# SYK模型の準位統計と Lyapunov spectrum

Level statistics and Lyapunov spectrum of the SYK model

「離散的手法による場と時空のダイナミクス」研究会2018

東北大学、2018年9月11日 14:00 – 15:15

京都大学理学研究科　手塚真樹 (Masaki TEZUKA)

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- Introduction: the Sachdev-Ye-Kitaev (SYK) model
  - Experiment proposal [Danshita-Hanada-Tezuka PTEP 2017]
- Energy spectrum of the SYK model and quantum chaos
  - [Cotler et al., JHEP 2017], [Gharibyan et al., JHEP 2018]
  - Extension of the model [Garcia-Garcia et al., PRL 2018, arXiv:1801.03204]
- Lyapunov spectra of chaotic systems
  - Classical systems [Hanada-Shimada-Tezuka PRE 2018]
  - Quantum systems e.g. SYK [Gharibyan-Hanada-Swingle-MT, arXiv:1809.01671]
- Summary

# Plan of talk

$$\hat{H}_{\text{SYK}} = \frac{\sqrt{3!}}{N^{3/2}} \sum_{1 \le a<b<c<d \le N} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d$$

black hole

holography

non-gravitational quantum system

cold atom realization

Danshita, Hanada and MT, PTEP 2017 [1606.02454]

maximally chaotic (chaos limit)

maximally chaotic (chaos limit)

$\lambda_{\text{Lyapunov}}$

numerical analysis; relation to other scrambling systems

random matrix behavior of finite-time Lyapunov spectrum

[Classical] Hanada, Shimada and MT, PRE 2018 [1702.02197]

[Quantum] Gharibyan, Hanada, Swingle and MT, 1809.01671

modifications to study chaos / integrable transition, many-body localization

García-García et al., PRL 2018 [1707.02197]

García-García and MT, 1801.03204

Cotler et al., JHEP 2017 [1611.04650]

Gharibyan et al., JHEP 2018 [1803.08050]

# Collaborators

Jordan Saul Cotler, Guy Gur-Ari, Masanori Hanada, Joseph Polchinski,
Phil Saad, Stephen H. Shenker, Douglas Stanford, Alexandre Streicher
JHEP 1705, 118 (2017) (arXiv:1611.04650)

Antonio M. García-García, Bruno Loureiro, Aurelio Romero-Bermúdez
arXiv:1801.03204 PRL **120**, 241603 (2018) (arXiv:1707.02197)

Ippei Danshita
PTEP 2017, 083I01 (arXiv:1606.02454) also with M. Hanada
日本物理学会誌「最近の研究から」2018年8月号 p. 569-574

Hidehiko Shimada

Hrant Gharibyan

JHEP 1807, 124 (2018) (arXiv:1803.08050)
also with M. Hanada and S. H. Shenker;
arXiv:1809.01671 also with M. Hanada and Brian Swingle

Phys. Rev. E **97**, 022224 (2018)
also with M. Hanada

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# Introduction: the SYK model

Subir Sachdev 叶锦武 Алексей Китаев

$$\hat{H} = \frac{\sqrt{3!}}{N^{3/2}} \sum_{1\le a<b<c<d\le N} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d$$

$\{\hat{\chi}_a, \hat{\chi}_b\} = \delta_{ab}$

Gaussian random

$\langle J_{abcd}\rangle^2 = J^2 = 1$

Analytically solvable in $N \gg 1$ limit, and in the $T \to 0$ limit,

- Satisfies the "chaos bound" [Kitaev's talks at KITP; S. Sachdev, PRX **5**, 041025 (2015); …]
  - "chaos bound": Under "physically reasonable" assumption, $\lambda_{\mathrm{L}} \le \frac{2\pi k_{\mathrm{B}} T}{\hbar}$

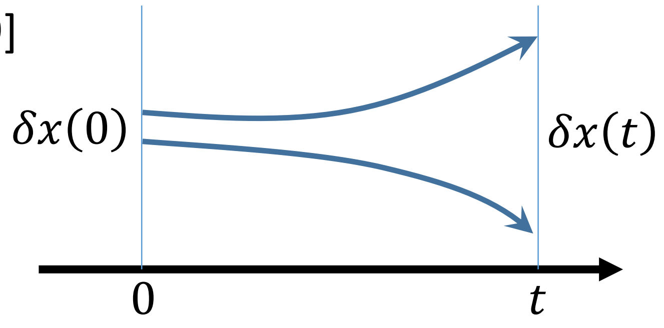
    [J. Maldacena, S. H. Shenker, and D. Stanford, JHEP **1608**, 106 (2016)]

Lyapunov exponent $\lambda_{\mathrm{L}}$ : Defined using out-of-time ordered correlator (OTOC)

[N. Wiener 1938][Larkin & Ovchinnikov 1969]

$$C_T(t) = -\langle[\hat{x}(t), \hat{p}(0)]^2\rangle$$

$$\sim \{x(t), p(0)\}_{\mathrm{PB}}{}^2 = \left(\frac{\partial x(t)}{\partial x(0)}\right)^2 \to e^{2\lambda_{\mathrm{L}} t}$$



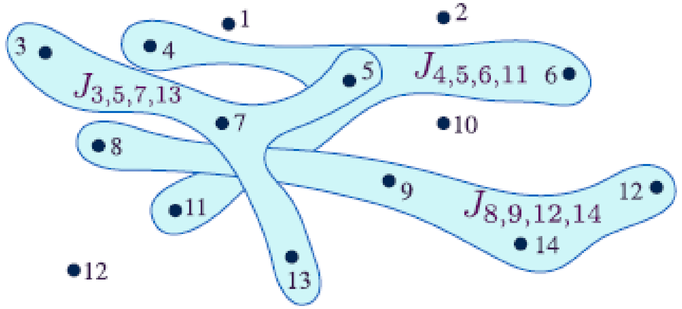

$$C(t) = \langle[W(t), V(t=0)]^2\rangle$$
$$= \langle W(t)V(0)W(t)V(0)\rangle + \cdots$$

- Corresponding system with gravity?: Not conclusive so far
  - Models of 1+1D gravity: Jackiw-Teitelboim model, CGHS model, etc.

# Holographic connection to black hole (BH) physics

[Sachdev PRL 2010, PRX 2015; Maldacena-Stanford, Hosur-Qi-Roberts-Yoshida; Polchinski-Rosenhaus, …]

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$




$$\mathcal{Q} = \frac{1}{N} \sum_i \left\langle c_i^\dagger c_i \right\rangle .$$

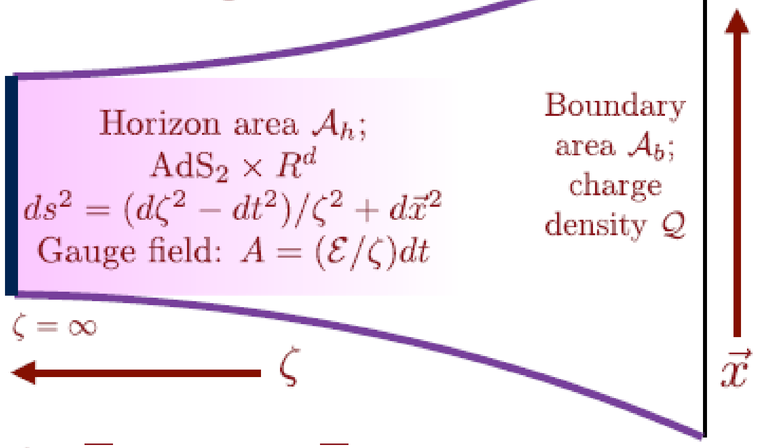
$$-\left\langle c_i(\tau) c_i^\dagger(0) \right\rangle \sim \begin{cases} -\tau^{-1/2} & , \ \tau > 0 \\ e^{-2\pi\mathcal{E}} \, |\tau|^{-1/2} , & \tau < 0. \end{cases}$$

Known "equation of state" determines $\mathcal{E}$ as a function of $\mathcal{Q}$

Microscopic zero temperature entropy density $\mathcal{S}$ obeys

$$\frac{\partial \mathcal{S}}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$$

Einstein-Maxwell theory + cosmological constant




$$\mathcal{L} = \overline{\psi}\Gamma^\alpha D_\alpha \psi + m\overline{\psi}\psi$$

$$-\left\langle \psi(\tau) \overline{\psi}(0) \right\rangle \sim \begin{cases} -\tau^{-1/2} & , \ \tau > 0 \\ e^{-2\pi\mathcal{E}} \, |\tau|^{-1/2} , & \tau < 0. \end{cases}$$

"Equation of state" relating $\mathcal{E}$ and $\mathcal{Q}$ depends upon the geometry of spacetime far from the $AdS_2$

Black hole thermodynamics (classical general relativity) yields

$$\frac{\partial \mathcal{S}_{BH}}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$$

[S. Sachdev, Phys. Rev. X **5**, 041025 (2015)]

# Feynman diagrams

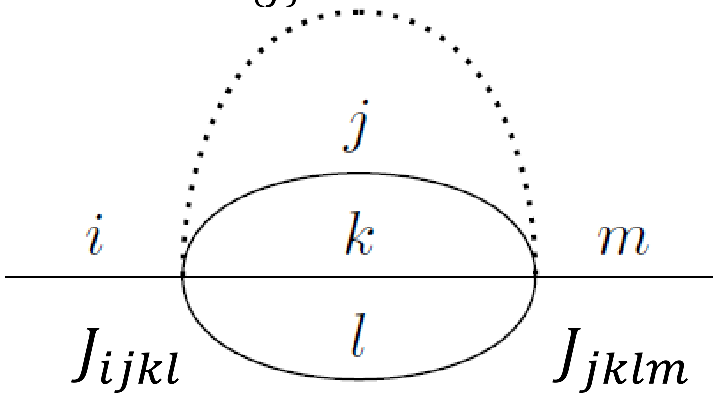
[J. Polchinski and V. Rosenhaus, JHEP 1604 (2016) 001]
[J. Maldacena and D. Stanford, PRD **94**, 106002 (2016)]

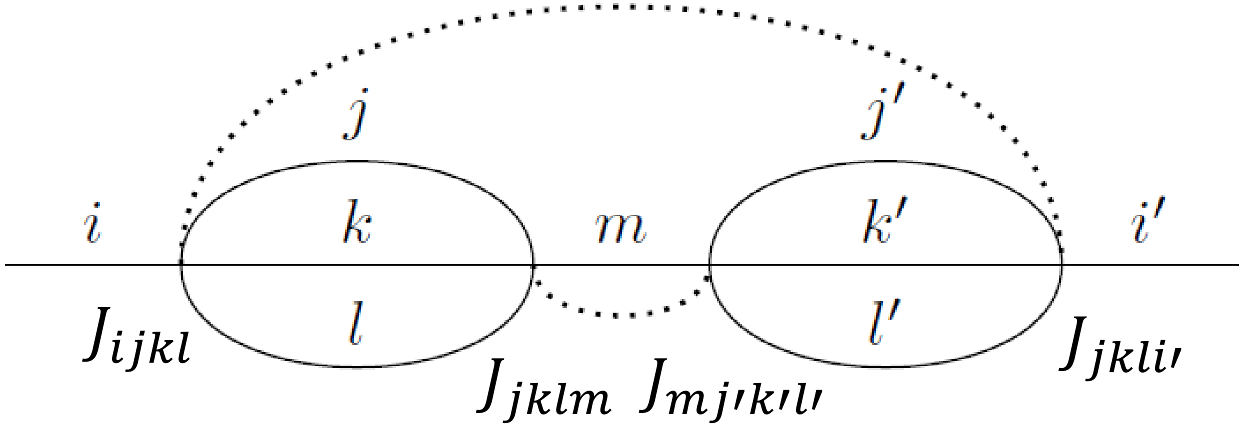
$$\hat{H} = \frac{\sqrt{3!}}{N^{3/2}} \sum_{1 \le a < b < c < d \le N} J_{abcd} \hat{\chi}_a \hat{\chi}_b \hat{\chi}_c \hat{\chi}_d$$

$$\{\hat{\chi}_a, \hat{\chi}_b\} = \delta_{ab}$$

$$\langle J_{abcd} \rangle^2 = J^2 = 1$$

Sample average $\langle \cdots \rangle_{\{J\}}$




$$\sum_{jkl} \langle J_{ijkl} J_{jklm} \rangle_{\{J\}} = \frac{N^3}{3!} \delta_{im}$$

$$\sum_{m \neq i} \sum_{jklj'k'l'} \langle J_{ijkl} J_{jklm} J_{mj'k'l'} J_{j'k'l'i'} \rangle_{\{J\}} \propto N^4 \delta_{ii'}$$

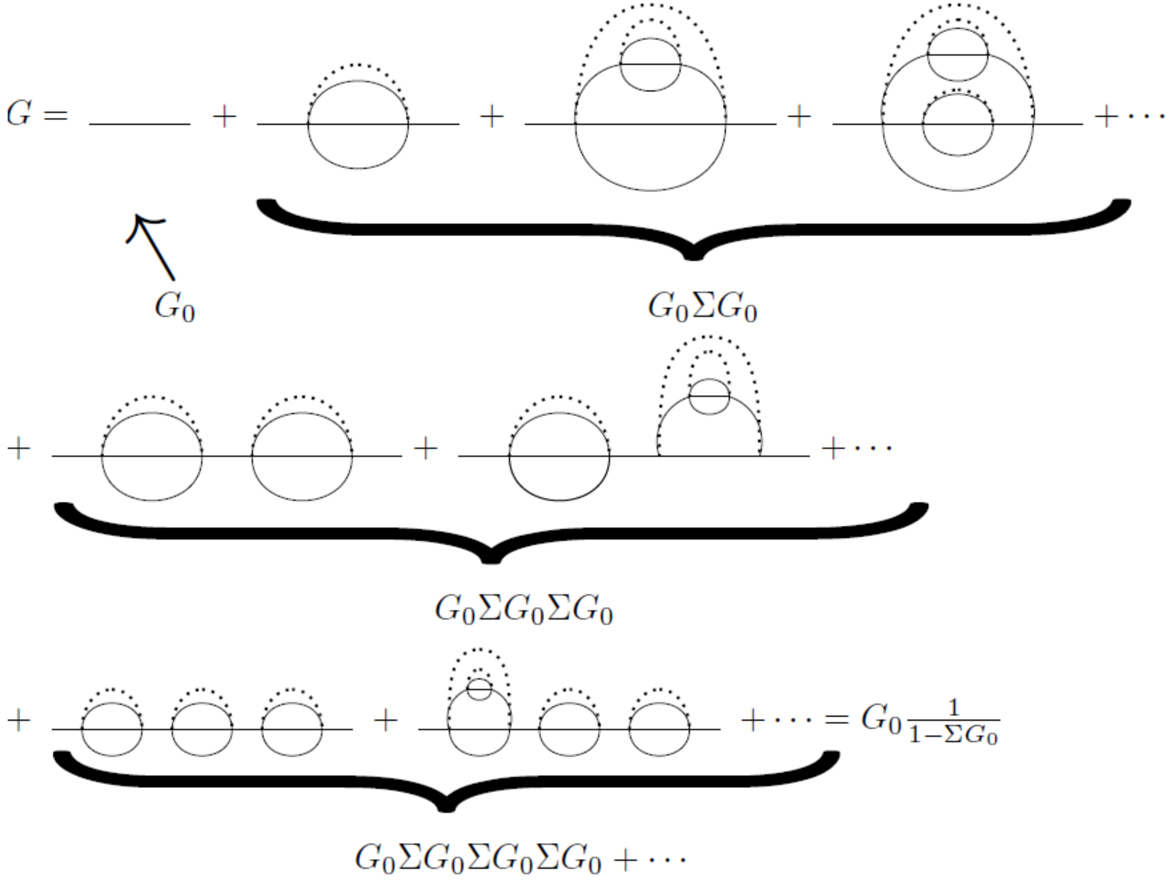
→ $O(N^0)$ contribution

→ $O(N^{-2})$ contribution

Large-$N$: “Melon diagrams” dominate

[J. Polchinski and V. Rosenhaus, JHEP 1604 (2016) 001]
[J. Maldacena and D. Stanford, PRD **94**, 106002 (2016)]

