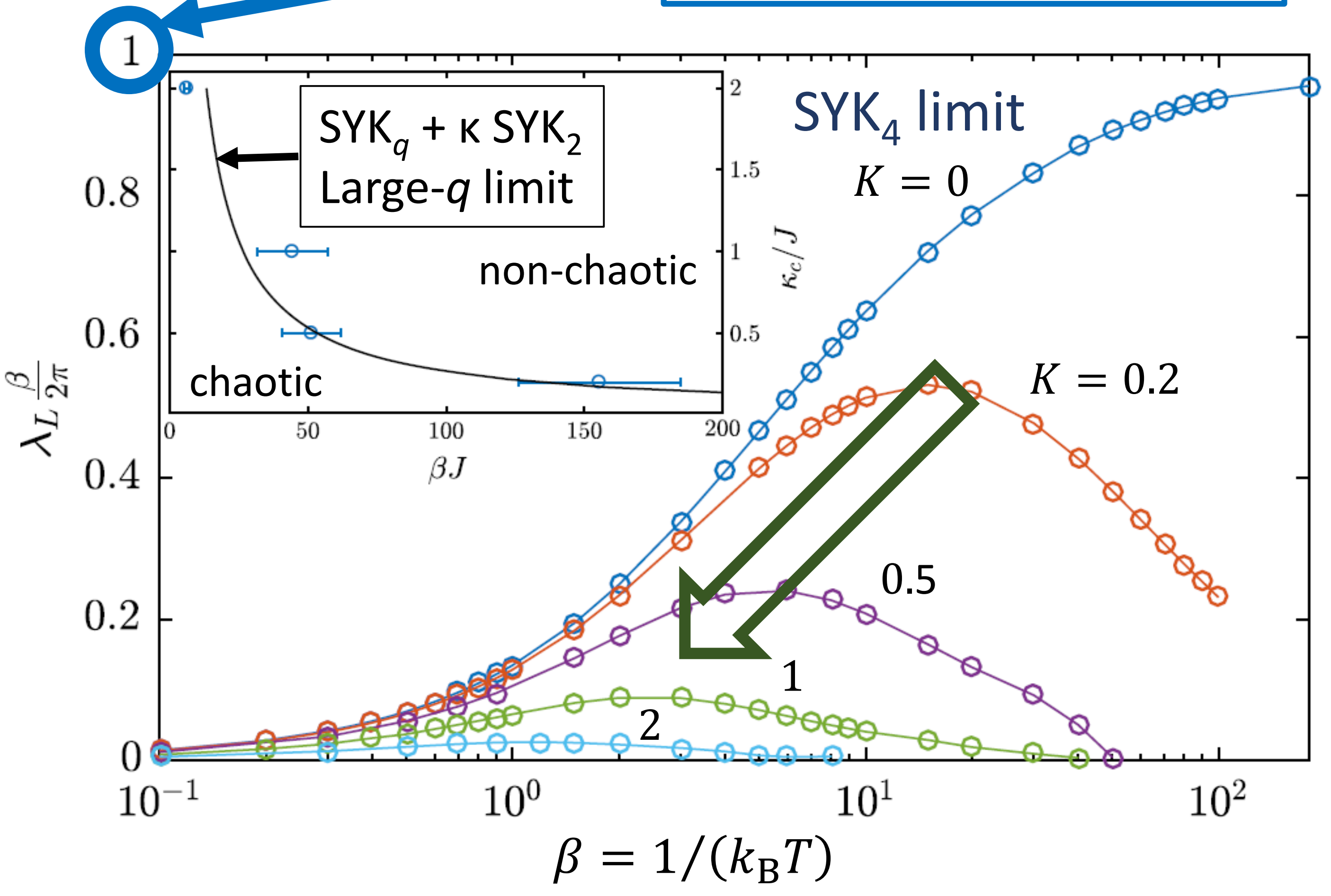
$$K_R(t_1,t_2,t_3,t_4) = G_R(t_1)G_R(t_2)[3J^2G_{lr}^2(t_3-t_4)+\kappa^2],$$

$$G_{lr}(\omega) = \frac{2ie^{-\frac{\beta}{2}\omega}}{1+e^{-\beta\omega}}\,\mathrm{Im}(G_R(\omega))$$

Lyapunov exponent is obtained by solving

$$f(\omega') = \left|G^R\left(\omega' + i\frac{\lambda_L}{2}\right)\right|^2\left[\kappa^2 f(\omega') + 3\int\frac{d\omega}{2\pi}g_{lr}(\omega'-\omega)f(\omega)\right]$$
$$\omega' = \omega_1 - i\lambda_L/2$$

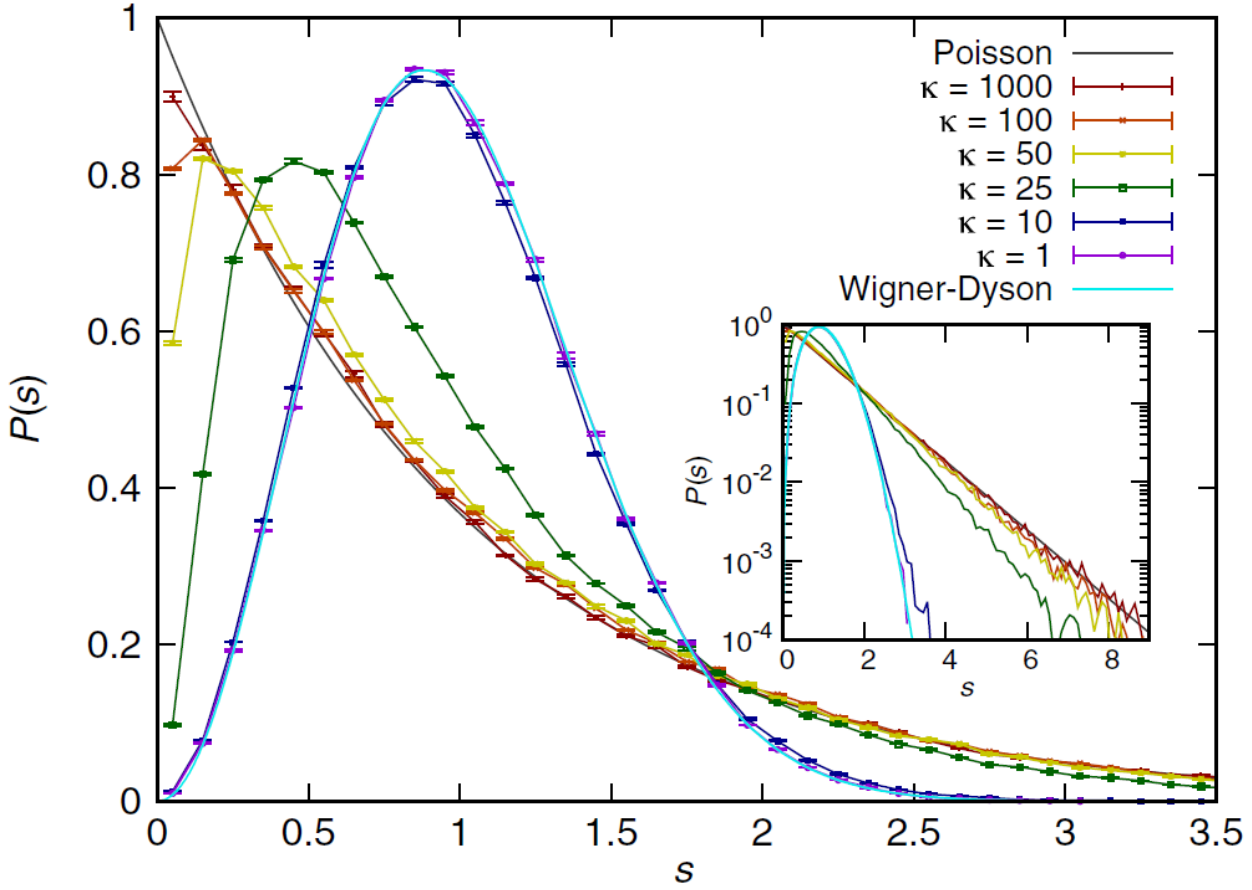
Chaos bound [Maldacena, Shenker, and Stanford 2016]



Deviation from the chaos bound as SYK$_2$ component is introduced

PRL **120**, 241603 (2018) (arXiv:1707.02197)

Small K: RMT-like behavior of energy spectrum



N=30, Central 10 % of eigenvalues

$P(s)$: level spacing distribution

Ratio of consecutive level spacing $E_{i+1} - E_i$
to the local mean level spacing Δ
(requires unfolding of the spectrum)