# $O(N^0)$ terms




$$G(1-\Sigma G_0) = G_0$$

$$G^{-1} = {G_0}^{-1} - \Sigma$$

$$G(i\omega)^{-1} = \boxed{i\omega} - \Sigma(i\omega) \qquad \Sigma = J^2 G^3$$

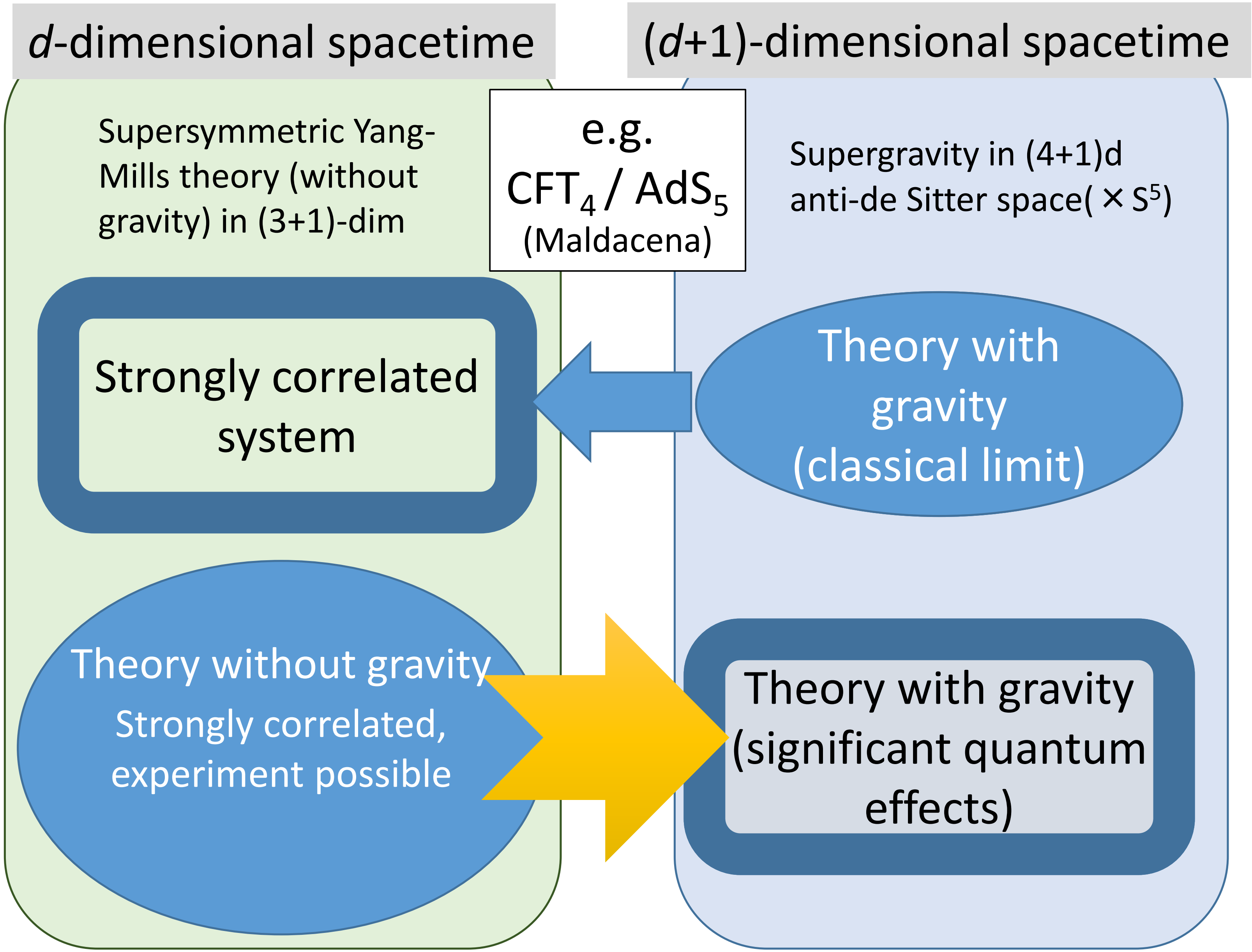
Low energy ($\omega, T \ll J$): ignore $i\omega$ and we have

$$\int dt G(t_1,t)\Sigma(t,t_2) = -\delta(t_1,t_2)$$

$$J^2 \int dt G(t_1,t) G(t,t_2)^3 = -\delta(t_1,t_2)$$

$$G(t) = -\left(\frac{1}{4\pi J^2}\right)^{1/4} \frac{\mathrm{sgn}(t)}{\sqrt{t}}$$

# Holography: how we use it




◎ Not limited to classical limit

➔ Several supporting evidences

e.g. check of the leading gravity correction for the black hole mass [M. Hanada, Y. Hyakutake, G. ishiki, and J. Nishimura, Science **344**, 882 (2013)]

Many "AdS/CMT" applications

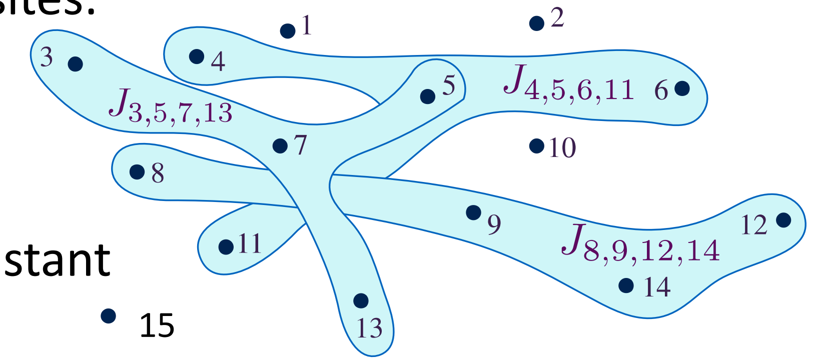
This work:
approach quantum gravity by realizing corresponding non-gravity models in cold gases

# (complex fermion) Sachdev-Ye-Kitaev model

A. Kitaev, KITP talks (2015);
S. Sachdev, PRX (2015)

is a model of *Q* spin-polarized fermions on *N* sites:

$$\hat{H} = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,l=1}^{N} J_{ij;kl} \hat{c}_i^\dagger \hat{c}_j^\dagger \hat{c}_k \hat{c}_l,$$




where $J_{ij;kl}$ is a complex random coupling constant that satisfies

$$J_{ij;kl} = -J_{ji;kl} = -J_{ij;lk}, \quad J_{ij;kl} = J^*_{kl;ij}, \quad \overline{\mathrm{Re}\ J_{ij;kl}} = \overline{\mathrm{Im}\ J_{ij;kl}} = 0, \quad \text{and}$$

$$\overline{(\mathrm{Re}\ J_{ij;kl})^2} = \begin{cases} J^2/2 & (\{i,j\} \neq \{k,l\}) \\ J^2 & (\{i,j\} = \{k,l\}) \end{cases}, \quad \overline{(\mathrm{Im}\ J_{ij;kl})^2} = \begin{cases} J^2/2 & (\{i,j\} \neq \{k,l\}) \\ 0 & (\{i,j\} = \{k,l\}) \end{cases}.$$

- No supersymmetry (bosons ⬄ fermions) needed
- Not relativistic, no antiparticles
- Spinless fermions can be used

➔ Experimental realization?

Different from other approaches to (classical) gravity in cold gases

Hawking radiation in sonic analogue of BEC
[J. Steinhauer, Nature Phys. **10**, 864 (2014); **12**, 959 (2016)]
cf. theory [W.G. Unruh, PRL **46**, 1351 (1981)]

Sakharov Oscillations in quenched BEC
[C.-L. Hung, V. Gurarie, and C. Chin, Science **341**, 1213 (2013)]

Creating the SY state of the SYK model in experiment:
equivalent to creating the dual quantum black hole!

# Experiment proposal for SYK

arXiv:1606.02454
[I. Danshita, M. Hanada, and MT, PTEP 2017]

Here we take the complex fermion version:

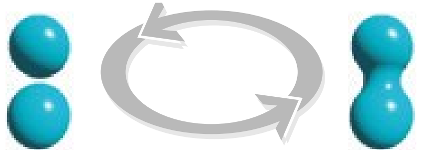
$$\hat{H} = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,l=1}^{N} J_{ij;kl} \hat{c}_i^\dagger \hat{c}_j^\dagger \hat{c}_k \hat{c}_l,$$

$$\hat{H}_{\rm m} = \sum_{s=1}^{n_s} \left\{ \nu_s \hat{m}_s^\dagger \hat{m}_s + \sum_{i,j} g_{s,ij} \left( \hat{m}_s^\dagger \hat{c}_i \hat{c}_j - \hat{m}_s \hat{c}_i^\dagger \hat{c}_j^\dagger \right) \right\}.$$

Trace out molecular d.o.f.:

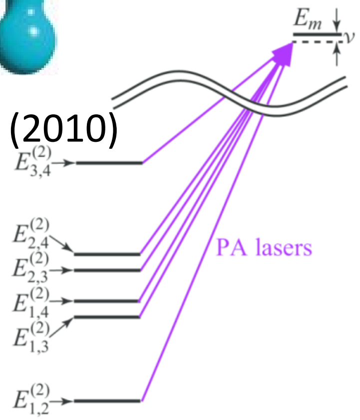
$$\hat{H}_{\rm eff} = \sum_{s,i,j,k,l} \frac{g_{s,ij} g_{s,kl}}{\nu_s} \hat{c}_i^\dagger \hat{c}_j^\dagger \hat{c}_k \hat{c}_l$$

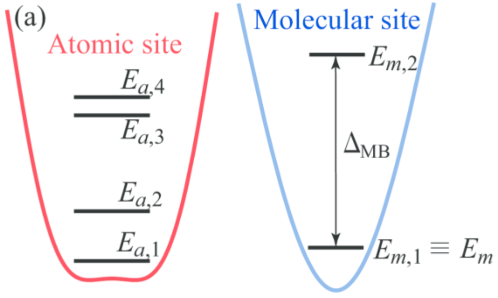
Photoassociation / dissociation



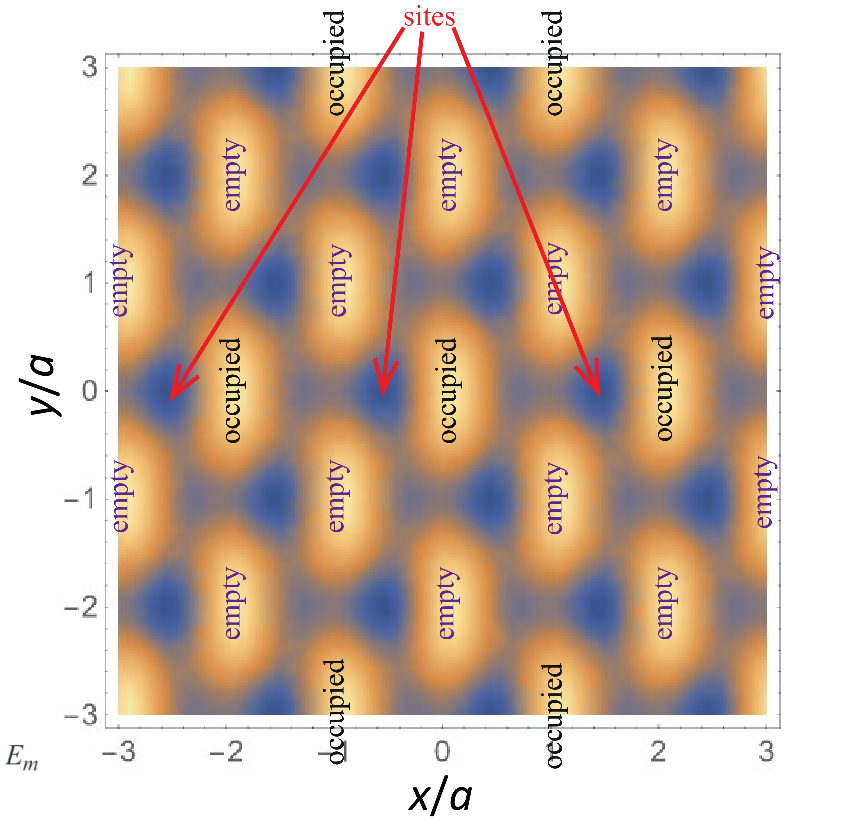

B. Butscher et al., NP (2010)

Sum of the energies of two atoms



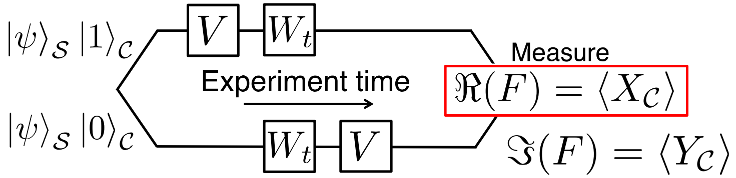

Optical lattice of non-degenerate sites




OTOC measurement protocol

$$F(t) = \langle \hat{W}^\dagger(t) \hat{V}^\dagger(0) \hat{W}(t) \hat{V}(0) \rangle$$

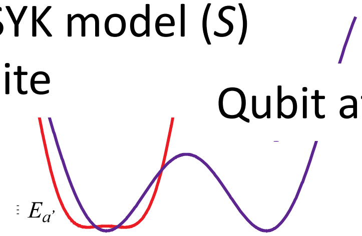
"Interferometric protocol"
(Swingle et al., PRA 2016)

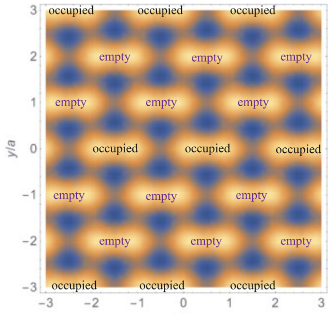



$$|\Psi\rangle = \hat{W}(t)\hat{V}(0)|\psi\rangle_S |1\rangle_C + \hat{V}(0)\hat{W}(t)|\psi\rangle_S |0\rangle_C$$

SYK model ($S$) site

Qubit atom potential

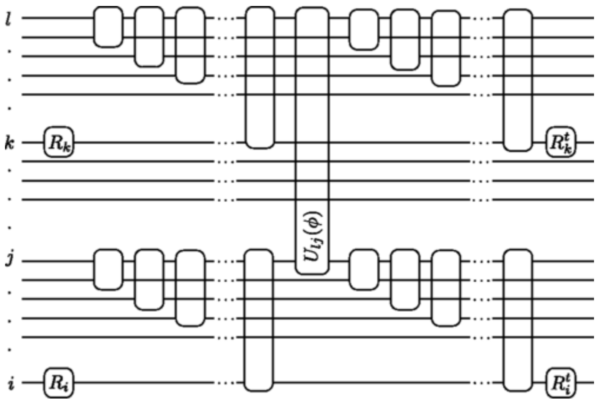


# Other proposals (after our arXiv:1606.02454)

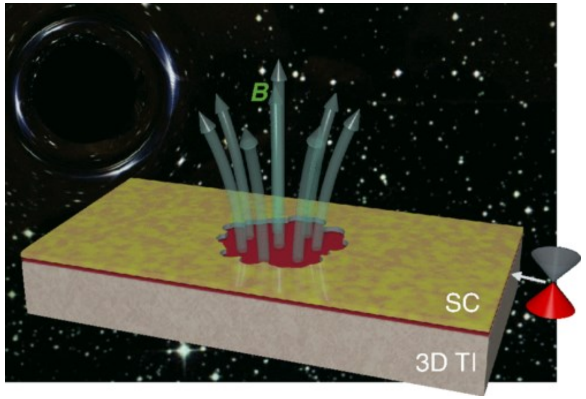
Review: M. Franz and M. Rozali, “Mimicking black hole event horizons in atomic and solid-state systems” arXiv:1808.00541 to appear in Nature Materials

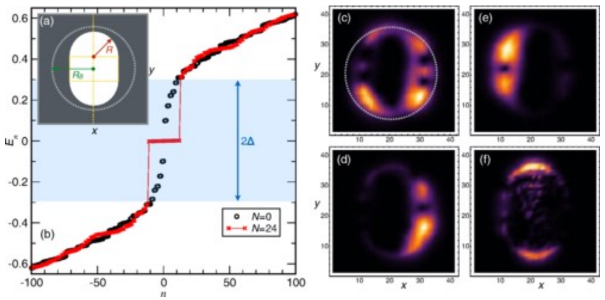
arXiv:1607.08560




L. García-Álvarez, I. L. Egusquiza, L. Lamata, A. del Campo, J. Sonner, and E. Solano, “Digital Quantum Simulation of Minimal AdS/CFT”, PRL **119**, 040501 (2017)