SYK_4 limit (small K): RMT

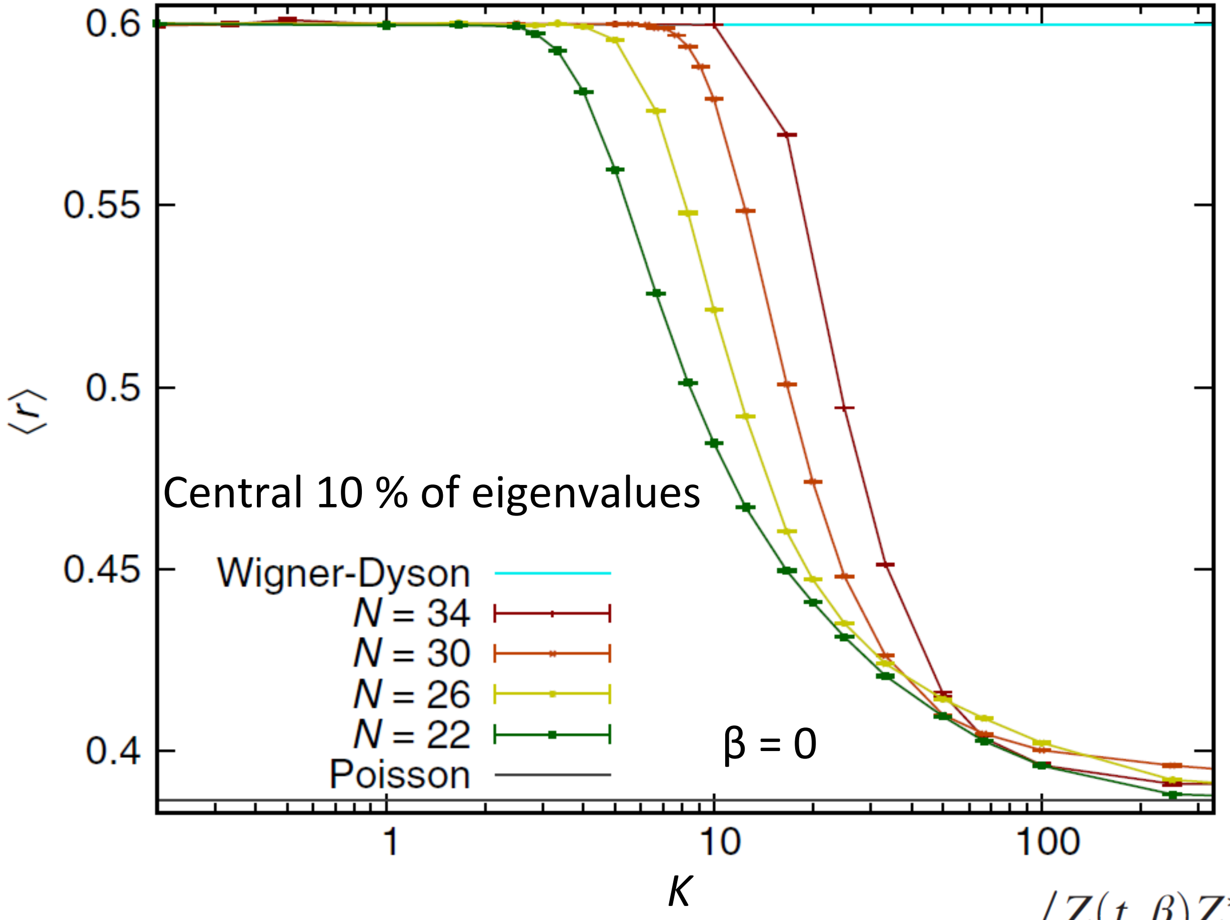
(GUE (Gaussian Unitary Ensemble) if $N \equiv 2 \pmod 4$)

SYK_2 (large K): Poisson (e^{-s})

Also see: T. Nosaka, D. Rosa, and J. Yoon, JHEP **1809**, 041 (2018) (arXiv:1804.09934) for other symmetry cases
cf. A. V. Lunkin, K. S. Tikhonov, and M. V. Feigel’man, 1806.11211; Y. Yu-Xiang, J. Ye, and W. M. Liu, 1809.07577, …

PRL **120**, 241603 (2018) (arXiv:1707.02197)

Small *K*: RMT-like behavior of energy spectrum



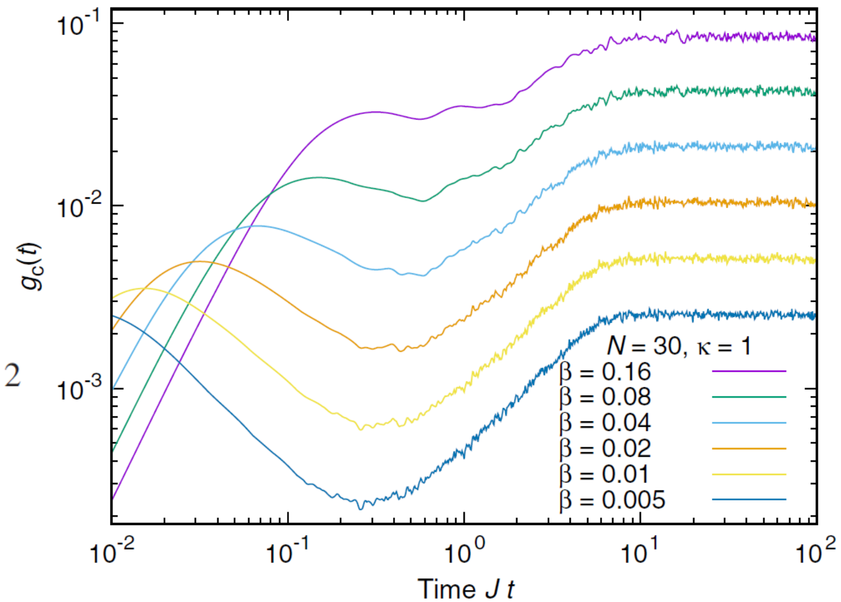
$\langle r \rangle$: average adjacent gap ratio

Average of $\frac{\min(E_{i+1}-E_i, E_{i+2}-E_{i+1})}{\max(E_{i+1}-E_i, E_{i+2}-E_{i+1})}$

SYK_4 ($K = 0$): RMT

(≈ 0.599 for GUE [Y. Y. Atas *et al.* PRL 2013])

SYK_2 (large *K*): Poisson ($2\log 2 - 1 \approx 0.386$)



Spectral form factor $g_c(t) \equiv \left\langle \frac{Z(t,\beta)Z^*(t,\beta)}{Z(0,\beta)^2} \right\rangle - \left| \left\langle \frac{Z(t,\beta)}{Z(0,\beta)} \right\rangle \right|^2$

shows robust ramp reflecting level rigidity

Summary of Part 1

$$\hat{H}_{\mathrm{SYK4}} = \sum_{1 \le a<b<c<d}^{N} J_{abcd} \hat{\chi}_a \hat{\chi}_b \hat{\chi}_c \hat{\chi}_d$$

"Stability of chaos in a generalized Sachdev-Ye-Kitaev model"

- Effect of one-body term

$$\hat{H} = \hat{H}_{\mathrm{SYK4}} + i \sum_{1 \le a<b}^{N} K_{ab} \hat{\chi}_a \hat{\chi}_b$$

  - No longer maximally chaotic
  - Random-matrix like spectra for weak perturbation
  - Temperature-dependent transition to non-chaotic behavior

Phys. Rev. Lett. **120**, 241603 (2018) (arXiv:1707.02197)

Interaction with randomness
+ locality
➔ many-body localization transition?

Other quantities for characterization of chaotic / non-chaotic phases?

2. Short-range SYK_4 + SYK_2: many-body localization

Antonio M. García-García and MT, arXiv:1801.03204

$$H = \sum_{1=i<j<k<l}^{N} \tilde{J}_{ijkl}(D)\, \chi_i\, \chi_j\, \chi_k\, \chi_l + i\kappa \sum_{1=i<j}^{N} \tilde{K}_{ij}(d)\, \chi_i\, \chi_j$$

For integer D, d

$$\tilde{J}_{ijkl}(D) = \begin{cases} \tilde{J}_{ijkl} & l-i<D \\ 0 & l-i \geq D \end{cases} \qquad \widetilde{K}_{ij}(d) = \begin{cases} \widetilde{K}_{ij} & j-i<d \\ 0 & j-i \geq d \end{cases}$$

$\tilde{J}_{ijkl}, \widetilde{K}_{ij}$: Gaussian random

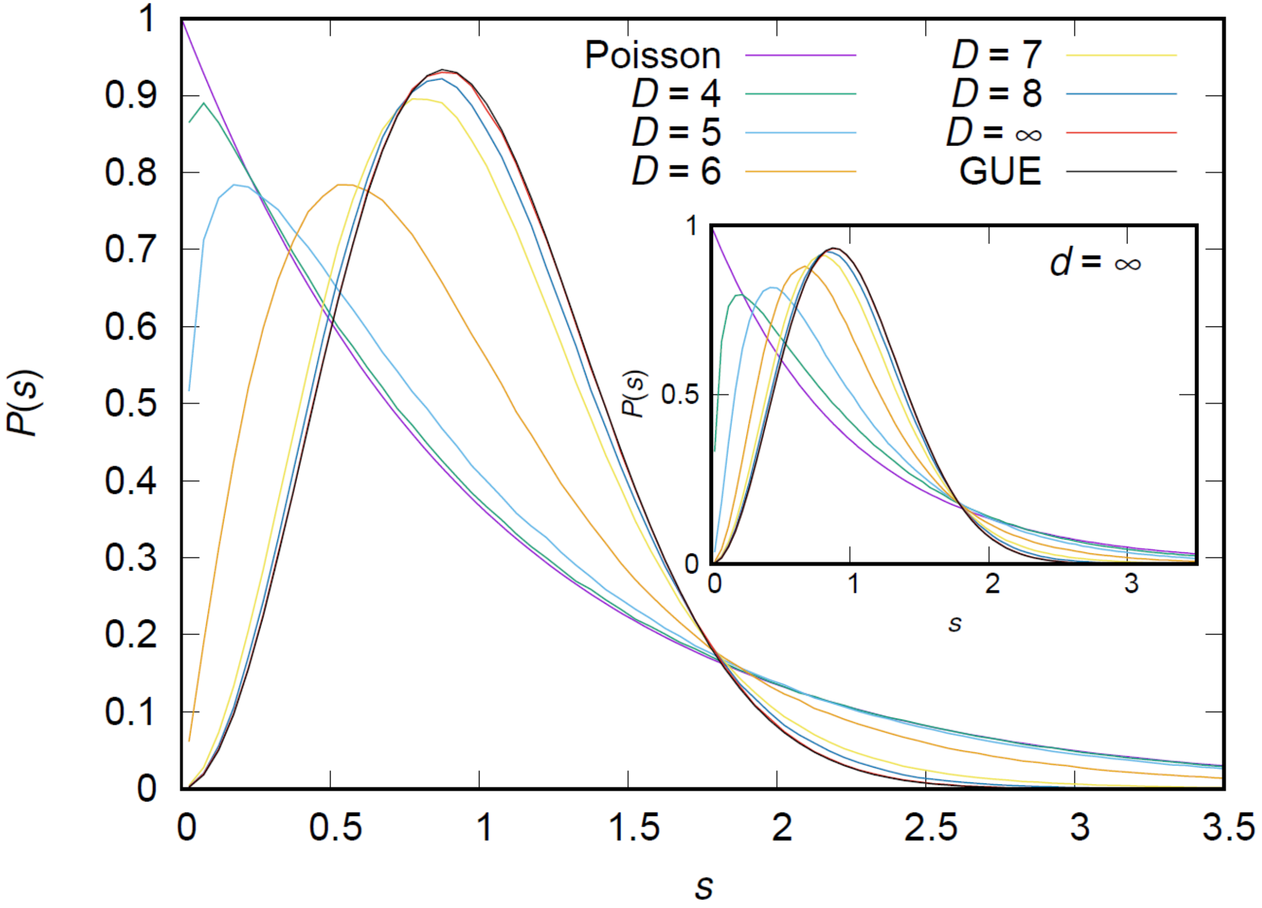
Also consider fractional $D = [D] + \widetilde{D}\ \left(0 < \widetilde{D} < 1\right)$:

If $l - i = [D]$, use non-zero $\tilde{J}_{ijkl}$ with probability $\widetilde{D}$, otherwise set to zero

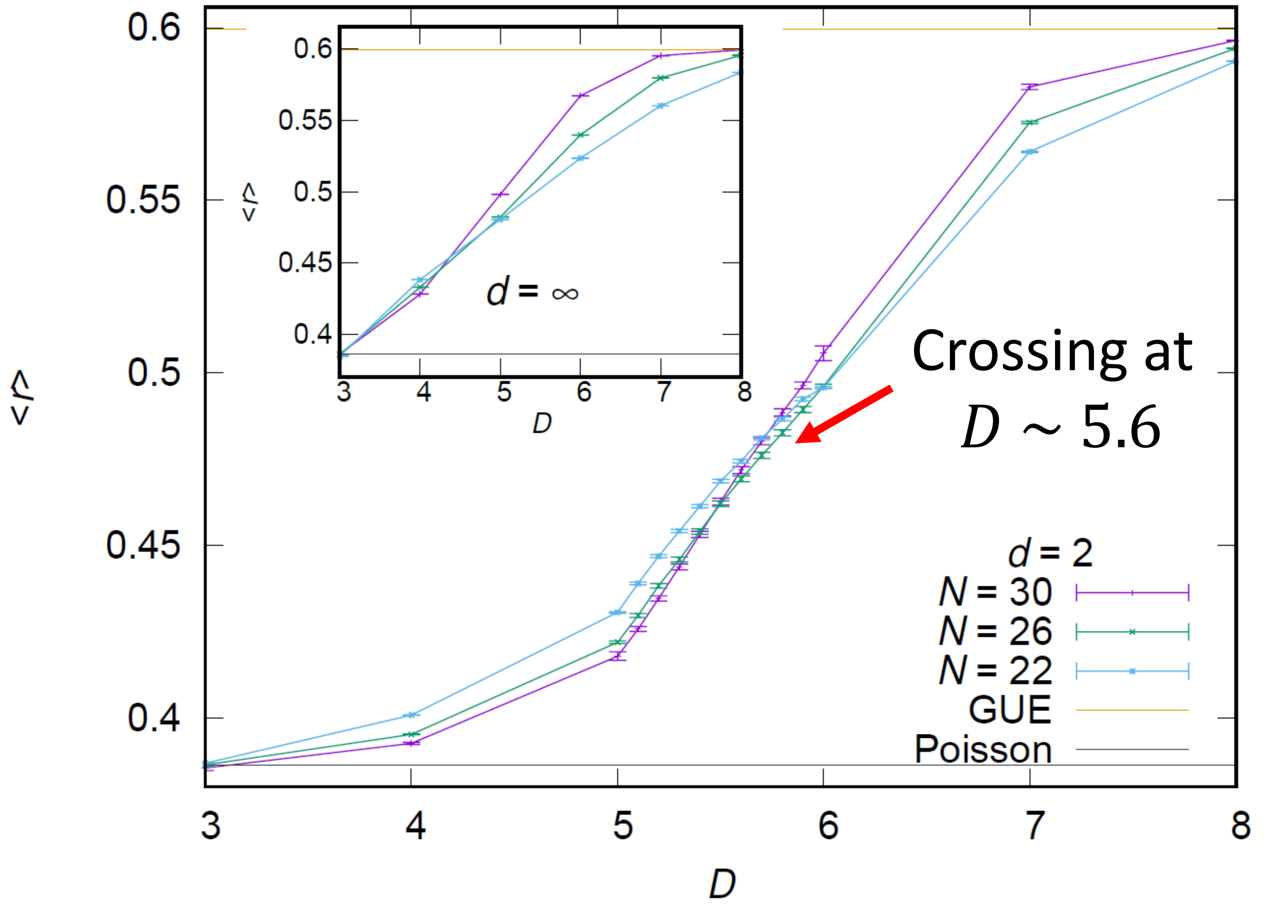
➔ We explore the possibility of a metal-insulator transition as D is decreased (following numerical data: for $\kappa = 1$)

Level separation distribution | Gap ratio as a function of D

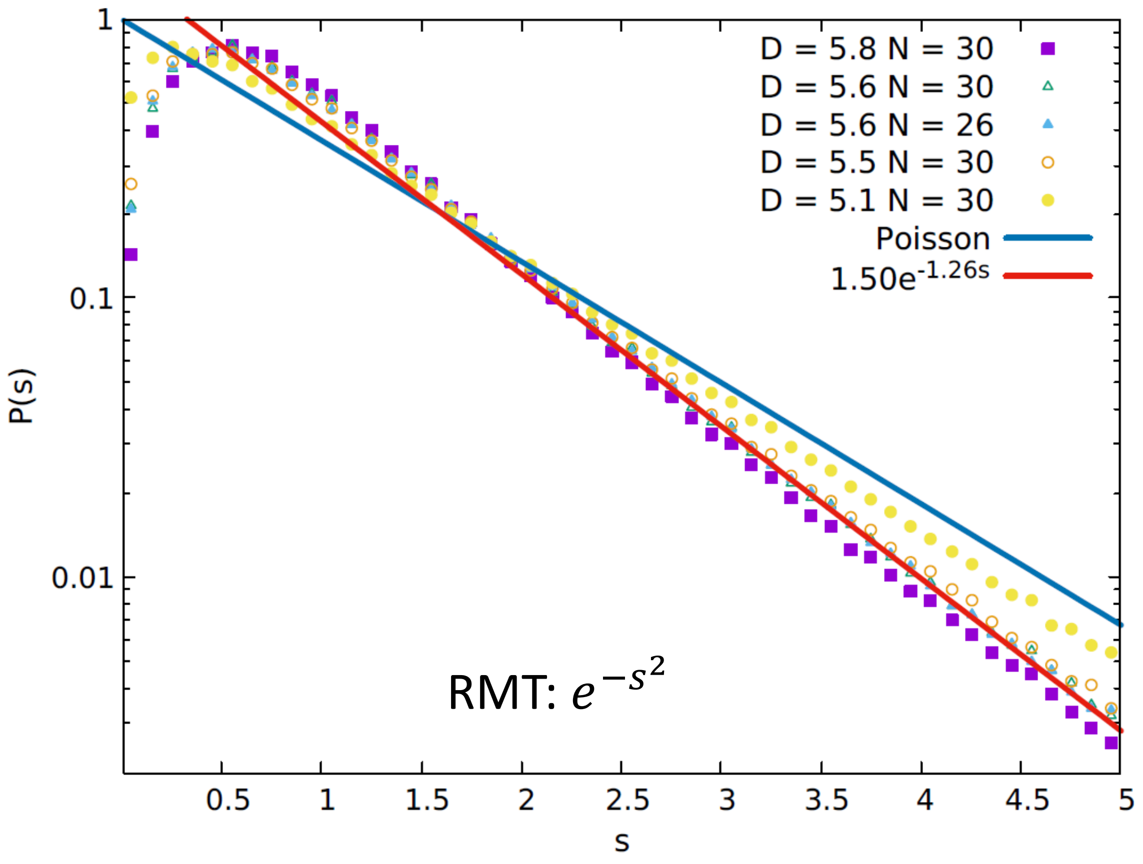
$$H = \sum_{1=i<j<k<l}^{N} \tilde{J}_{ijkl}(D)\,\chi_i\,\chi_j\,\chi_k\,\chi_l + i\kappa \sum_{1=i<j}^{N} \tilde{K}_{ij}(d)\,\chi_i\,\chi_j$$