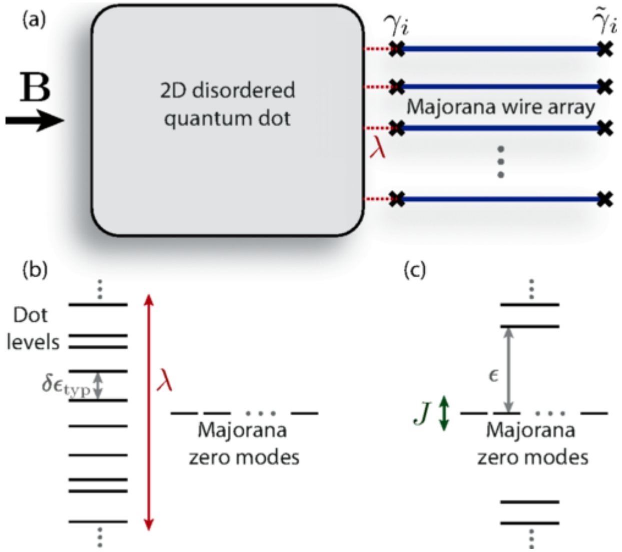
arXiv:1702.04426



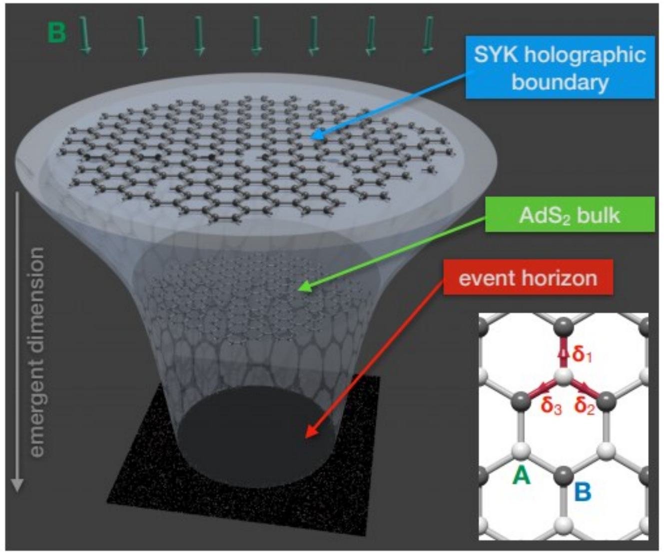

D. I. Pikulin and M. Franz, “Black Hole on a Chip: Proposal for a Physical Realization of the Sachdev-Ye-Kitaev model in a Solid-State System”, PRX **7**, 031006 (2017)

arXiv:1703.06890




Aaron Chew, Andrew Essin, and Jason Alicea, “Approximating the Sachdev-Ye-Kitaev model with Majorana wires”, PRB **96**, 121119(R) (2017)

arXiv:1802.00802




Anffany Chen, R. Ilan, F. de Juan, D.I. Pikulin, M. Franz, “Quantum holography in a graphene flake with an irregular boundary”, PRL **121**, 036403 (2018)

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numerical analysis; relation to other scrambling systems

Cotler et al., JHEP 2017 [1611.04650]

Gharibyan et al., JHEP 2018 [1803.08050]

# "Black Holes and Random Matrices"

[Cotler, Gur-Ari, Hanada, Polchinski, Saad, Shenker, Stanford, Streicher, and MT, JHEP 1705, 118 (2017) (arXiv:1611.04650)]

## Collaborators

Jordan Saul Cotler, Guy Gur-Ari, Masanori Hanada, Joseph Polchinski,

Phil Saad, Stephen H. Shenker, Douglas Stanford, Alexandre Streicher

## Videos (on "videosfromIAS")

Natifest – 17 September 2016 "Black Holes and Random Matrices" by **Stephan Shenker**

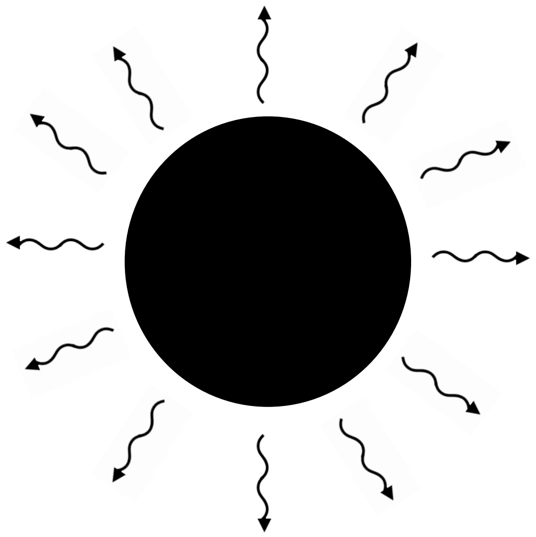
https://youtu.be/3qEoMBZLf10

26 October 2016 "The Sachdev-Ye-Kitaev quantum mechanics model, black holes, and random matrices" by **Douglas Stanford**

https://youtu.be/hK2S-pyAf0c

Here we present our numerical results for finite $N$, focusing on the spectral statistics

# Information loss problem of black holes



Assume gauge / gravity correspondence

Study the real-time dynamics of the gauge theory side

Finite size: What to learn from the discrete energy spectrum?

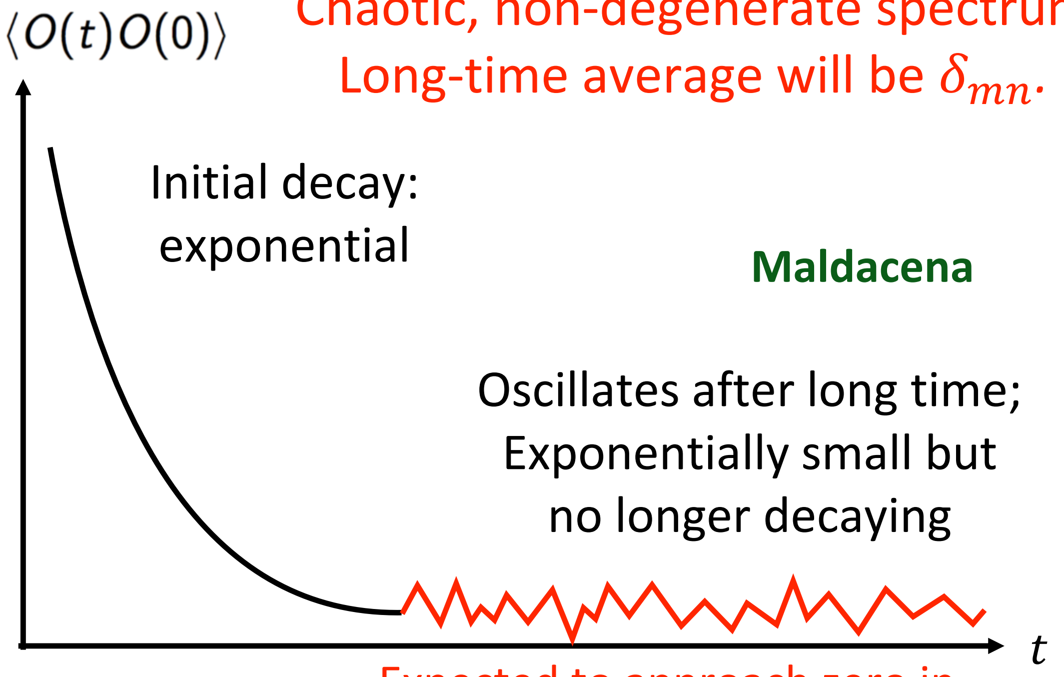
# Spectral form factor

$\beta = (k_B T)^{-1}$

Consider some real-time correlator:

$$\langle O(t)O(0)\rangle = \mathrm{tr}\left(e^{-\beta H}O(t)O(0)\right)/\mathrm{tr}e^{-\beta H}$$
$$= \sum_{m,n} e^{-\beta E_m}|\langle m|O|n\rangle|^2 e^{i(E_m-E_n)t} / \sum_n e^{-\beta E_n}$$

Chaotic, non-degenerate spectrum
Long-time average will be $\delta_{mn}$.




Initial decay: exponential

**Maldacena**

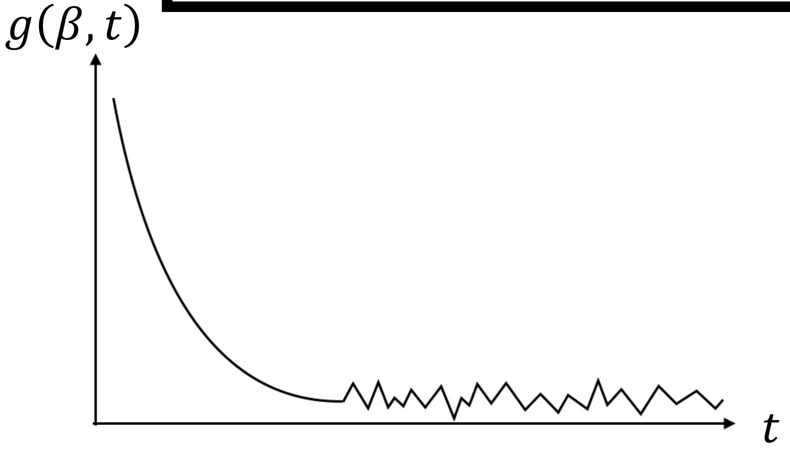
Oscillates after long time;
Exponentially small but
no longer decaying

Expected to approach zero in classical gravity;
Should reflect features of quantum gravity

Spectral form factor $g(\beta, t)$

$$\sum_{m,n} e^{-\beta(E_m+E_n)} e^{i(E_m-E_n)t} = Z(\beta+it)Z(\beta-it) = Z(t)Z^*(t)$$

$Z$: analytically continued partition function




(The initial behavior may differ from correlators)

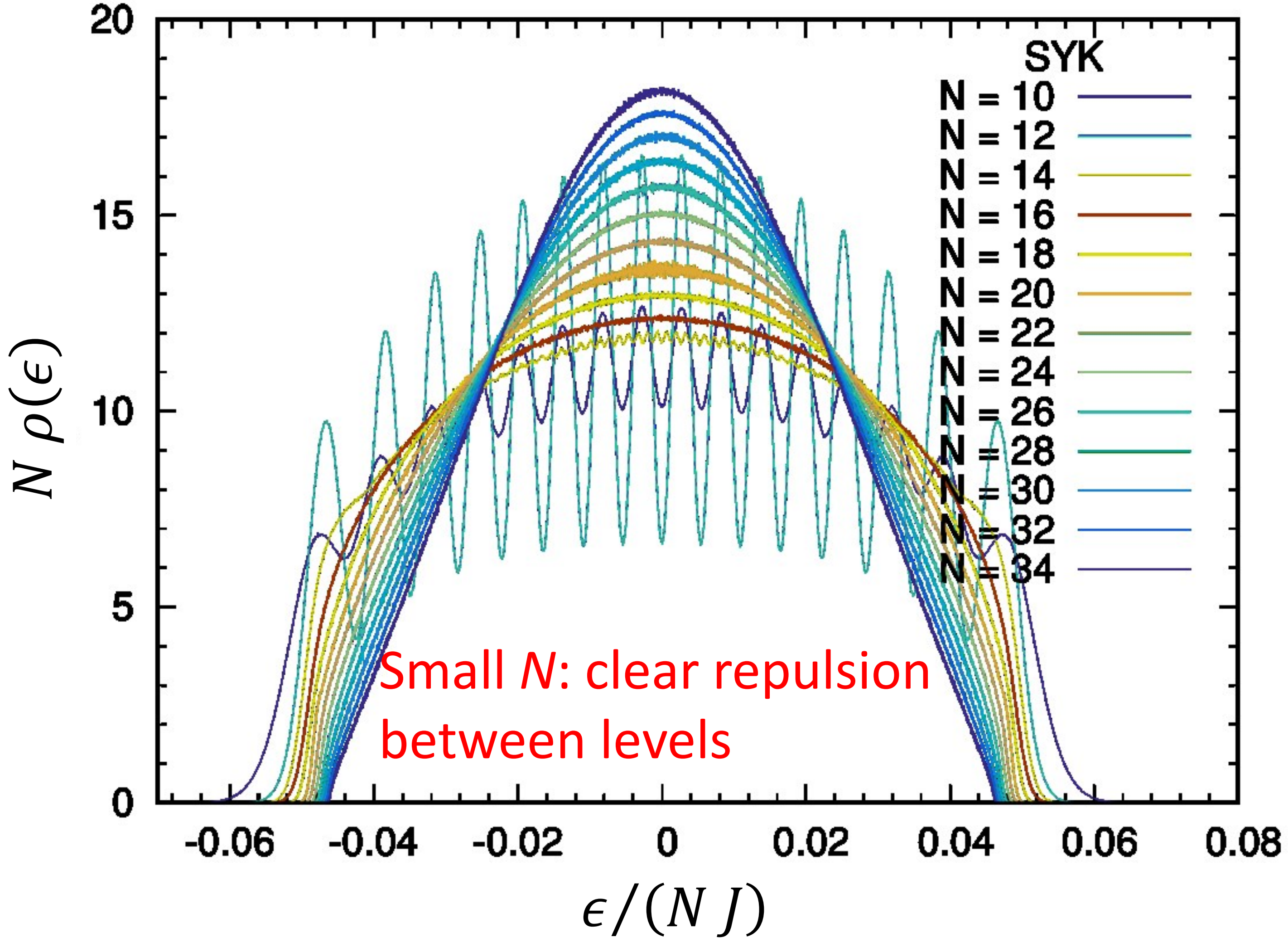
**Papadodimas-Raju; RMT literature**

$Z(\beta)^2$ at $t = 0$ $\rightarrow$ $Z(2\beta)$ at late time

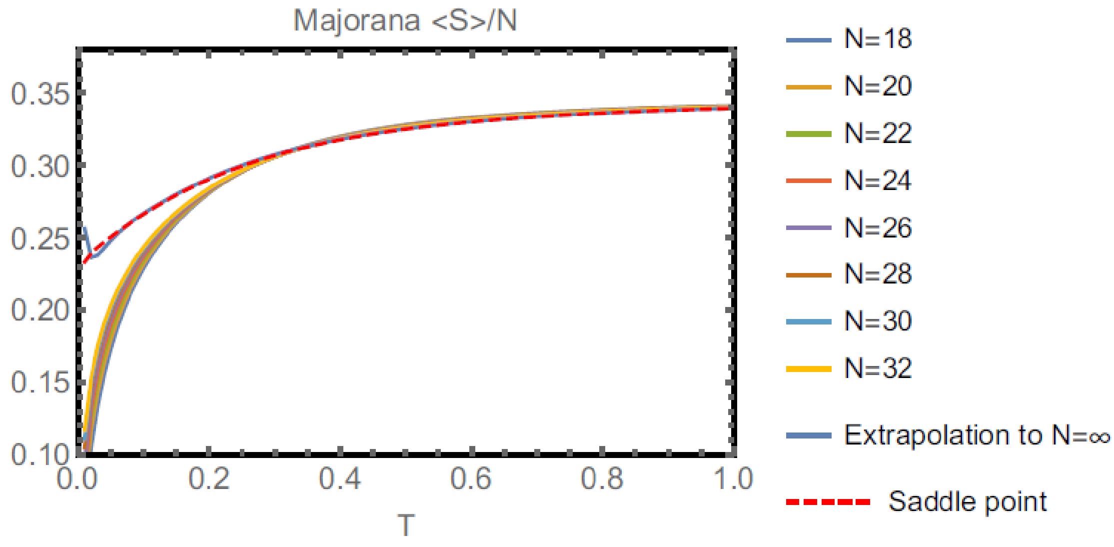
(Larger value than naively estimated from
the correspondence between Super Yang-Mills
and superstring theories)

# SYK, not too large $N$: Numerically diagonalize the Hamiltonian for energy spectrum

$$\hat{H} = \frac{\sqrt{3!}}{N^{3/2}} \sum_{1 \le a<b<c<d \le N} J_{abcd} \hat{\chi}_a \hat{\chi}_b \hat{\chi}_c \hat{\chi}_d \qquad J_{abcd} : \text{Gaussian and variance } \sigma^2 = J^2$$




Small $N$: clear repulsion between levels




Large $N$ limit: the entropy remains finite at low $T$ limit

# Gaussian random matrices

$$\left(a_{ij}\right)_{i,j=1}^{L}$$

$$a_{ij} = a_{ji}^*$$

Real (β=1): Gaussian Orthogonal Ensemble (GOE)
Complex (β=2): G. Unitary E. (GUE)
Quaternion (β=4): G. Symplectic E. (GSE)

Density $\propto e^{-\frac{\beta K}{4}\mathrm{Tr}H^2} = \exp\left(-\frac{\beta K}{4}\sum_{i,j}^{K}\left|a_{ij}\right|^2\right)$

[F. J. Dyson, J. Math. Phys. **3**, 1199 (1962)]

Eigenvalue distribution

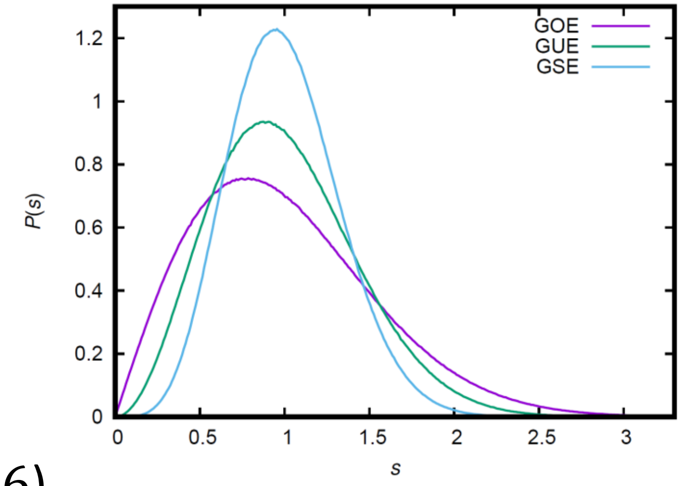
Level repulsion

$$p(e_1, e_2, \dots, e_K) \propto \prod_{1\le i<j\le K} \left|e_i - e_j\right|^{\beta} \prod_{i=1}^{K} e^{-\beta K {e_i}^2/4}$$

$P(s)$ : level separation distribution

Uncorrelated: Poisson ($e^{-s}$)

GOE/GUE/GSE: $\propto s^{\beta}$ at small $s$, $e^{-s^2}$ tail




$$r = \frac{\min(e_{i+1} - e_i\,,\ \ e_{i+2} - e_{i+1})}{\max(e_{i+1} - e_i\,,\ \ e_{i+2} - e_{i+1})}$$

$\langle r \rangle$ : mean neighboring gap ratio

Uncorrelated: Poisson ($2\log 2 - 1 \approx 0.386$)

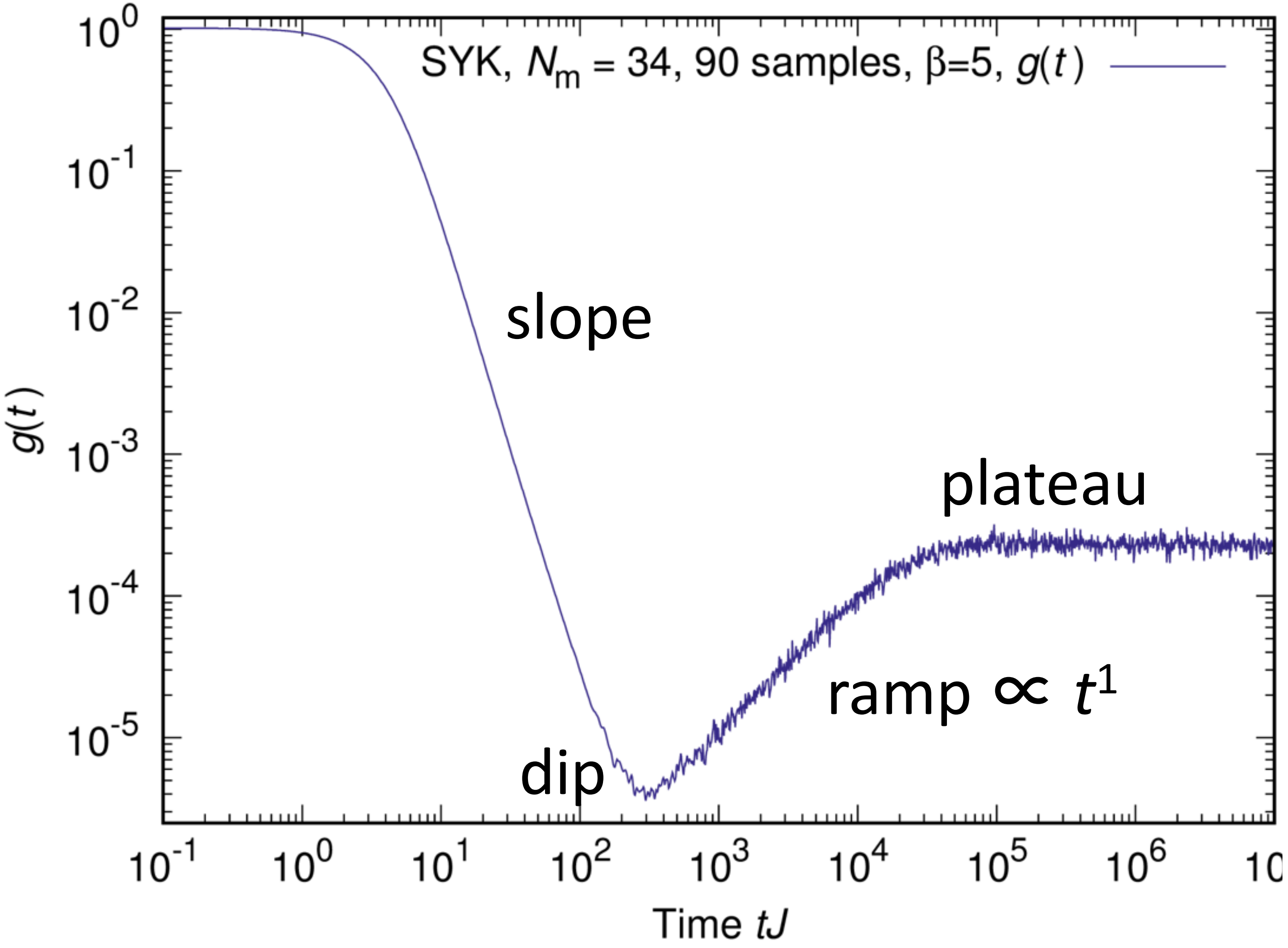
GOE/GUE/GSE: larger (*e.g.* $0.599$ for GUE [Y. Y. Atas *et al.* PRL 2013])

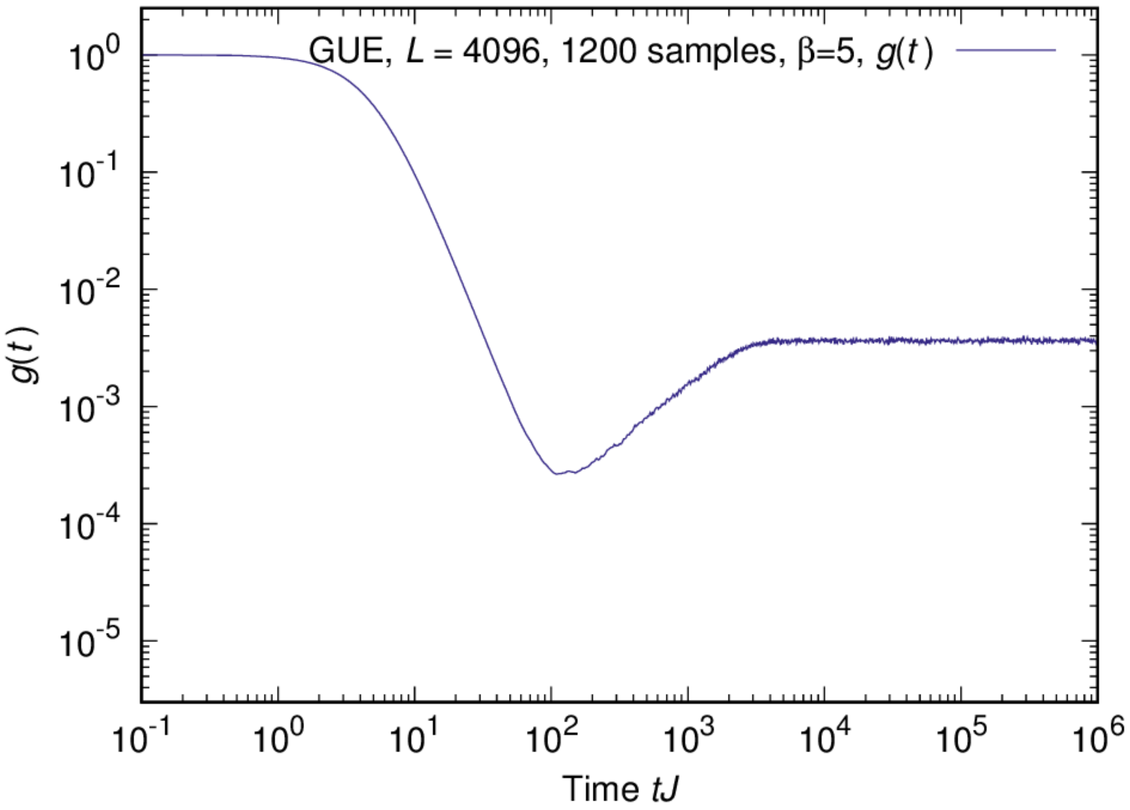
# Spectral form factor

$$g(\beta, t) = \frac{\langle |Z(\beta, t)|^2 \rangle_J}{\langle Z(\beta) \rangle_J^{\,2}}$$

$$Z(\beta, t) = Z(\beta + \mathrm{i}t) = \mathrm{Tr}\left(\mathrm{e}^{-\beta \hat{H} - \mathrm{i}\hat{H}t}\right)$$

$$\beta = (k_\mathrm{B} T)^{-1}$$



# Slope-dip-ramp-plateau structure of $g(\beta, t)$

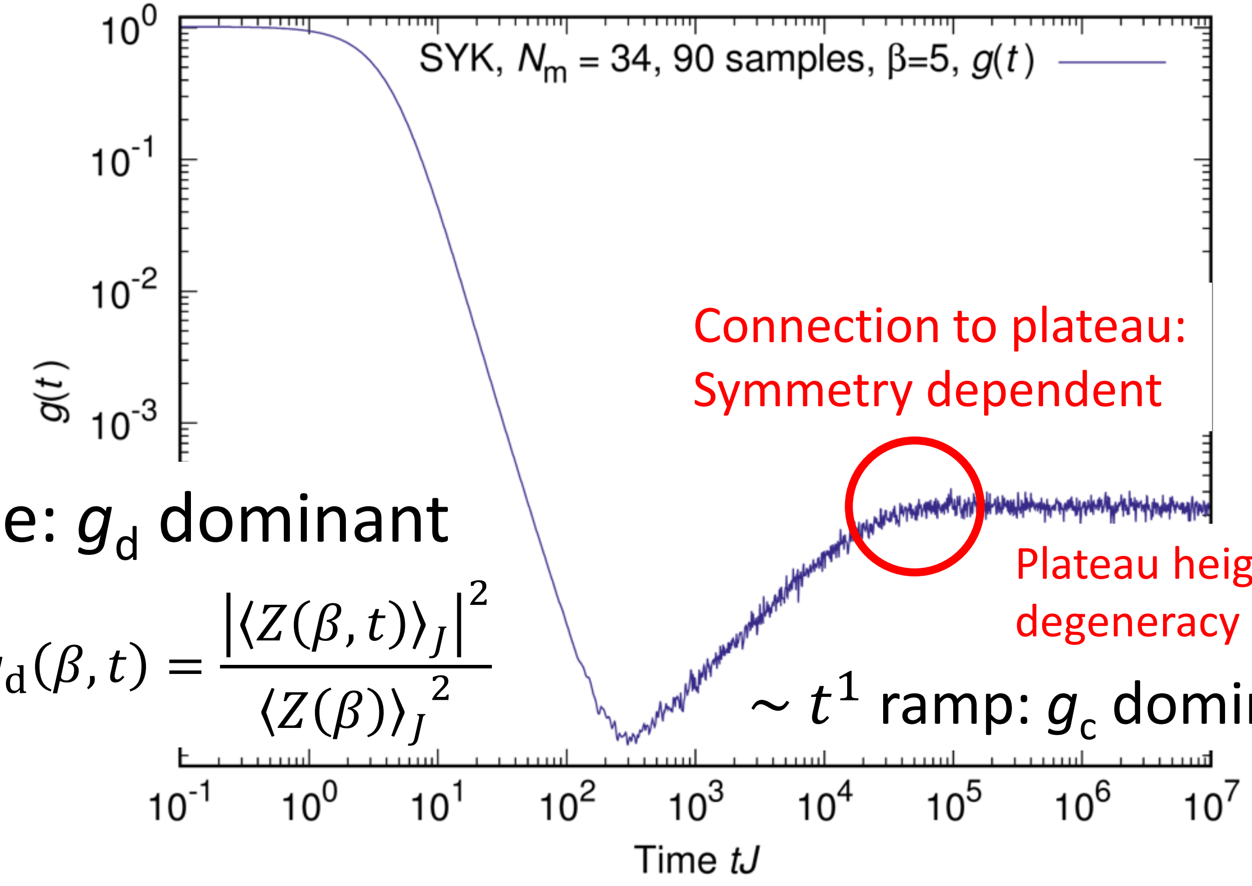
Early time: $\sim t^{-3}$
(depends on the overall density of states)

$$Z(\beta, t) = \mathrm{Tr}\left(\mathrm{e}^{-\beta \hat{H} - \mathrm{i}\hat{H}t}\right)$$

$$g(\beta, t) = \frac{\langle |Z(\beta, t)|^2 \rangle_J}{\langle Z(\beta) \rangle_J^{\ 2}}$$

$$g_{\mathrm{c}}(\beta, t) = \frac{\langle |Z(\beta, t)|^2 \rangle_J - \left|\langle Z(\beta, t) \rangle_J\right|^2}{\langle Z(\beta) \rangle_J^{\ 2}}$$

$$\sim \iint d\lambda_1 d\lambda_2 \langle \delta\rho(\lambda_1)\delta\rho(\lambda_2) \rangle e^{\mathrm{i}t(\lambda_1 - \lambda_2)}$$




Connection to plateau:
Symmetry dependent

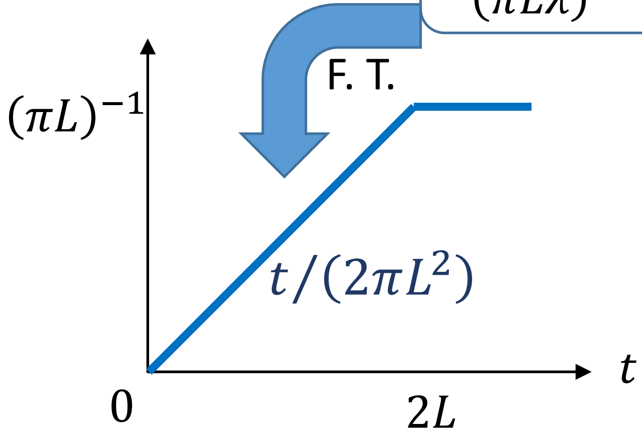
Slope: $g_{\mathrm{d}}$ dominant

$$g_{\mathrm{d}}(\beta, t) = \frac{\left|\langle Z(\beta, t) \rangle_J\right|^2}{\langle Z(\beta) \rangle_J^{\ 2}}$$

Plateau height:
degeneracy

$\sim t^1$ ramp: $g_{\mathrm{c}}$ dominant

$$R(\lambda) = \langle \delta\rho(\lambda_1)\delta\rho(\lambda_1 - \lambda) \rangle = -\frac{\sin^2 L\lambda}{(\pi L \lambda)^2} + \frac{1}{\pi L}\delta(\lambda)$$

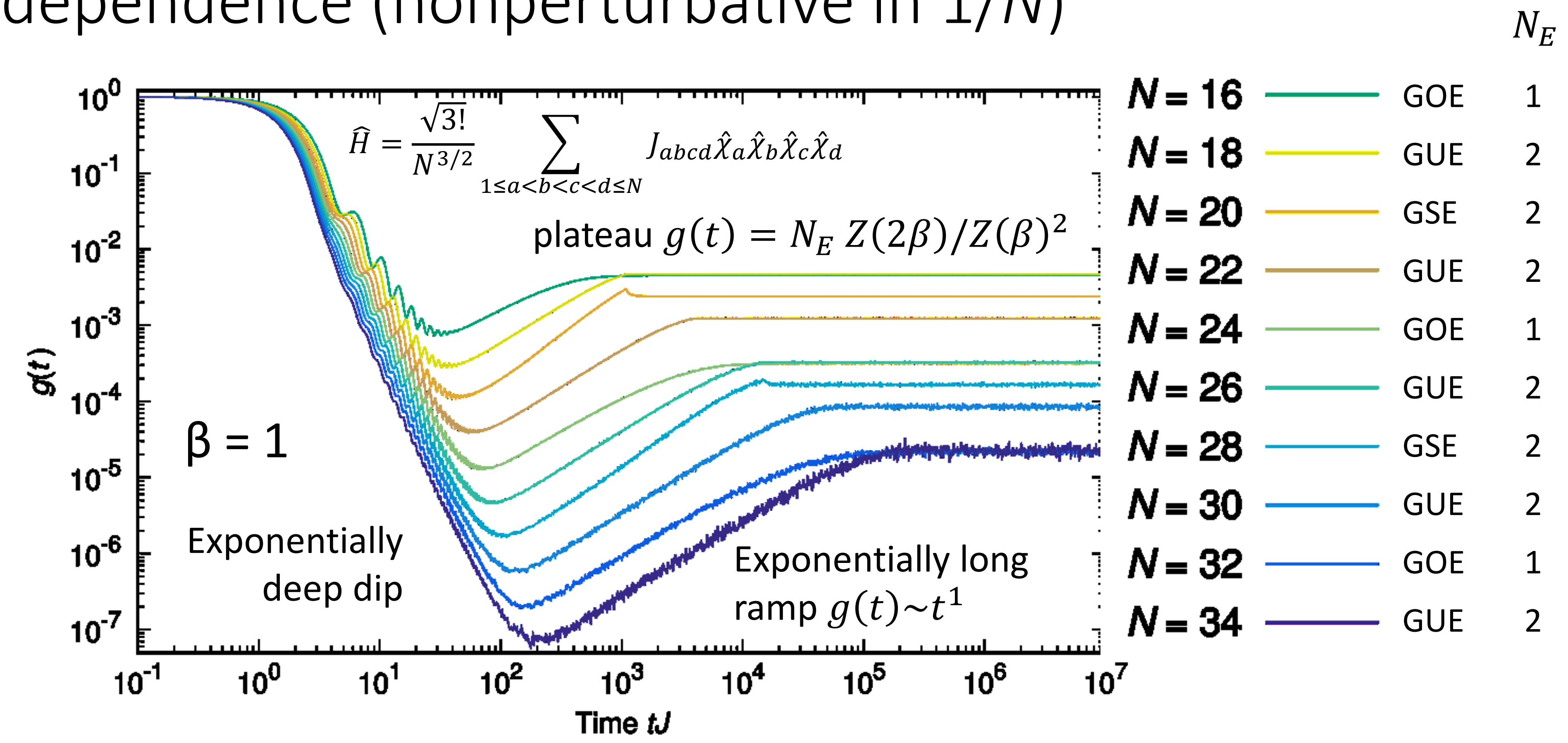



in RMT

Long, linear ramp $\sim t^1$

➔ Strong spectral rigidity in the SYK model

# $g(t)$: $N$ dependence (nonperturbative in $1/N$)




SPT (Symmetrically Protected Topological) phase classification for class BDI: Z → $Z_8$ due to interaction
[L. Fidkowski and A. Kitaev: PRB **81**, 134509 (2010); PRB **83**, 075103 (2011)]