d = 2: nearest neighbor only

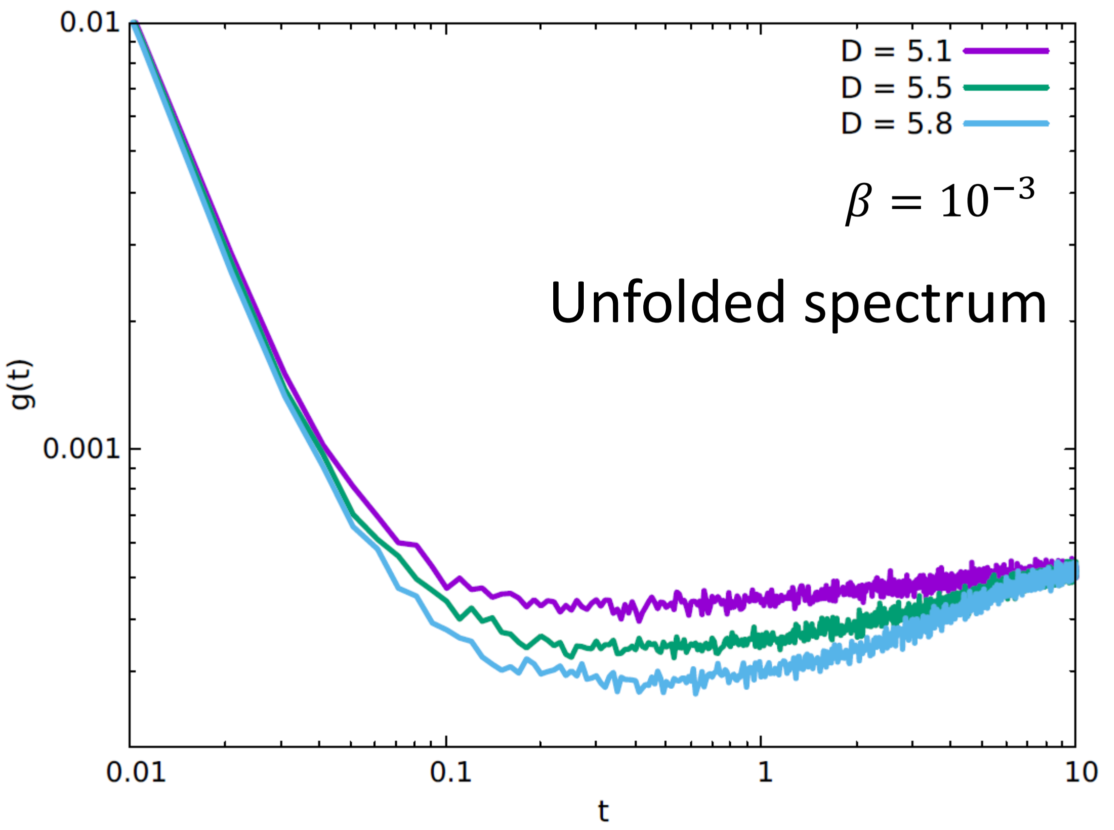


Tail of level separation distribution



Decay at long distance approaches Poisson as interaction (SYK_4) range D is decreased

Spectral form factor

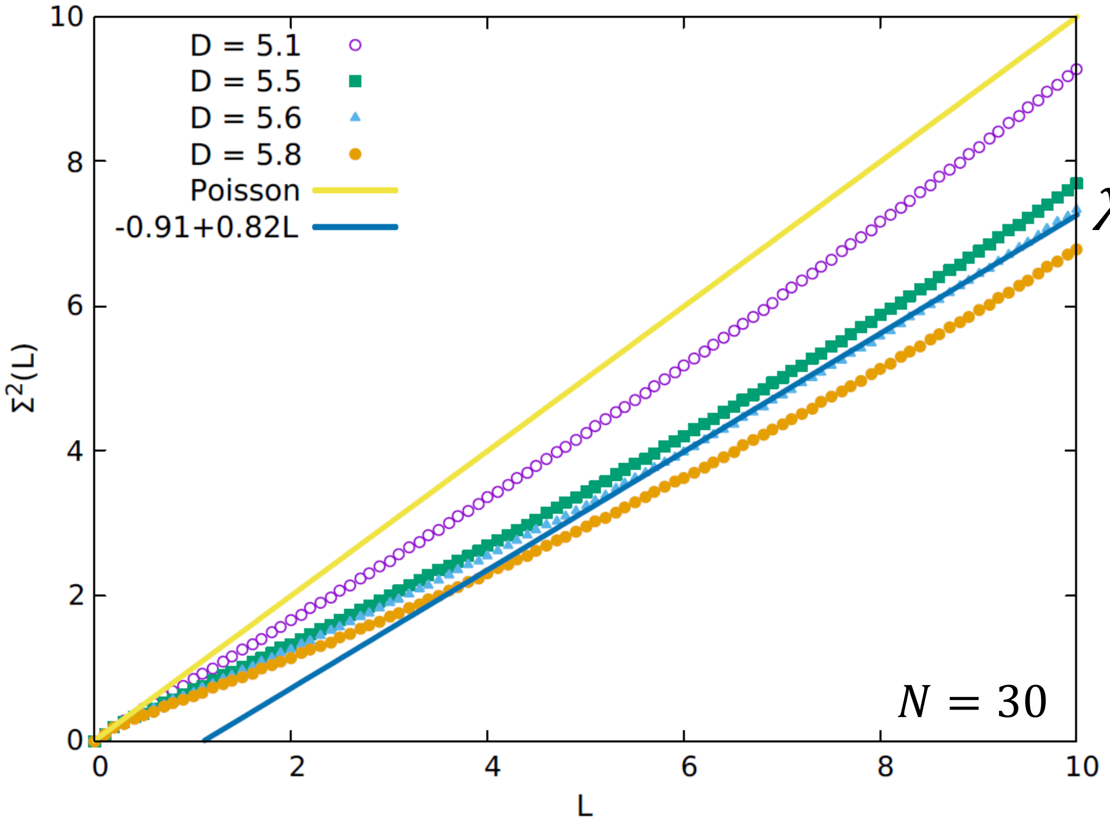


t^1 ramp disappears as D is decreased

$N = 30$

Number variance

$\Sigma^2(L)$: Variance of number of levels within L(average level separation)



Logarithmic increase for quantum chaotic case
Linear with slope $\chi < 1$ for Anderson localization

$\chi \sim 0.82$ for $D = 5.6$

$\chi \sim 0.27$ in 3D cubic lattice,
increase and approach 1 for higher dimensions
[Zharekeshev and Kramer 1995][Schreiber and Grussbach 1996]
[García-García and Cuevas 2007]

Summary of Part 2: Short-range SYK$_4$ + SYK$_2$: many-body localization [1801.03204]

$$H = \sum_{1=i<j<k<l}^{N} \tilde{J}_{ijkl}(D)\,\chi_i\,\chi_j\,\chi_k\,\chi_l + i\kappa \sum_{1=i<j}^{N} \tilde{K}_{ij}(d)\,\chi_i\,\chi_j$$

For integer D, d

$$\tilde{J}_{ijkl}(D) = \begin{cases} \tilde{J}_{ijkl} & l-i<D \\ 0 & l-i \geq D \end{cases}$$

$$\tilde{K}_{ij}(d) = \begin{cases} \tilde{K}_{ij} & j-i<d \\ 0 & j-i \geq d \end{cases}$$

Also consider fractional $D = [D] + \widetilde{D}\ (0 < \widetilde{D} < 1)$:

If $l - i = [D]$, use non-zero $\tilde{J}_{ijkl}$ with probability $\widetilde{D}$, otherwise set to zero

Metal → insulator transition as D is decreased ($d = 2$: at $D \sim 5.6$ for $\kappa = 1$)

Gap ratio, level separation tail, spectral form factor, number variance consistently support a many-body localization

Out-of-Time-Ordered Correlation

3. Quantum Lyapunov spectrum (cf. OTOC)

$$\delta x_i(t) = M_{ij}\delta x_j(0)$$

Canonically conjugate variables x, p at different times

$$\{x(t), p(0)\}_{\mathrm{PB}}{}^2 = \left(\frac{\partial x(t)}{\partial x(0)}\right)^2 \to e^{2\lambda_{\mathrm{L}} t}$$

at large t

$$M_{ij} = \frac{\delta x_i(t)}{\delta x_j(0)}$$

$$L = \left(\frac{\delta x_i(t)}{\delta x_j(0)}\right)^2$$

Singular values **at finite *t***: $\{s_k(t)\} = \{e^{\lambda_k t}\}$

Hanada, Shimada, and MT: PRE **97**, 022224 (2018)
For classical chaos, $\{\lambda_k(t)\}$ obeys RMT statistics

Corresponding quantity in quantum systems: for bosonic V, W : $C_T(t) = -\left\langle\left[\hat{V}(t), \hat{W}(0)\right]^2\right\rangle$

[Norbert Wiener 1938]
[Larkin & Ovchinnikov 1969]

Out-of-time correlator is included $\left\langle\hat{V}(t)\hat{W}(0)\hat{V}(t)\hat{W}(0)\right\rangle$ etc.

For systems of fermions
e.g. Sachdev-Ye-Kitaev (SYK) model?

$$\hat{H} = \sum_{1\le a<b<c<d\le N} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d$$

$\hat{\chi}_a$: Majorana fermion, $\hat{M}_{ab}(t=0) = \{\hat{\chi}_a, \hat{\chi}_b\} = \delta_{ab}$

Anticommutator

$$\hat{M}_{ab}(t) = \{\hat{\chi}_a(t), \hat{\chi}_b(0)\}$$

$$\hat{L}_{ab}(t) = \sum_{j=1}^{N} \hat{M}_{ja}(t)\hat{M}_{jb}(t)$$

Our definition: for state ϕ (e.g. eigenstate)