Level statistics of the many-body system ← Corresponding (dense) random matrix
[Y.-Z. You, A. W. W. Ludwig, and Cenke Xu, PRB **95**, 115150 (2017)]

| $N_\chi$ (mod 8) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| qdim | 1 | $\sqrt{2}$ | 2 | $2\sqrt{2}$ | 2 | $2\sqrt{2}$ | 2 | $\sqrt{2}$ |
| lev. stat. | GOE | GOE | GUE | GSE | GSE | GSE | GUE | GOE |

# degeneracy

[L. Fidkowski and A. Kitaev: PRB **83**, 075103 (2011)]
[W. Fu and S. Sachdev: PRB **94**, 035135 (2016)]
[Y. Z. You, A. W. W. Ludwig, and C. Xu: PRB **95**, 115150 (2017)]

$$P = K\prod_{j=1}^{N_\mathrm{D}}\left(\hat{c}_j^\dagger + \hat{c}_j\right), \hat{c}_j = \frac{(\chi_{2j-1} + \mathrm{i}\chi_{2j})}{\sqrt{2}}, N_\mathrm{D} = N/2$$

$K$: Complex conjugate
$\hat{c}_j$: Complex (Dirac) fermion

$$\left(\hat{c}_j^\dagger + \hat{c}_j\right)^2 = \{\hat{c}_j^\dagger, \hat{c}_j\} = 1, \qquad \left(\hat{c}_j^\dagger + \hat{c}_j\right)\left(\hat{c}_k^\dagger + \hat{c}_k\right) = -\left(\hat{c}_k^\dagger + \hat{c}_k\right)\left(\hat{c}_j^\dagger + \hat{c}_j\right) \ (j \neq k)$$

$$P^2 = (-1)^{N_\mathrm{D}(N_\mathrm{D}-1)/2} = \begin{cases} +1 & (N_\mathrm{D} \bmod 4 = 0,1) \\ -1 & (N_\mathrm{D} \bmod 4 = 2,3) \end{cases}$$

$$P\hat{c}_j^\dagger P = \eta\hat{c}_j, P\hat{c}_j P = \eta\hat{c}_j^\dagger, \eta = (-1)^{(N_\mathrm{D}-1)}P^2 = \begin{cases} +1 & (N_\mathrm{D} \bmod 4 = 1,2) \\ -1 & (N_\mathrm{D} \bmod 4 = 0,3) \end{cases}$$

$$\therefore P\chi P = \eta\chi, PHP = \eta^4 H = H$$

$$[H, P] = 0$$

$\chi\chi\chi\chi$ preserves the parity of complex fermions

$N \equiv 0 \pmod 8$ : $P$ maps e⇔e, o⇔o and $P^2 = 1$ (not degenerate)
$N \equiv 2 \pmod 8$ : $P$ maps e⇔o and $\langle \mathrm{even}|\chi|\mathrm{odd}\rangle$ finite
$N \equiv 4 \pmod 8$ : $P$ maps e⇔e, o⇔o and $P^2 = -1$
$N \equiv 6 \pmod 8$ : $P$ maps e⇔o but $\langle \mathrm{even}|\chi|\mathrm{odd}\rangle = 0$

(N ≡ 2, 4, 6 mod 8): Twofold degenerate eigenstates

# Correlation functions

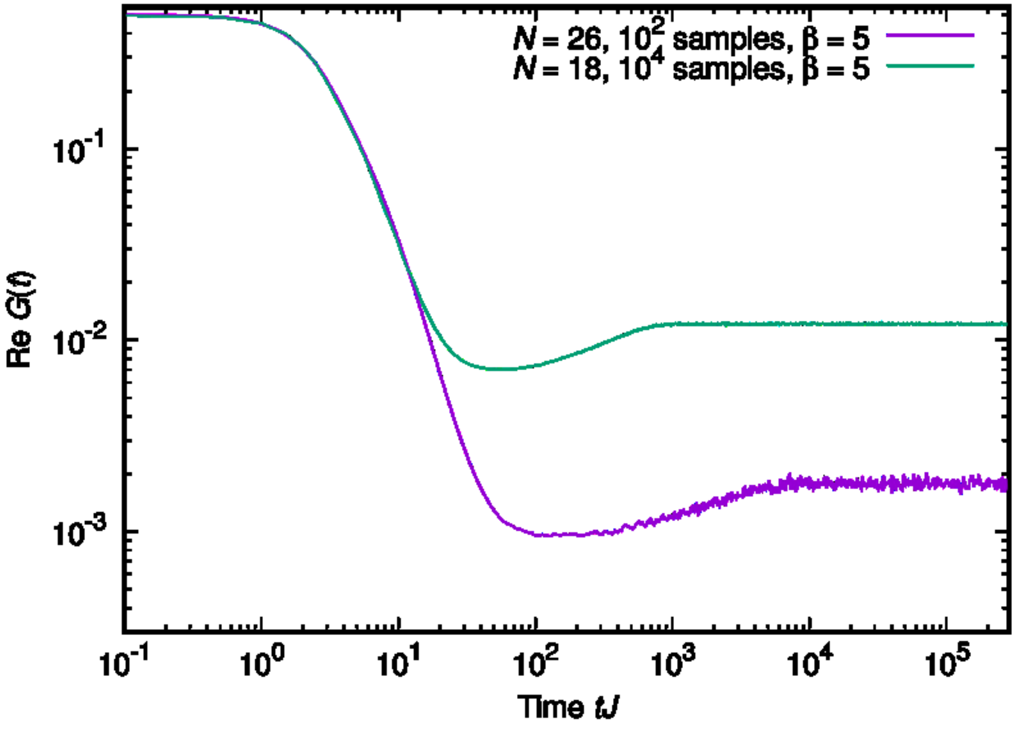
$N \equiv 0 \pmod 8$ : $P$ maps e⇔e, o⇔o and $P^2 = 1$ (no degeneracy)
$N \equiv 2 \pmod 8$ : $P$ maps e⇔o, $\langle \text{even}|\chi|\text{odd}\rangle$ finite
$N \equiv 4 \pmod 8$ : $P$ maps e⇔e, o⇔o and $P^2 = -1$ (degeneracy)
$N \equiv 6 \pmod 8$ : $P$ maps e⇔o but $\langle \text{even}|\chi|\text{odd}\rangle = 0$

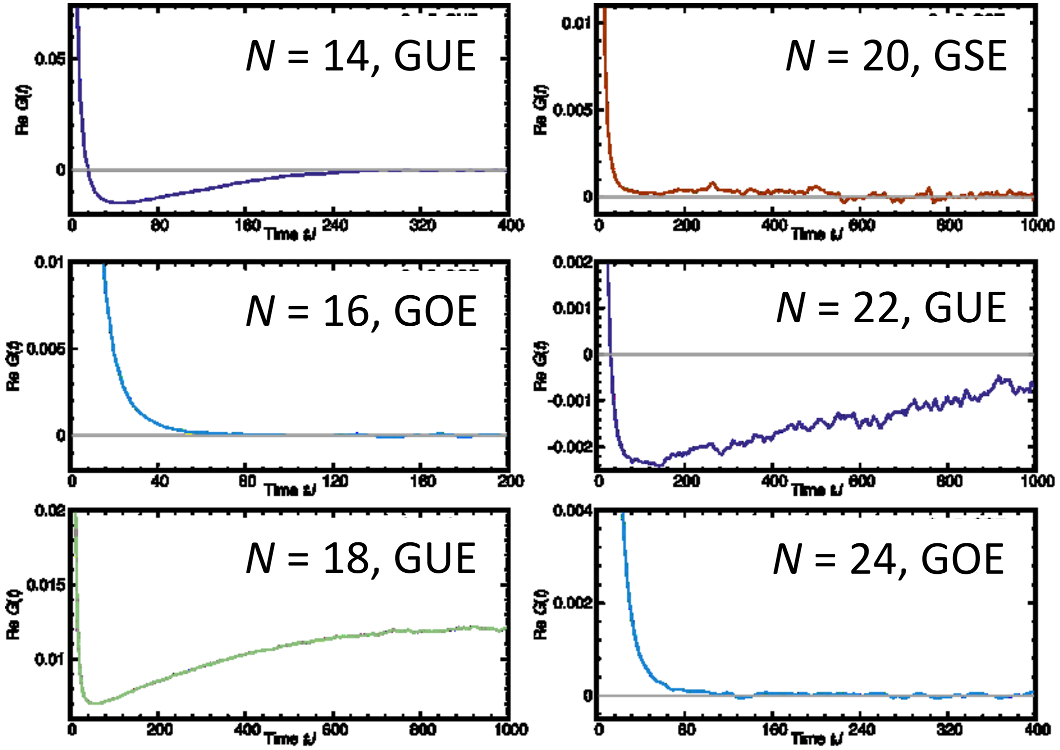
$$G(t) = \langle \chi_a(t)\chi_a(0)\rangle$$

$N \equiv 2 \pmod 8$: dip-ramp-plateau similar to $g(\beta, t)$

$$g(\beta,t) \sim \left|\frac{Z(\beta,t)}{Z(\beta,t=0)}\right|^2 = \frac{1}{Z(\beta,t=0)^2}\sum_{m,n} \mathrm{e}^{-\beta(E_m+E_n)}\mathrm{e}^{\mathrm{i}(E_m-E_n)t}$$

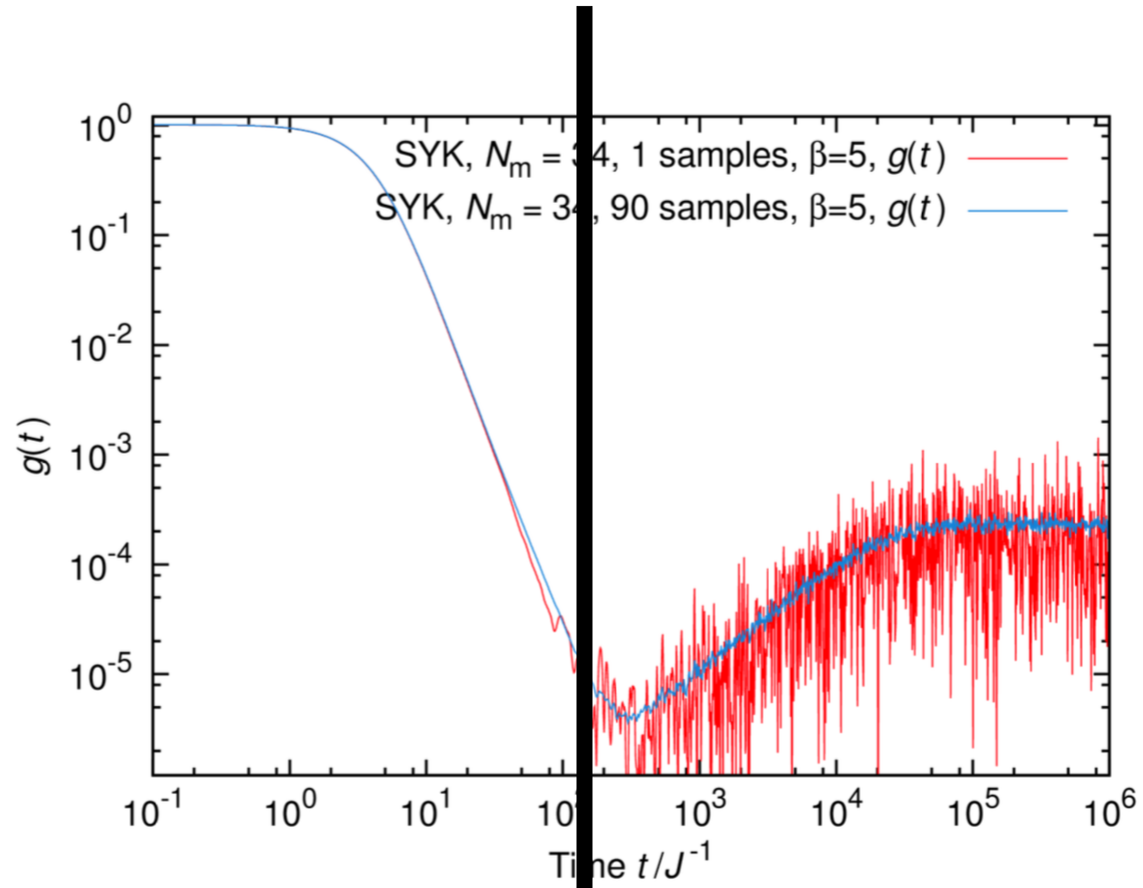
$$G(t) = \langle \hat{\chi}_a(t)\hat{\chi}_a(0)\rangle_\beta = \frac{1}{Z(\beta,t=0)}\sum_{m,n} \mathrm{e}^{-\beta E_m}|\langle m|\hat{\chi}_a|n\rangle|^2 \mathrm{e}^{\mathrm{i}(E_m-E_n)t}$$




$$P = K\prod_{j=1}^{N/2}\left(\hat{c}_j^\dagger + \hat{c}_j\right), \hat{c}_j = \frac{(\chi_{2j-1} + \mathrm{i}\chi_{2j})}{\sqrt{2}}$$