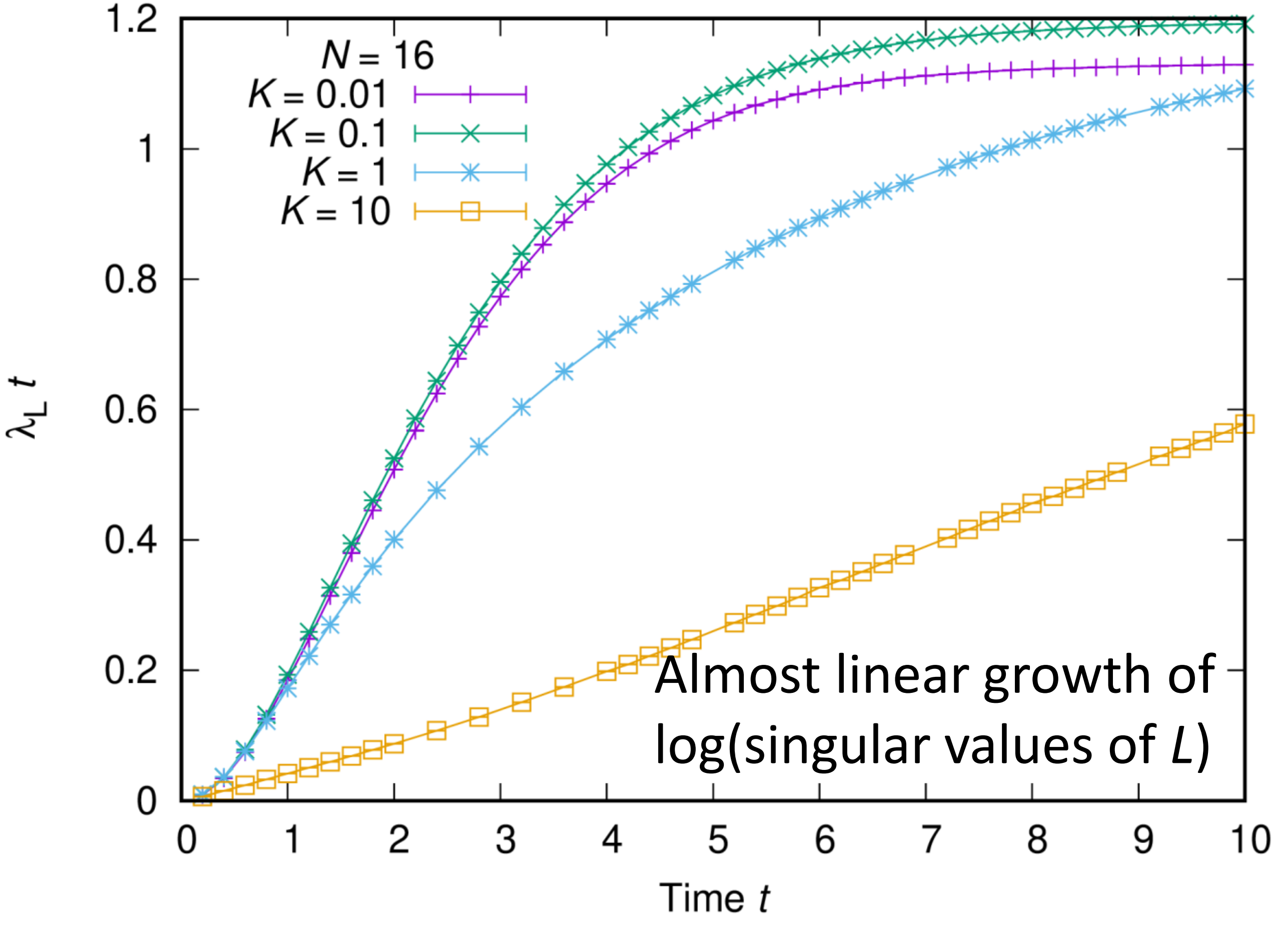
For matrix $\langle\phi|\hat{L}_{ab}(t)|\phi\rangle$, obtain singular values $\{s_k(t)\}_{k=1}^{N}$

and the Lyapunov spectrum is defined as $\left\{\lambda_k(t) = \frac{\log s_k(t)}{2t}\right\}$.

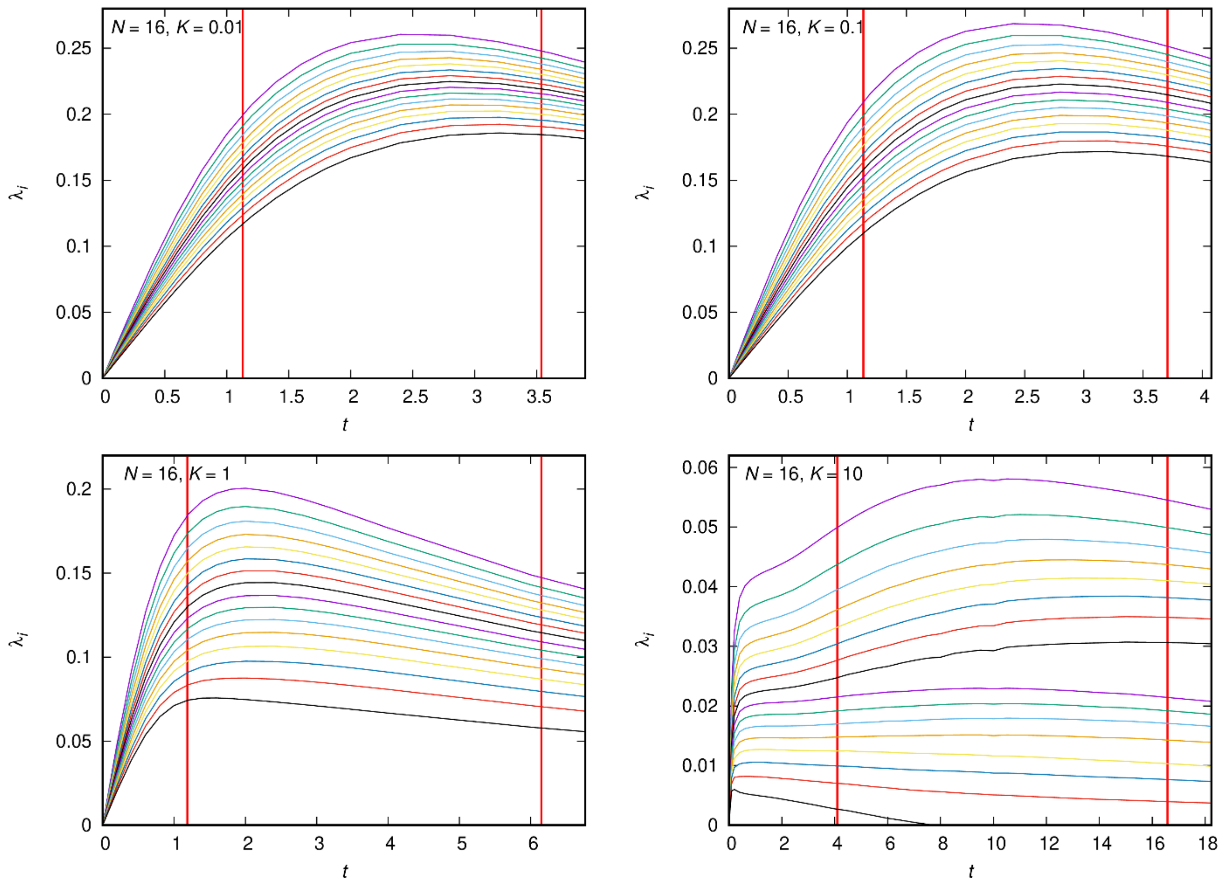
Other possibilities: see Rozenbaum-Ganeshan-Galitski, 1801.10591; Hallam-Morley-Green: 1806.05204

SYK: dependence on the SYK$_2$ coefficient K

$$\hat{H} = \sum_{1\le a<b<c<d}^{N} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d + i\sum_{1\le a<b}^{N} K_{ab}\hat{\chi}_a\hat{\chi}_b$$



Sample- and state-averaged full Lyapunov spectrum: time dependence



Close to constant between red lines
(20 % and 80 % of the saturated value of $\lambda_N t$)

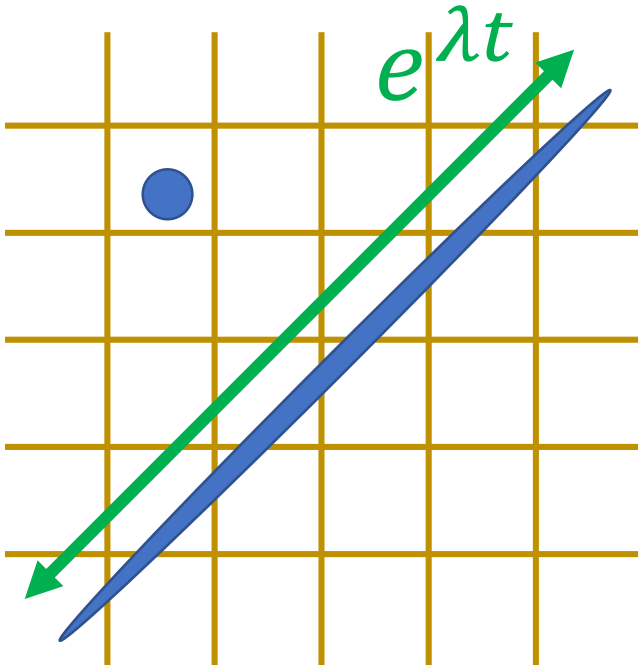
Classical KS entropy vs entanglement entropy production

Initial state: $|\psi(t=0)\rangle = \underbrace{|000\ldots000\rangle}_{N/2}$ in the complex fermion basis