# (Non-)self-averaging




Consistent with RMT

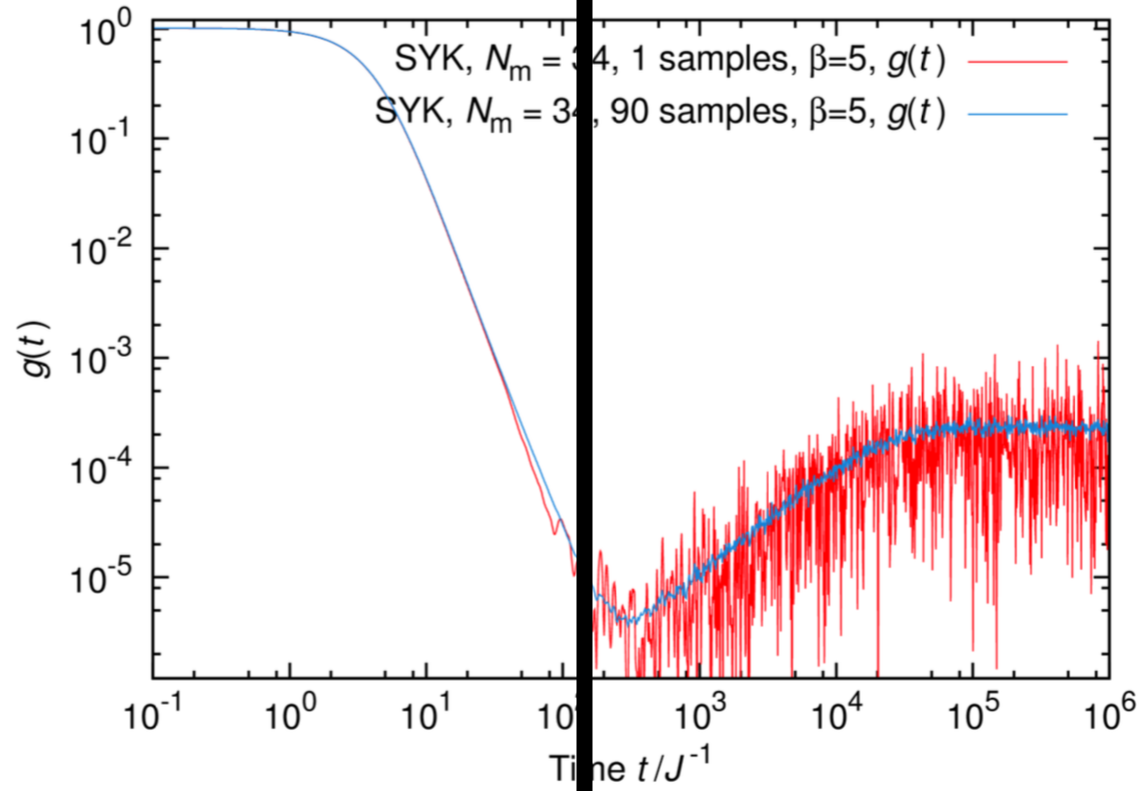
1 sample = many samples

**Self-averaging**

O(1) variance for 1 sample

**Not self-averaging**

# (Non-)self-averaging

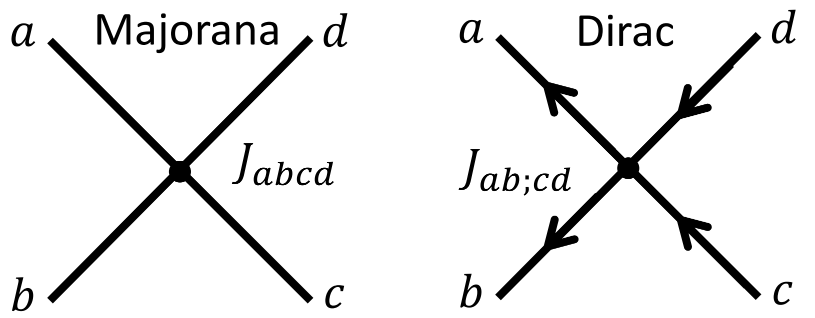



"Gravity"
$1/N$ expansion

"Random matrix theory"
$1/L \sim e^{-N}$ expansion

$L = 2^{N/2}$

# ここまでのまとめ




- $q$=4 Sachdev-Ye-Kitaev (SYK) 模型: 全ての組み合わせでのランダムな $q$ 点相互作用
  - 実験提案 [I. Danshita, M. Hanada, and MT PTEP 2017] など

[J. S. Cotler, G. Gur-Ari, M. Hanada, J. Polchinski, P. Saad, J. Saul, S. H. Shenker, D. Stanford, and MT]

- 有限の $N$ に対する数値的な結果 [1611.04650; JHEP 2017]
  - スペクトル形状因子 $g(\beta,t) \propto \langle |Z|^2 \rangle_J$, $g_c(\beta,t) \propto \langle |Z|^2 \rangle_J - |\langle Z \rangle_J|^2$ を分配関数を拡張した $Z(\beta,t)$ から求めた。
    - $g$: 初期の急速な減衰 (slope): $|\langle Z \rangle_J|^2$ の減衰による
    - Ramp + plateau: ランダム行列的な準位相関を反映; $N$ mod 8 (GOE, GUE, GSE)
  - 相関関数 $\langle \chi(t)\chi(0) \rangle$: 同様に slope-dip-ramp-plateau を示す ($N$ mod 8 = 2 のとき)

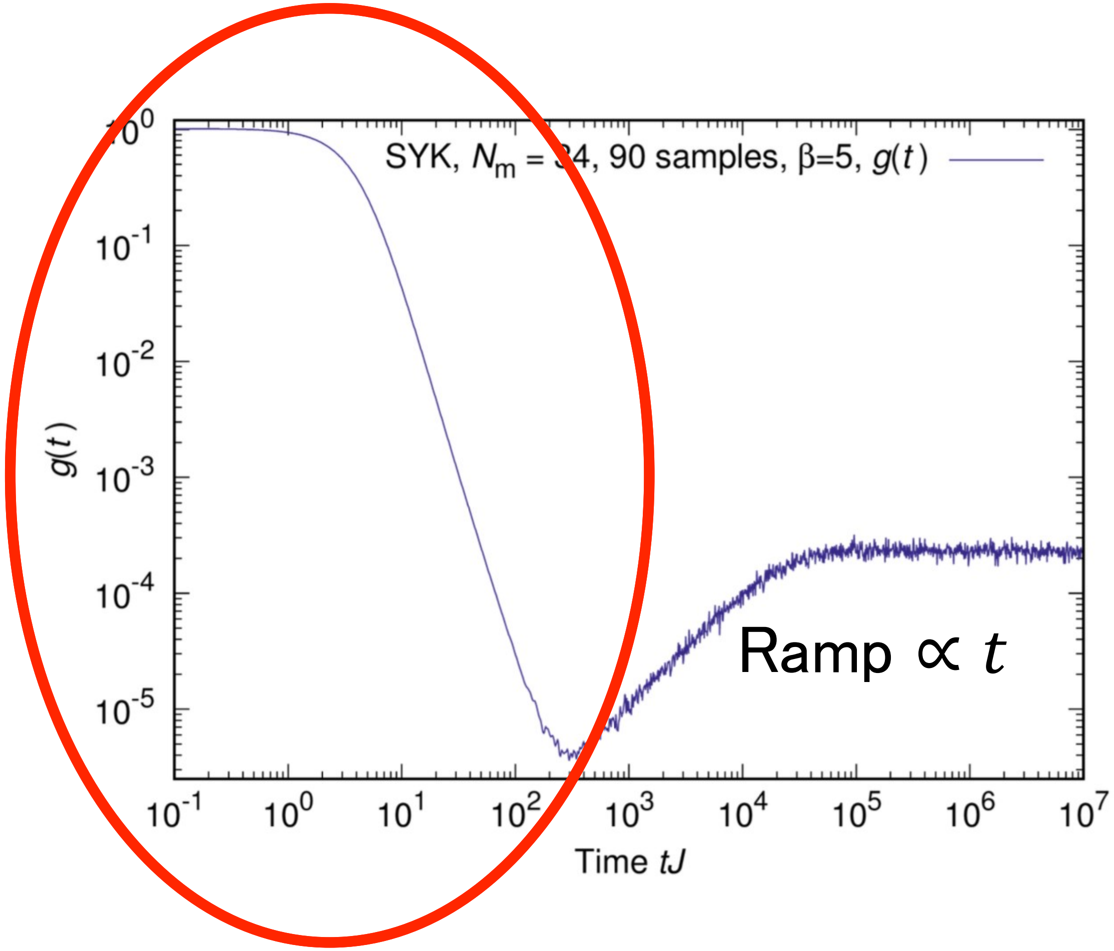
  - Ramp 以降は、自己平均化しない
  - Dip time と plateau time の比: $N$ とともに指数関数的に増大 (長距離相関を反映)

➔ $AdS_5/CFT_4$ ($\mathcal{N}$=4 SYM) でも同様に指数関数的に長い ramp を期待 (see our paper)

# “Onset of Random Matrix Behavior in Scrambling System”

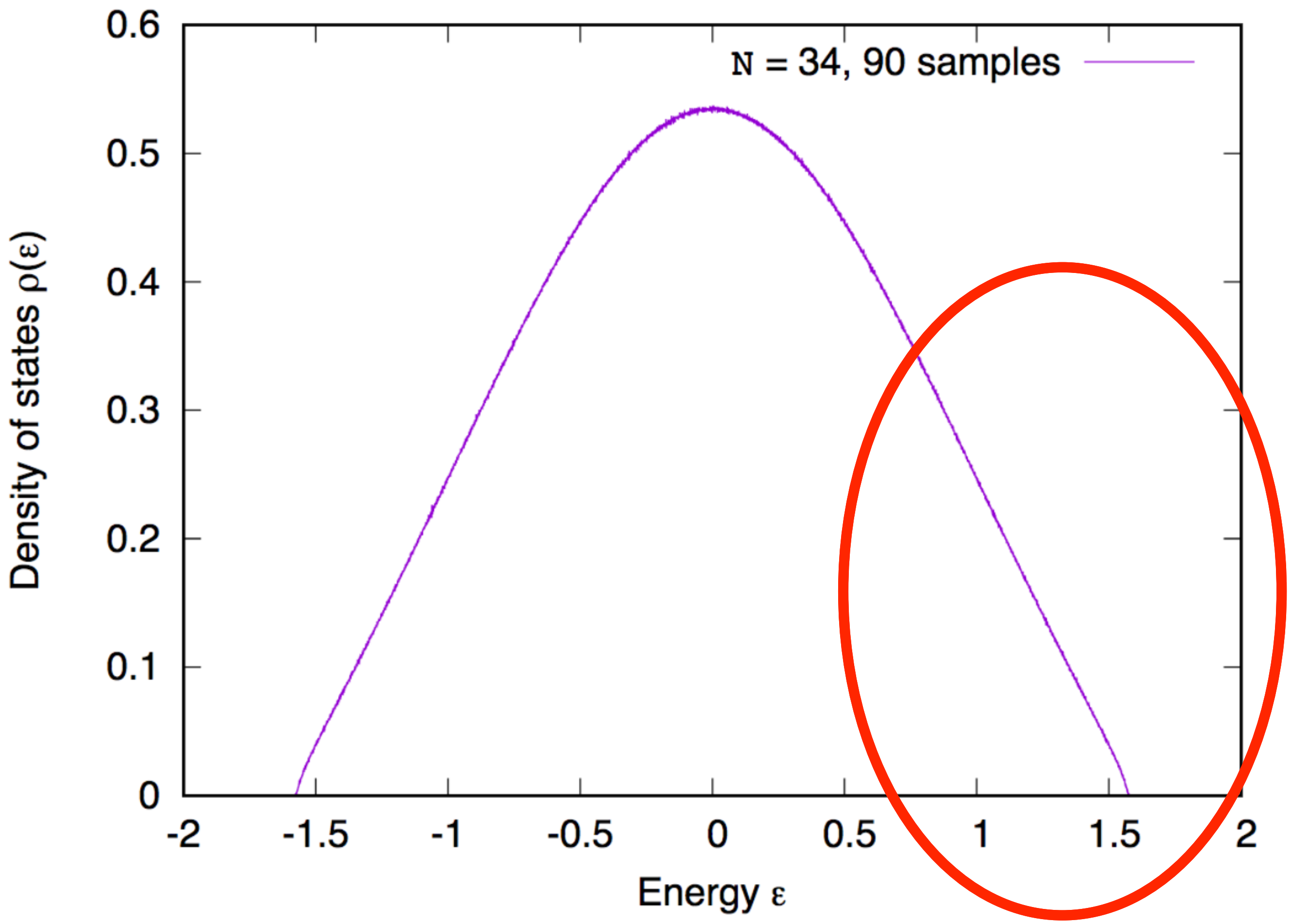
Hrant Gharibyan, Masanori Hanada, Stephen H. Shenker, and MT, arXiv:1803.08050
JHEP 1807, 124 (2018)