$$\hat{c}_j = \frac{(\chi_{2j-1} + \mathrm{i}\chi_{2j})}{\sqrt{2}}$$

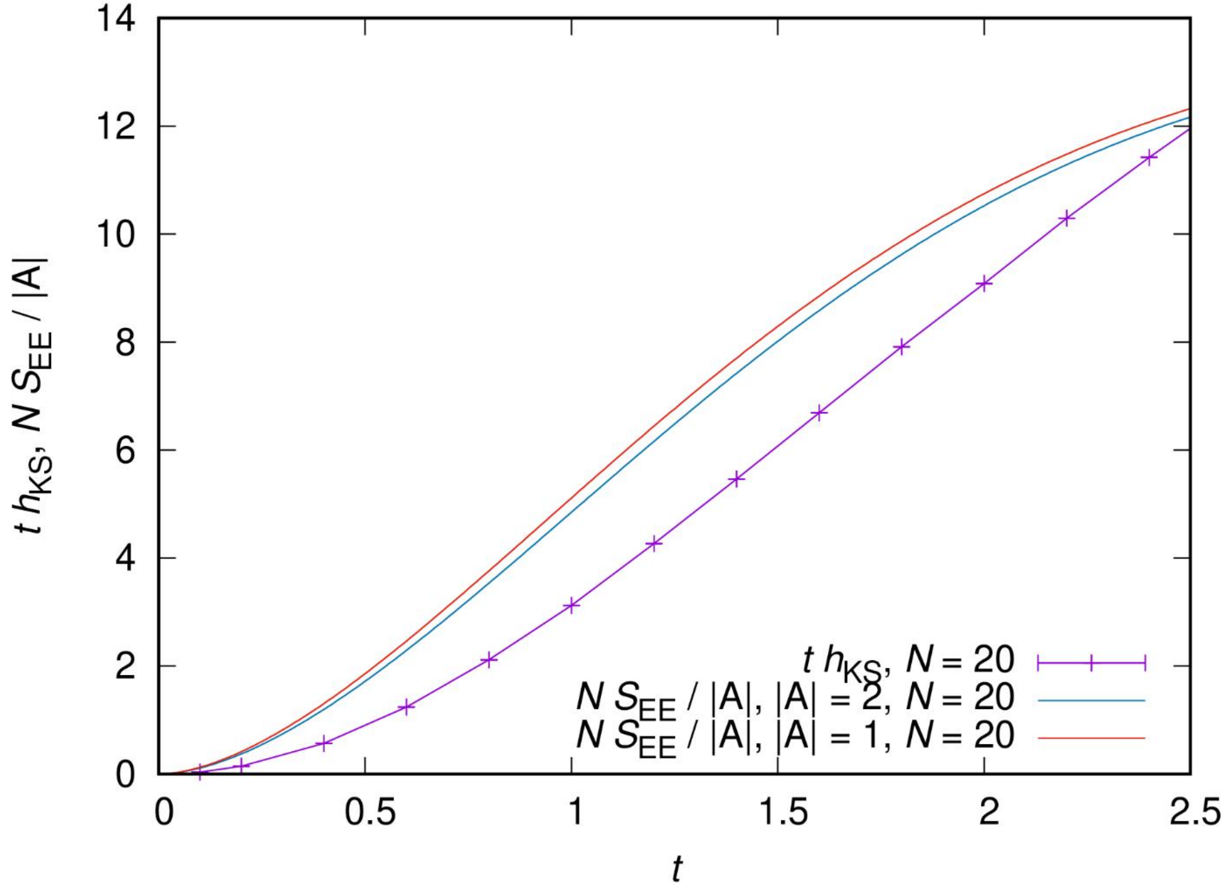
Coarse-grained entropy
= log(# of cells covering the region)
~ (sum of positive λ) t

Kolmogorov-Sinai entropy h_{KS}
= (sum of positive λ)
= entropy production rate



$$\rho_{\mathrm{A}}(t) = \mathrm{Tr}_{\mathrm{B}}\, \rho(t), \quad \rho(t) = |\psi(t)\rangle\langle\psi(t)|$$

$$S_{\mathrm{EE}}(t) = -\mathrm{Tr}\log(\rho_{\mathrm{A}}(t))$$



Classical KS entropy vs entanglement entropy production

Coarse-grained entropy
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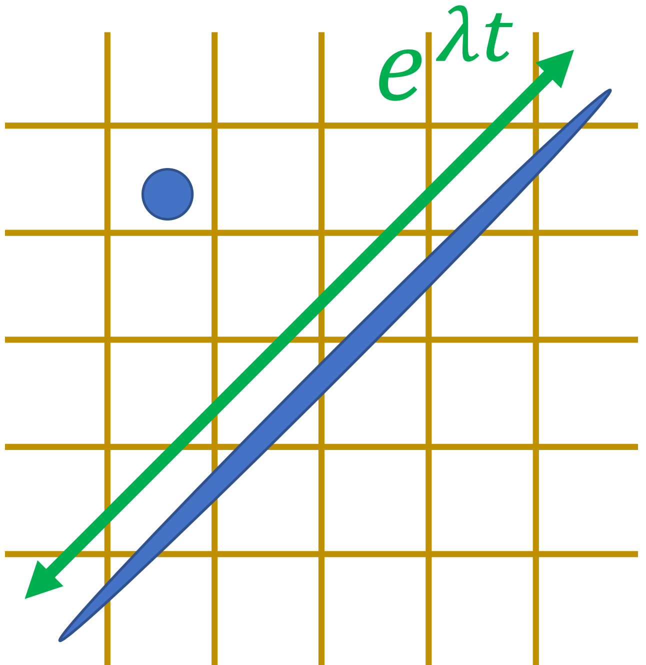
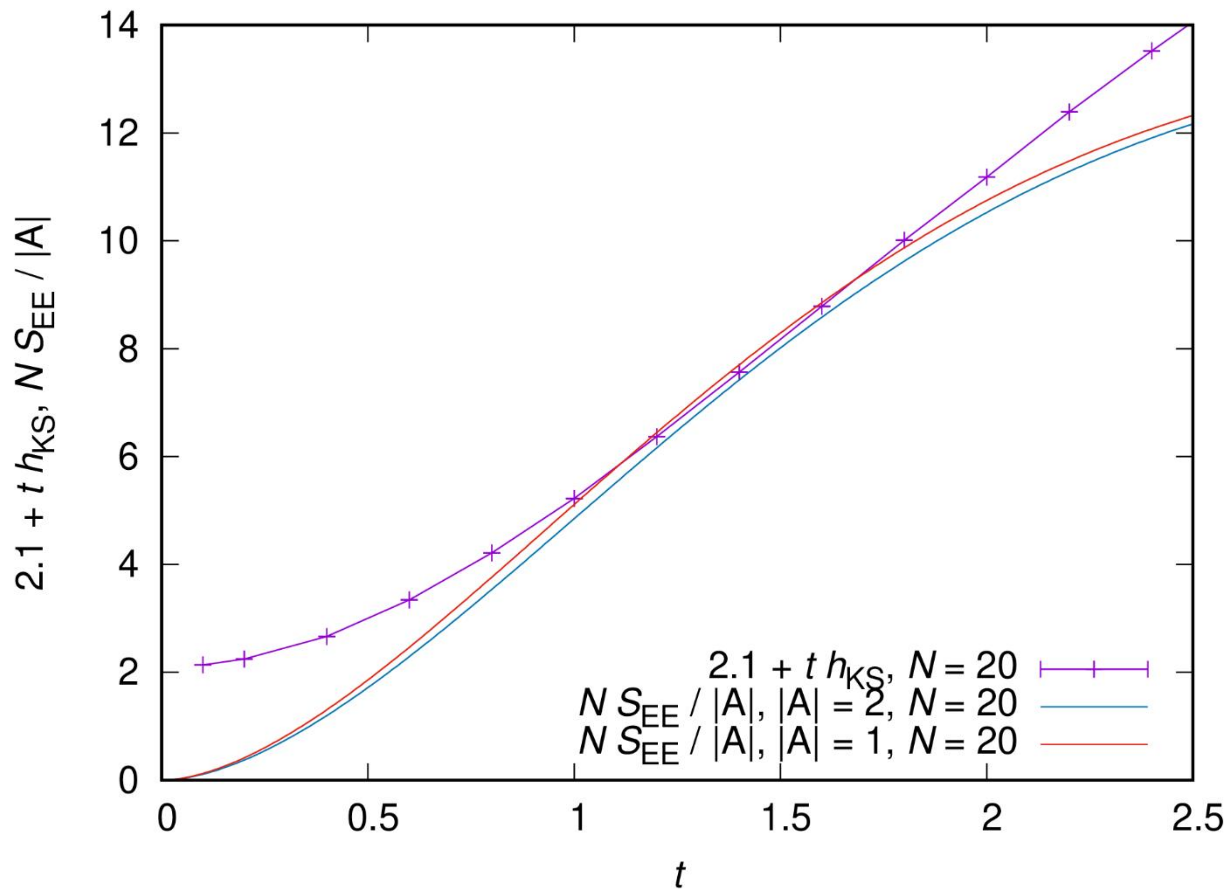


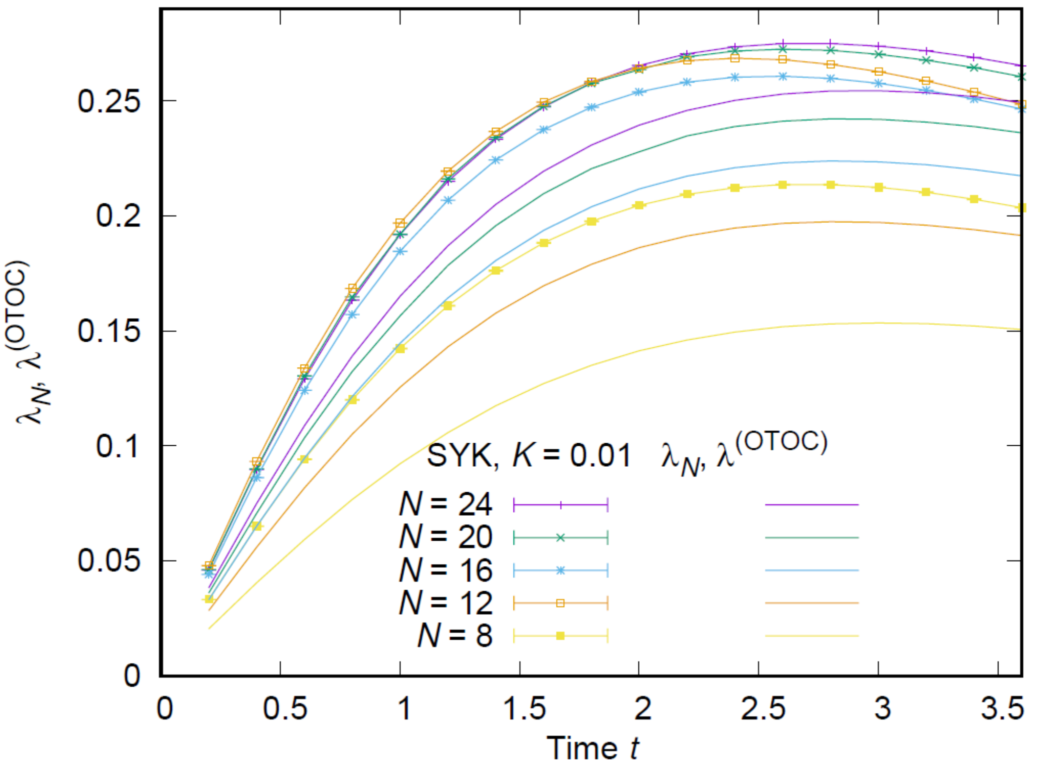


Similar time scale for saturation in SYK model; other models?

Conjecture on entropy production

SYK$_4$ limit

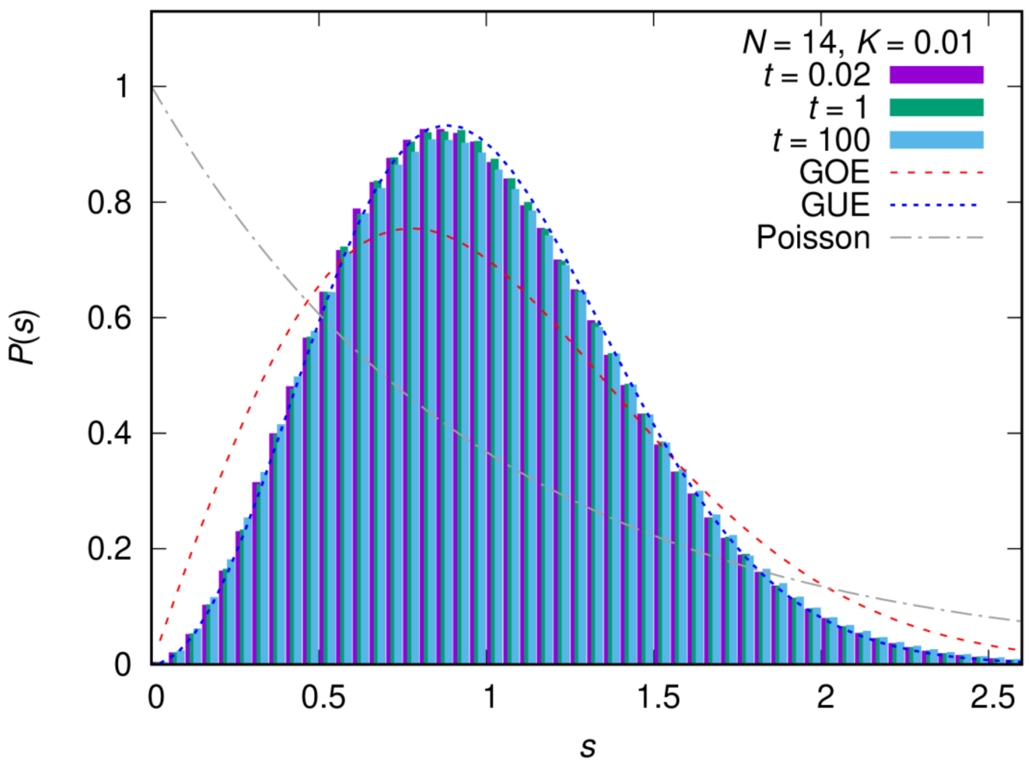
- λ_N and $\lambda_{\rm OTOC} = \frac{1}{2t}\log\left(\frac{1}{N}\sum_{i=1}^{N} e^{2\lambda_i t}\right)$ approach each other; difference decreases as $1/N$
- Same for λ_N and λ_1: all exponent → single peak
- All saturate the MSS bound at strong coupling (low T) limit
- Growth rate of entanglement entropy $\sim h_{\rm KS} =$ sum of positive (all) λ_i



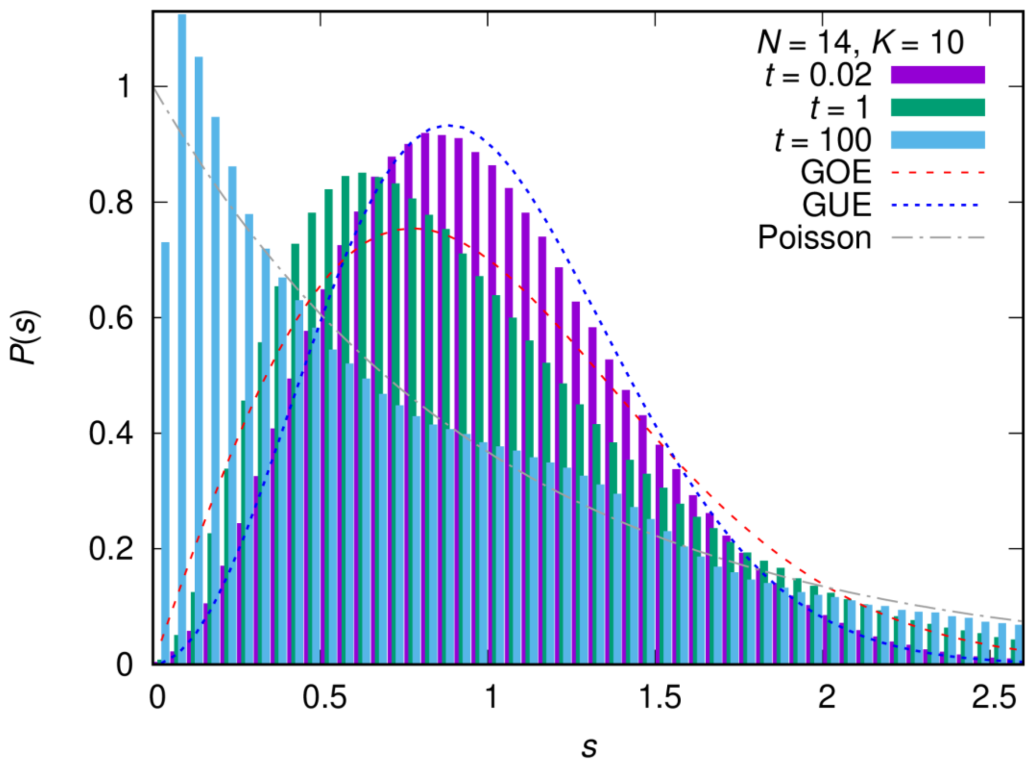
➔ Black holes: not only the fastest scramblers [Sekino and Susskind 2008], but also fastest entropy generators

Spectral statistics: SYK

(fixed-*i* unfolding:
unfold each gap $\lambda_{i+1} - \lambda_i$ by its average)



Close to SYK_4 :
remains GUE even after long time



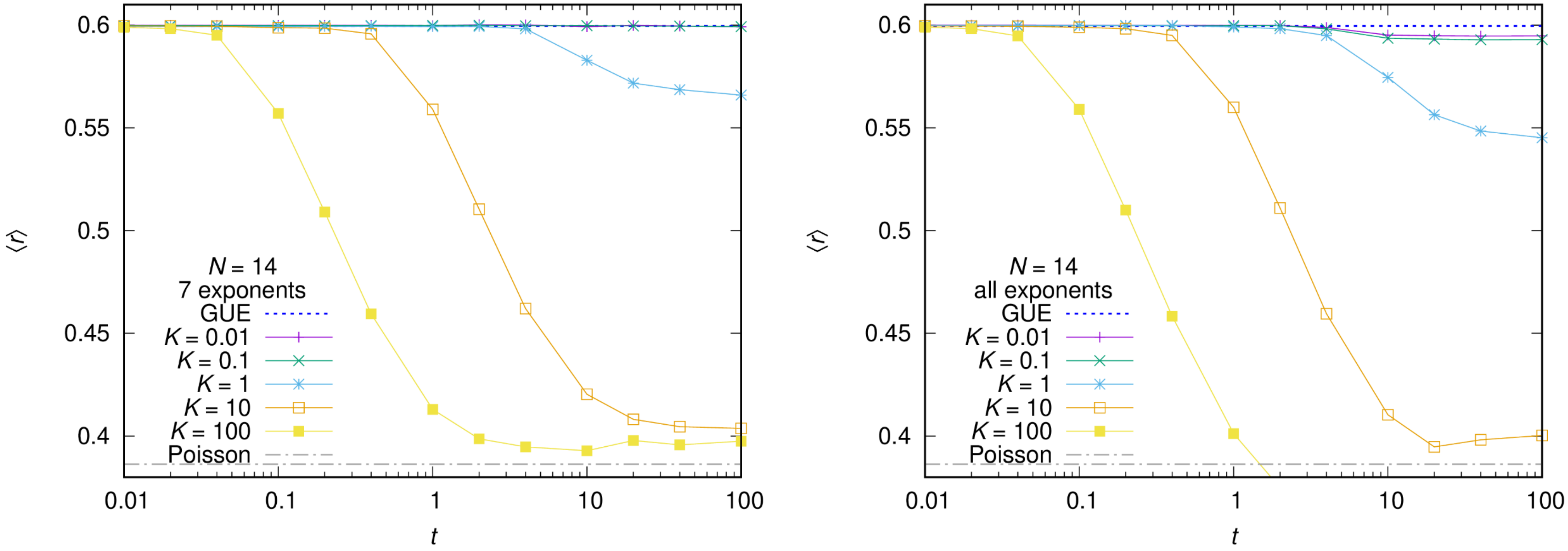
Strong one-body hopping term:
approaches Poisson

Spectral statistics: SYK

$\langle r \rangle$: average of the adjacent gap ratio $\frac{\min(\epsilon_{i+1}-\epsilon_i\,,\,\epsilon_{i+2}-\epsilon_{i+1})}{\max(\epsilon_{i+1}-\epsilon_i\,,\,\epsilon_{i+2}-\epsilon_{i+1})}$

Uncorrelated (Poisson): $2\log 2 - 1 \approx 0.386$

Correlated: larger (GOE: 0.5307, GUE: 0.5996 etc.) [Atas *et al.*, PRL 2013]

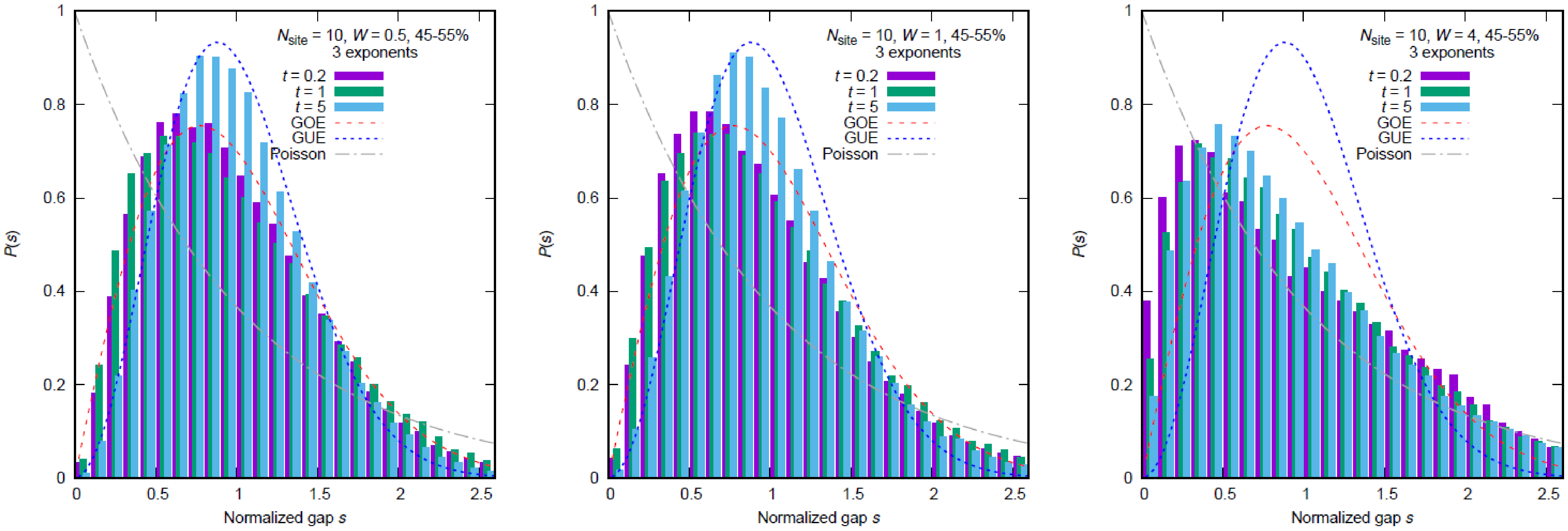


Spectral statistics: XXZ + random field

$$\hat{H} = \sum_{i}^{N} \hat{S}_i \cdot \widehat{S_{i+1}} + \sum_{i}^{N} h_i \widehat{S_i^z}$$

h_i: uniform distribution $[-W, W]$

$$\hat{M}_{ab}(t) = \left[\widehat{S_a^+}(t), \widehat{S_b^-}(0)\right]$$



W = 0.5: approaches GUE

W = 4: close to Poisson

Quantum Lyapunov spectrum distinguishes chaotic and non-chaotic phases

New possibility: characterization of chaos by singular value statistics of two-point functions

SYK, largest 3 exponents

Preliminary

Fixed-*i* unfolded log of singular values of $G_{ab}^{(\phi)} = \langle\phi|\hat{\psi}_a(t)\hat{\psi}_b(0)|\phi\rangle$

Work in progress with Gharibyan, Hanada, Swingle, …

New possibility: characterization of chaos by singular value statistics of two-point functions

XXZ, all non-trivial exponents

$$G_{ab}^{(\phi)} = \langle \phi | \widehat{\sigma^{+}}_{a}(t) \widehat{\sigma^{-}}_{b}(0) | \phi \rangle$$

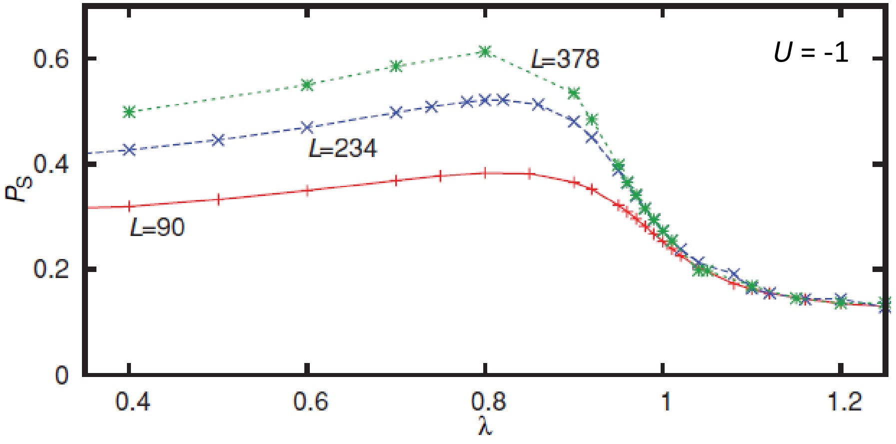
Preliminary

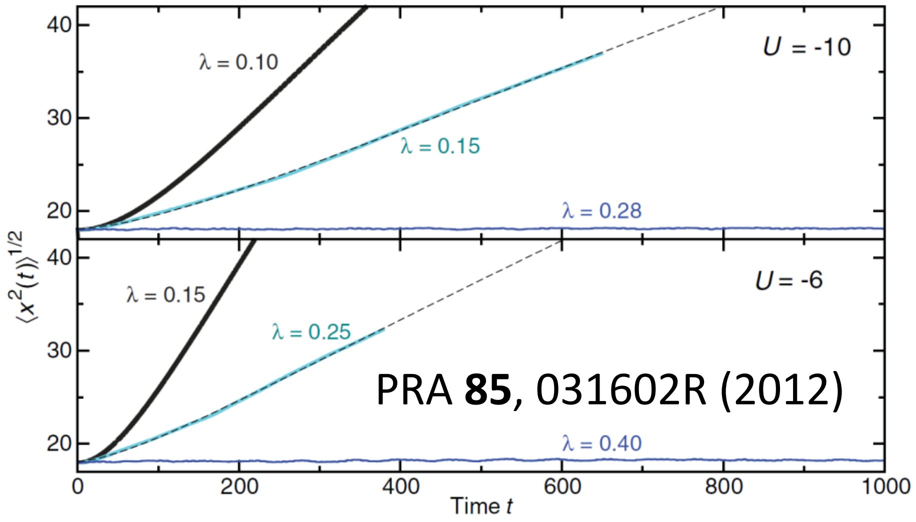
Work in progress

Our other works on localization + interaction

Quasiperiodic site level modulation in 1D $U < 0$ Hubbard model: superfluid-insulator transition and quench dynamics

MT & A. M. García-García: PRA **82**, 043613 (2010)





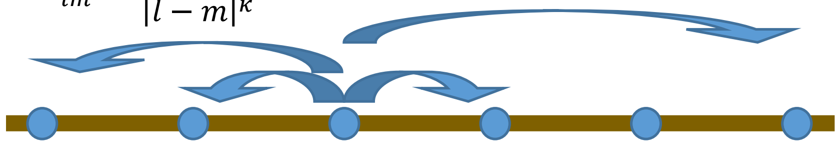
PRA **85**, 031602R (2012)

Higher effective dimension in 1D lattice with power-law interaction

A. M. Lobos, MT and A. M. García-García: PRB **88**, 134506 (2013)

1D Hubbard + power-law hopping: stabilization of long-range order

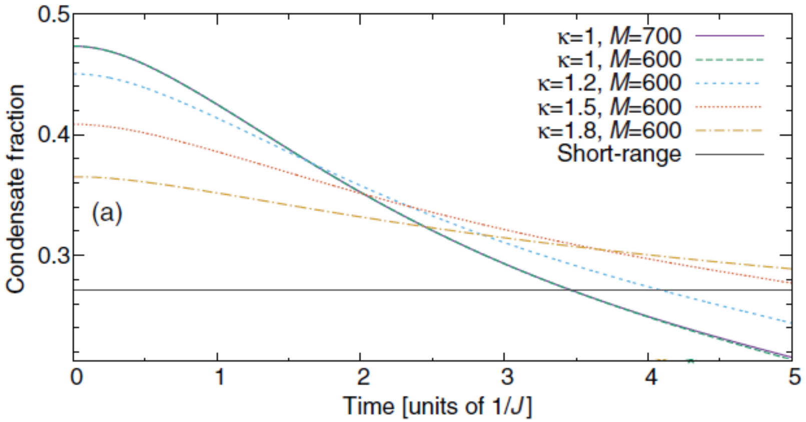
$$t_{lm} = \frac{t}{|l-m|^\kappa}$$



Dimensional quench dynamics by tDMRG

MT, A. M. García-García, and M. A. Cazalilla: PRA **90**, 053618 (2014)

Remove non-nearest-neighbor hoppings:
quench to 1D from larger effective dimension condensate



Summary of the talk

$$\hat{H}_{\mathrm{SYK4}} = \sum_{1\le a<b<c<d}^{N} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d$$

Part 1: effect of one-body term

$$\hat{H} = \hat{H}_{\mathrm{SYK4}} + i\sum_{1\le a<b}^{N} K_{ab}\hat{\chi}_a\hat{\chi}_b$$

- No longer maximally chaotic
- Random-matrix like spectra for weak perturbation
- Temperature-dependent transition to non-chaotic behavior

Phys. Rev. Lett. **120**, 241603 (2018)
(arXiv:1707.02197)

Part 2: short-range SYK model

- Many-body localization

arXiv:1801.03204

Part 3: quantum Lyapunov spectrum

- Analogue of time-dependent classical Lyapunov spectrum (Hanada-Shimada-MT PRE 2018)
- Defined by out-of-time-ordered correlator
- Random-matrix like behavior for SYK_4
- One-body term destroys RMT behavior after some time
- Comparison to random-field XXZ model (many-body localization)

arXiv:1809.01671

New possibility: characterization of chaos by two-point functions? (in progress)