# Where does the ramp start?



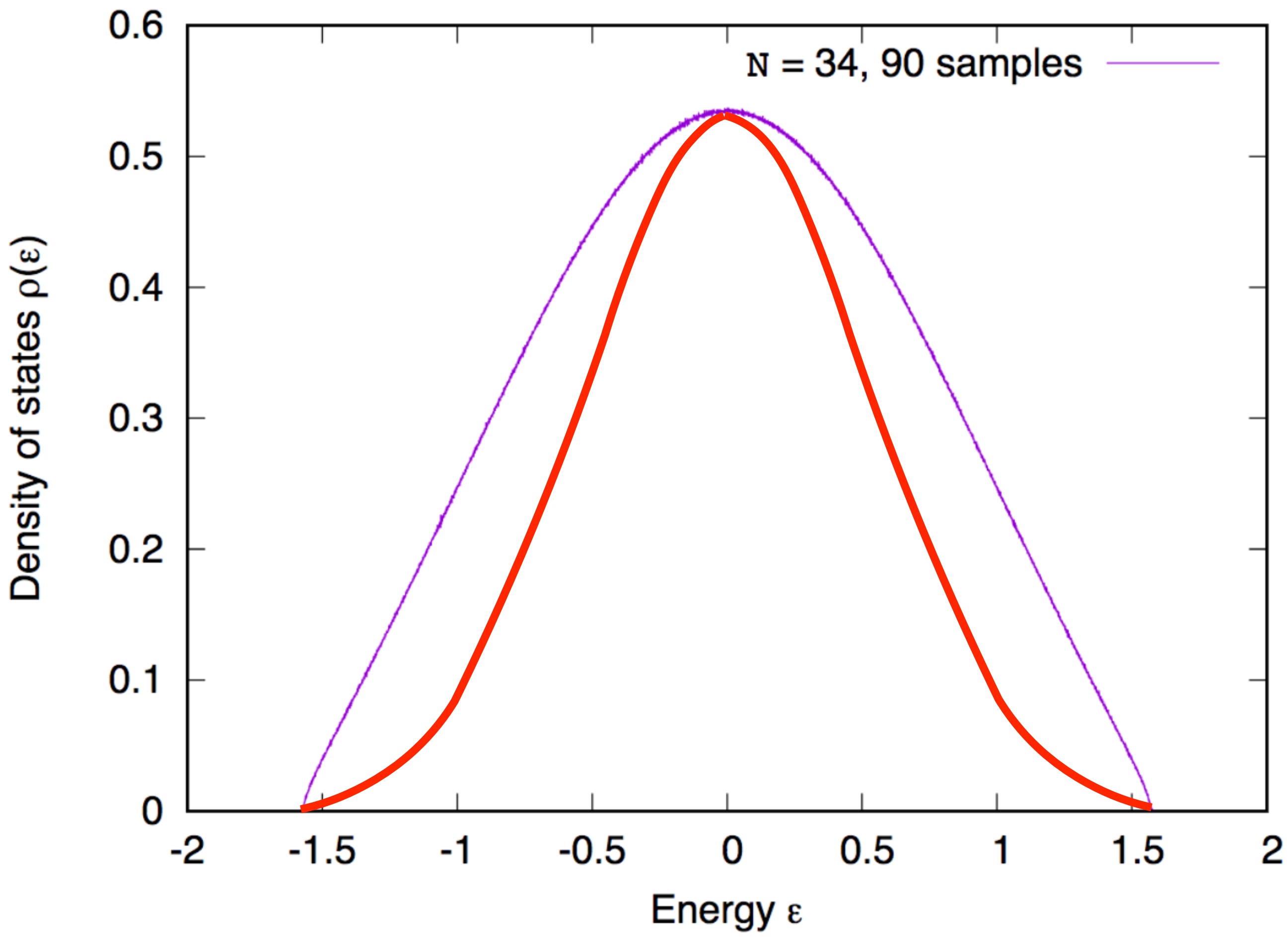

'Slope' hides the beginning of the slope

# Energy spectrum of the SYK model




‘Slope’ depends on the edge of the density of states

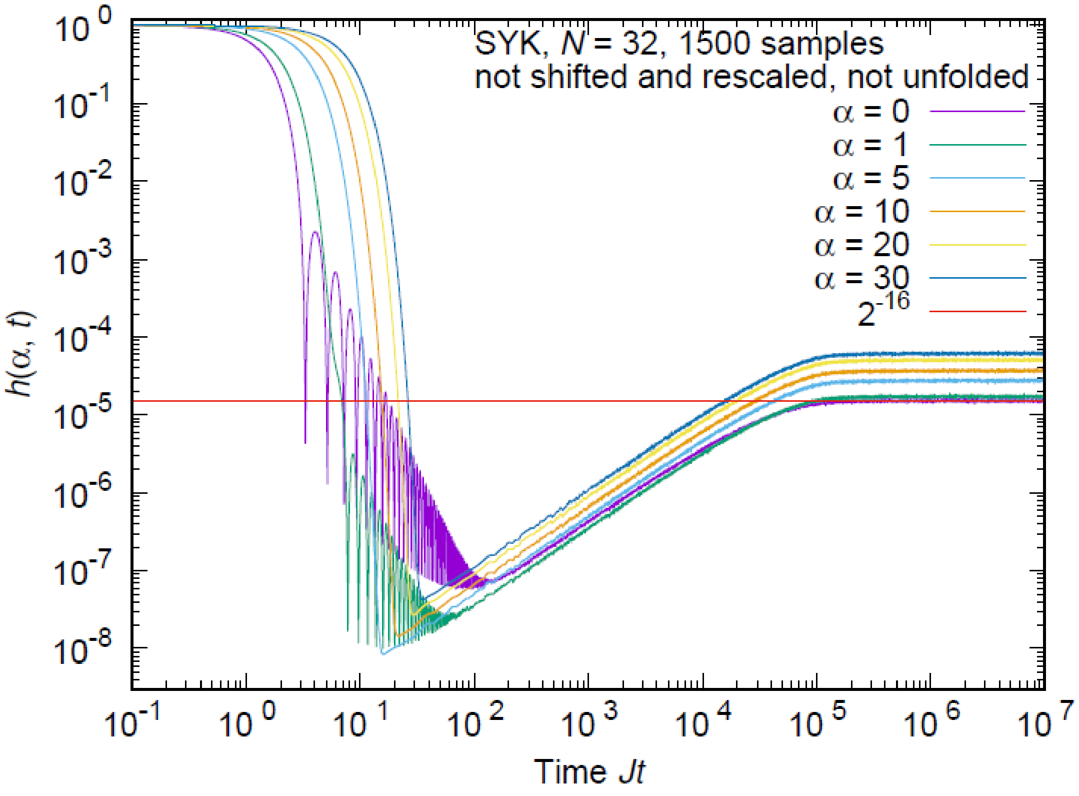
# Energy spectrum of the SYK model

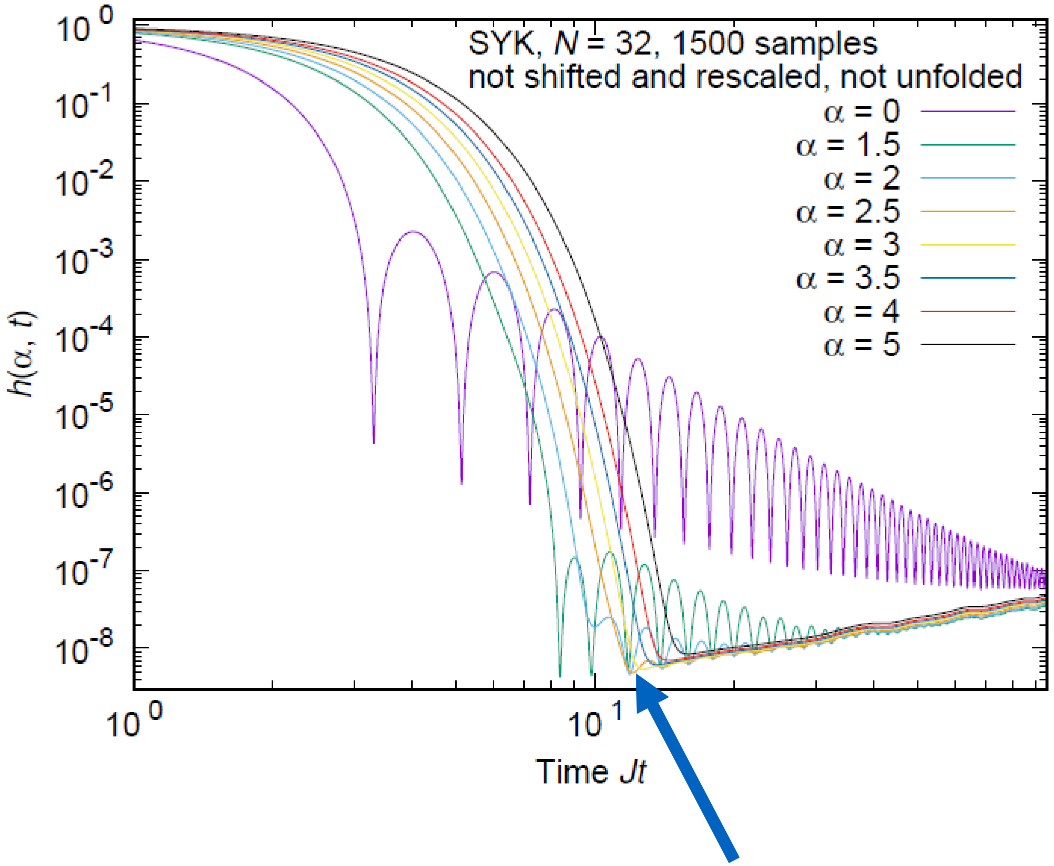



$$Y(\alpha,t)Y^*(\alpha,t) = \sum_{m,n} e^{-\alpha(E_n^2+E_m^2)} e^{+i(E_m-E_n)t}$$

# Spectral form factor

$$|Y(\alpha,t)|^2 = \left|\sum_i e^{-\alpha E_i^2 - itE_i}\right|^2$$

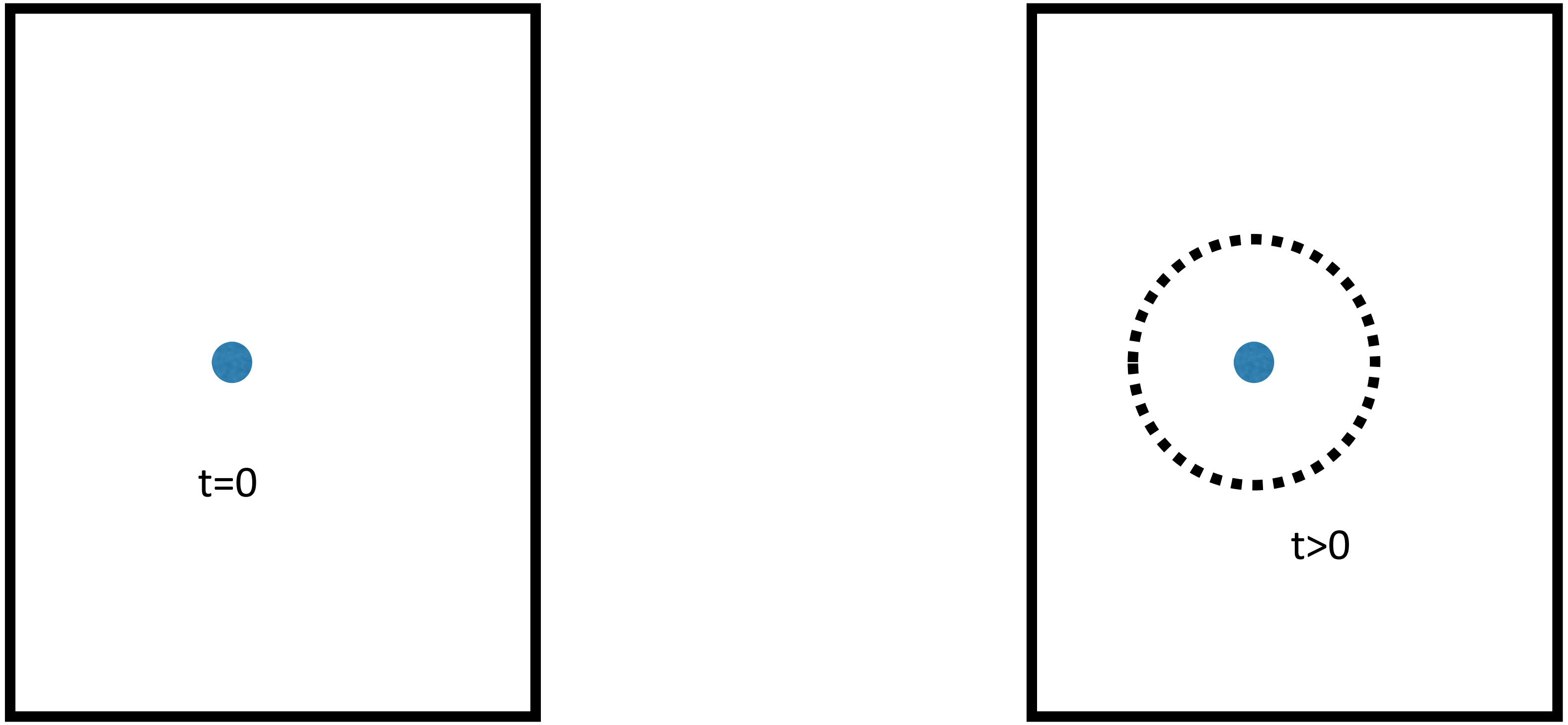



$t_{\min}$: almost constant? ($8 < t_{\min} < 15$ for $10 \leq N \leq 34$)

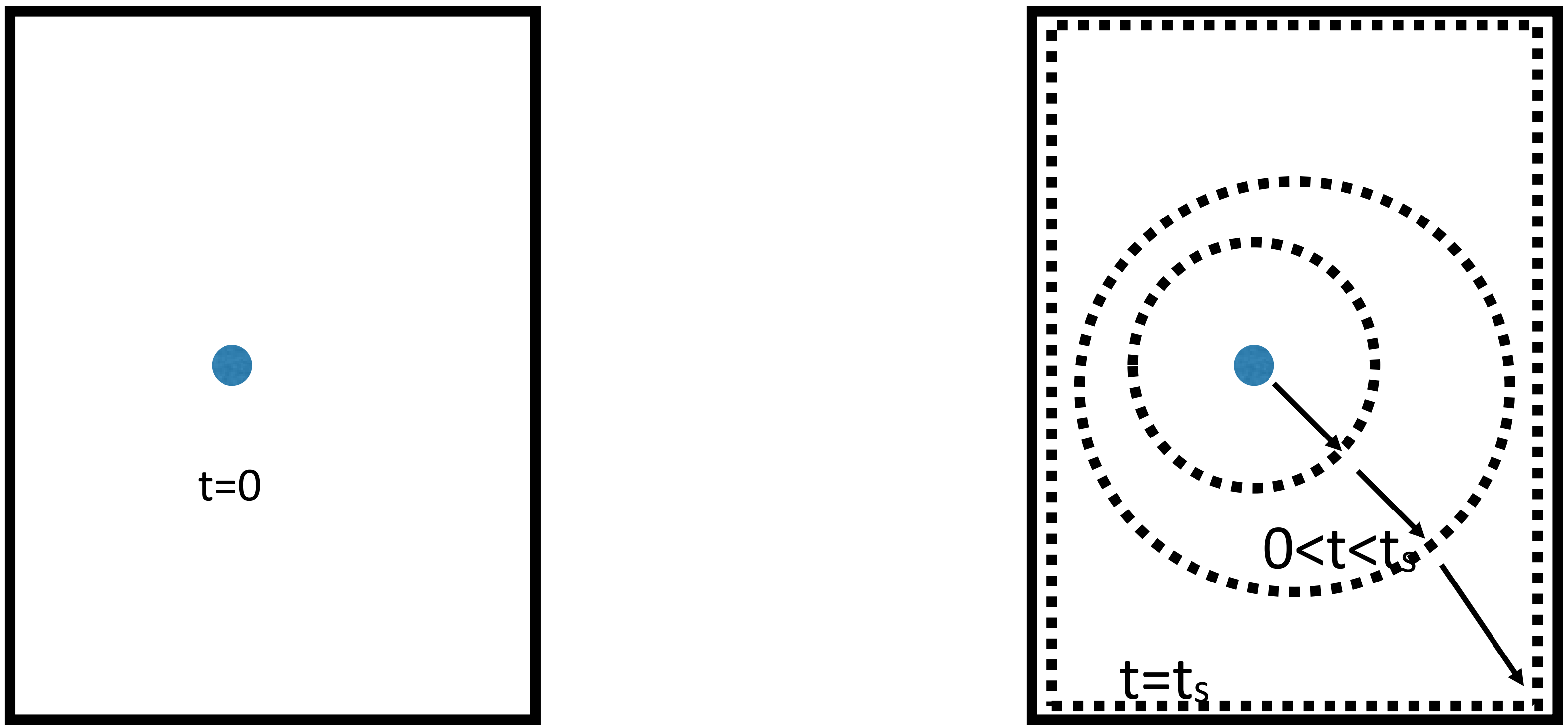
$t_{\min} = 12.5$ for $\alpha = 2.9$

# Scrambling




After some time, non-local measurements are needed for information on the local perturbation at $t = 0$ (“information scrambling”)

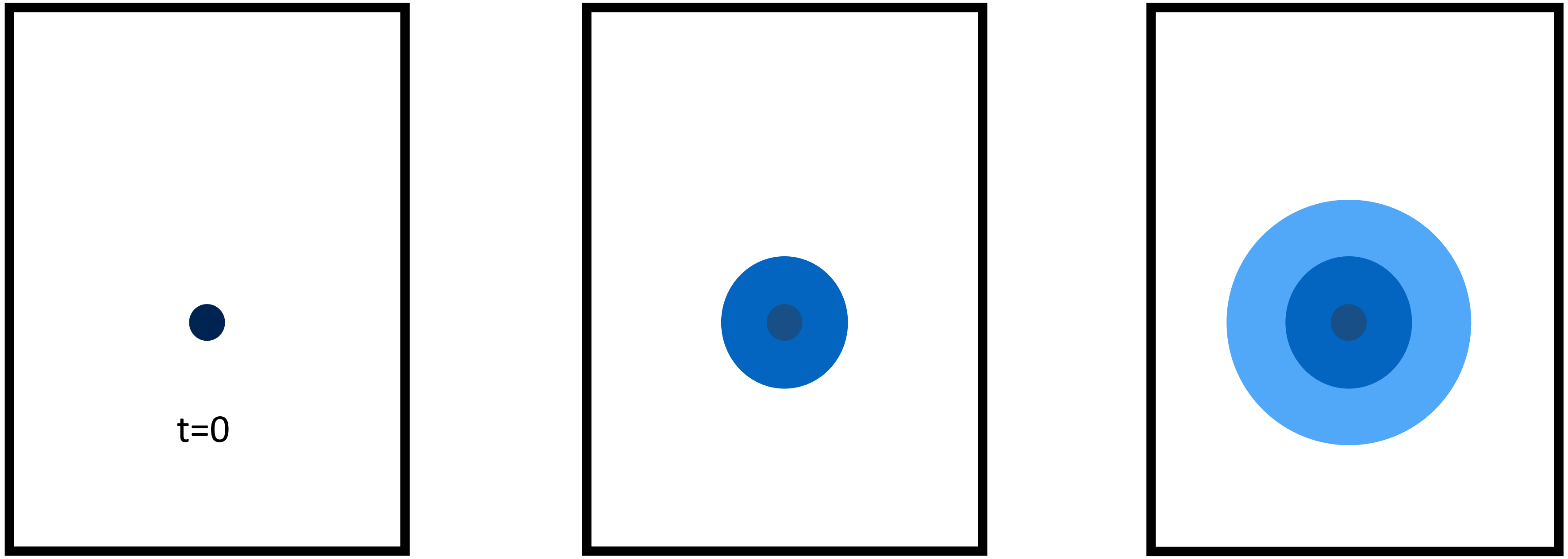
# Scrambling




After $t=t_s$ , information has been scrambled with the entire system

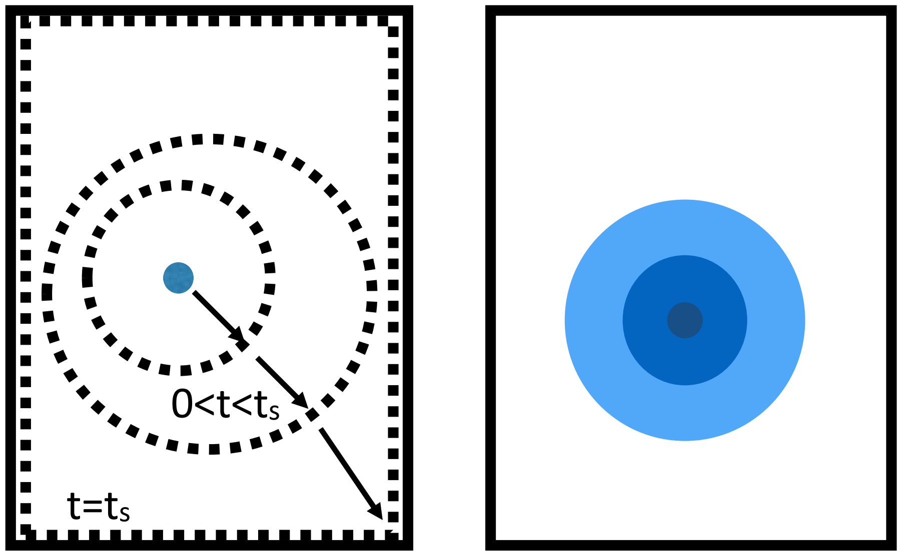
‘scrambling time’

# Diffusion




Conserved quantity (e.g. charge) diffuses, eventually (after diffusion time $t_d$ , also called the Thouless time) will be uniformly distributed

# Scrambling or diffusion?



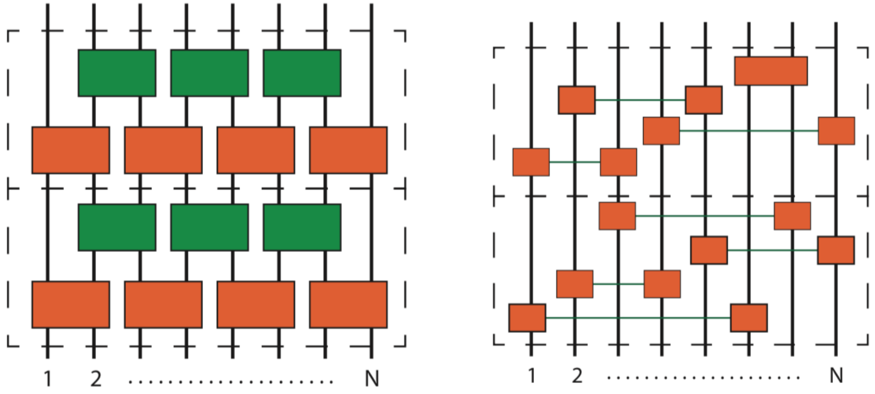

G. Gharibyan, M. Hanada, S. H. Shenker, and MT, JHEP **1807**, 124 (2018) (arXiv:1803.08050)

In this talk, we show the following examples:

- Known case: band matrix (single particle hopping)
- Numerical results on spin systems

- RMT universality observed after 'ramp time' $t_{\mathrm{ramp}}$
- Physical interpretation?
- Relationship to BH information paradox? scrambling? diffusion?
- Our results: ramp time seems to be determined by diffusion, not by scrambling

see also: Random circuit-based discussion in our paper



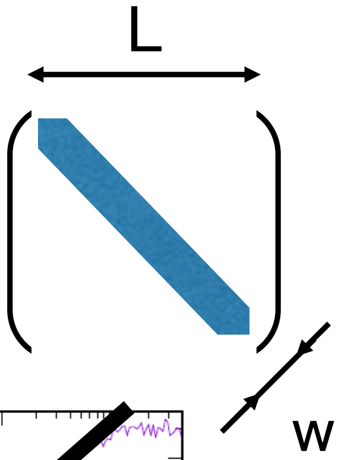
# Random band matrix

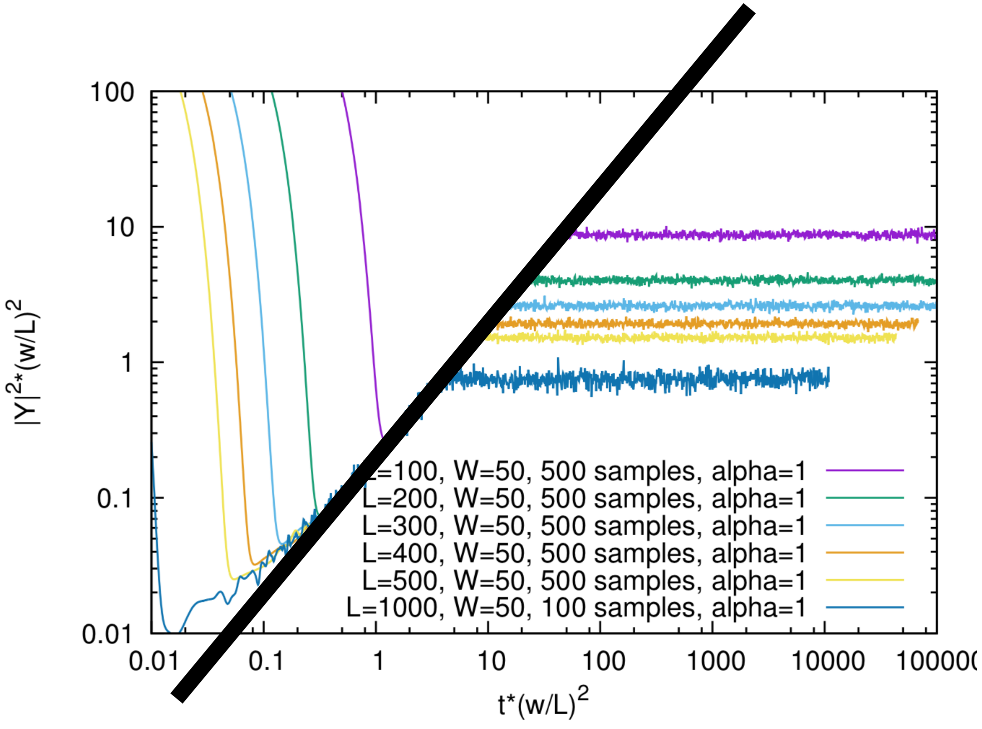
see L. Erdös and A. Knowles, "The Altshuler-Shklovskii Formulas for Random Band Matrices I: the Unimodular Case," Comm. Math. Phys. **333**, 1365 (2015) for derivation of the scaling

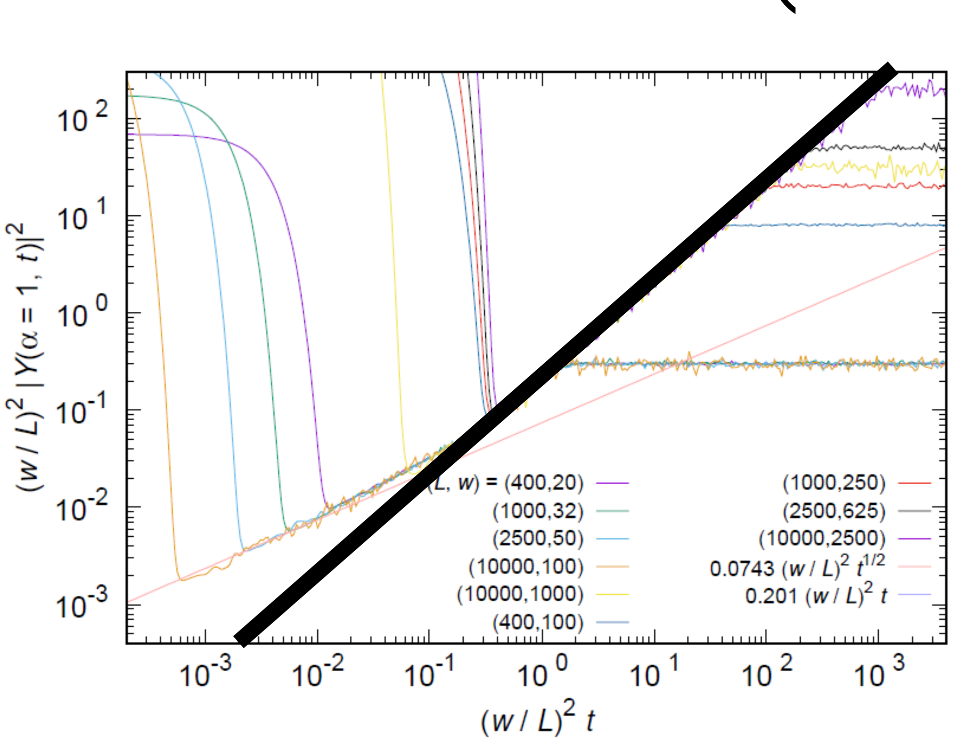
(Single particle hopping: diffusion is defined)

$$|Y(\alpha,t)|^2 = \left|\sum_i e^{-\alpha E_i^2 - itE_i}\right|^2$$

$$M_{ij} = M_{ij}^{(0)} \cdot \frac{e^{-\frac{(i-j)^2}{2w^2}}}{\sqrt{w}}$$




$t_{\text{scrambling}} \sim (L/w)$

$t_{\text{diffusion}} \sim (L/w)^2$

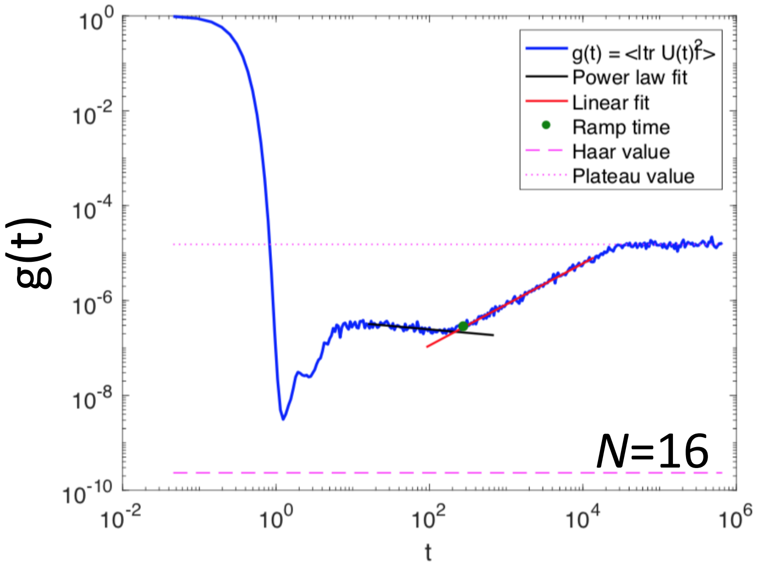
$t_{\text{ramp}} \sim (L/w)^2$

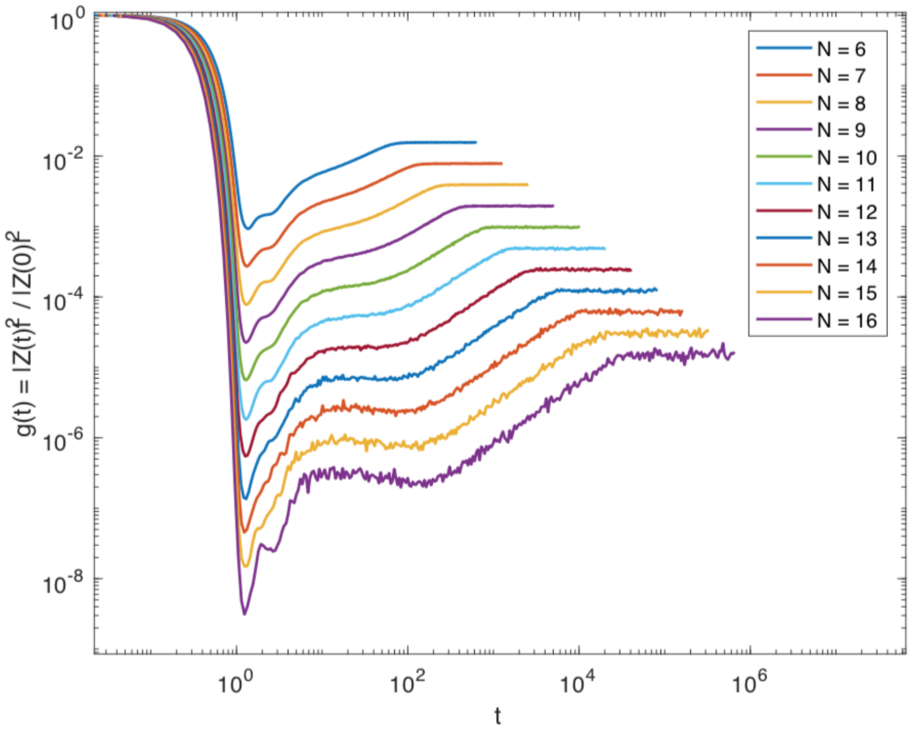
# $S = 1/2$ spin chain

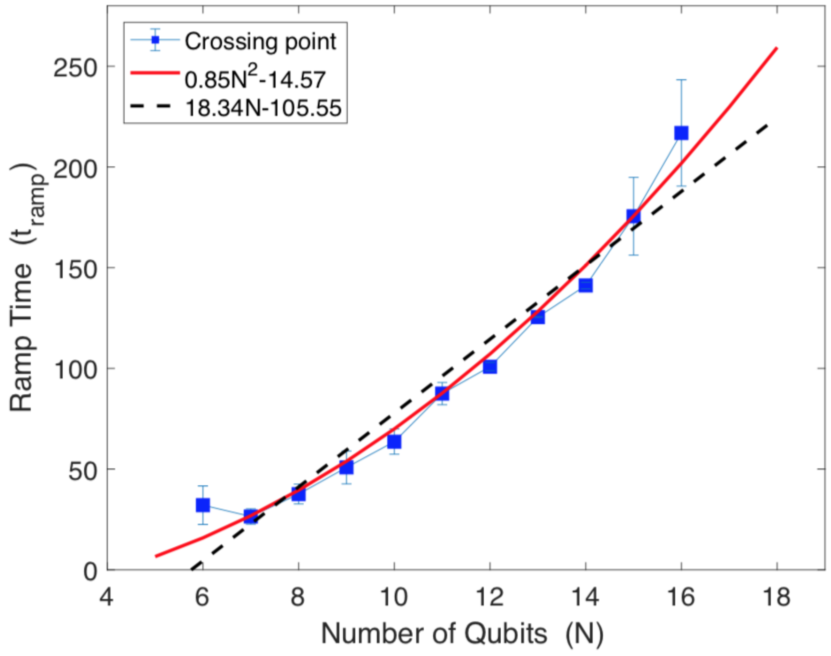
$$H = \frac{1}{4} \sum_{i=1}^{N-1} \sum_{\alpha,\beta=0}^{3} J_i^{\alpha\beta} \sigma_i^{\alpha} \otimes \sigma_{i+1}^{\beta}$$

$J$: normal distribution

$$\{\sigma^0, \sigma^1, \sigma^2, \sigma^3\} = \{I, X, Y, Z\}$$




Better overlap with diffusion time ($N^2$)

Hrant Gharibyan, Masanori Hanada, Stephen H. Shenker, and MT, JHEP **1807**, 124 (2018) (arXiv:1803.08050)

# この節のまとめ

- SYK模型のようなスクランブリング系: ランダム行列理論的なふるまいは、保存量が充分拡散した時間スケールで現れる
  cf. D.A. Roberts, D. Stanford and A. Streicher, "Operator growth in the SYK model", JHEP 06 (2018) 122 [arXiv:1802.02633]; A. Altland and D. Bagrets, Quantum ergodicity in the SYK model, Nucl. Phys. B 930 (2018) 45 [arXiv:1712.05073]

- 蒸発するブラックホール

$$t_{\text{ramp}} \sim \log N < N^{\#} \sim t_{\text{evaporation}}$$

- 重力側での導出?

➔ もう1つの universality — 短時間でのリアプノフスペクトル?
(Hanada-Shimada-MT PRE 2018, Gharibyan-Hanada-Swingle-MT, arXiv:1809.01671)

# Plan of talk

$$\hat{H}_{\mathrm{SYK}} = \frac{\sqrt{3!}}{N^{3/2}} \sum_{1\le a<b<c<d\le N} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d$$

black hole

holography

non-gravitational quantum system

cold atom realization

Danshita, Hanada and MT, PTEP 2017 [1606.02454]

maximally chaotic (chaos limit)

maximally chaotic (chaos limit)

$\lambda_{\mathrm{Lyapunov}}$

random matrix behavior of finite-time Lyapunov spectrum

[Classical] Hanada, Shimada and MT, PRE 2018 [1702.02197]

[Quantum] Gharibyan, Hanada, Swingle and MT, 1809.01671

modifications to study chaos / integrable transition, many-body localization

García-García et al., PRL 2018 [1707.02197]

García-García and MT, 1801.03204

numerical analysis; relation to other scrambling systems

Cotler et al., JHEP 2017 [1611.04650]

Gharibyan et al., JHEP 2018 [1803.08050]

# Preparation: Stability of chaos in the SYK model

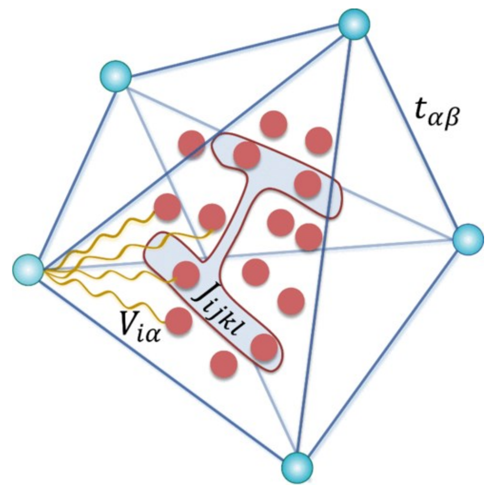
Antonio M. García-García, Bruno Loureiro, Aurelio Romero-Bermúdez, and Masaki Tezuka, Phys. Rev. Lett. **120**, 241603 (2018) (arXiv:1707.02197)

## Various modifications of SYK have been studied

e.g.

$$Q = i \sum_{1 \leqslant i < j < k \leqslant N} C_{ijk} \psi^i \psi^j \psi^k$$

- Supersymmetric SYK [Fu, Gaiotto, Maldacena, and Sachdev 2016]
- Non-random couplings [Witten, 1610.09758]
- Higher-dimensional generalizations [Gu, Qi, and Stanford 2017]
  [Davison, Fu, Georges, Gu, Jensen, and Sachdev 2017]




[S. Banerjee and E. Altman, Phys. Rev. B **95**, 134302 (2017)]

Addition of new Fermi species can induce a transition to a Fermi liquid [Banerjee and Altman 2017] or metal-insulator transition [S.-K. Jian and H. Yao 1703.02051], additional interaction can induce a metal-insulator transition [C.-M. Jian, Bi, and Xu 1703.07793], …

# Motivation and our model

Q.: Minimum requirements for chaotic behavior? (→ gravity interpretation?)
Here we study a simple model with analytical + numerical methods

$$\hat{H} = \overbrace{\sum_{1\le a<b<c<d}^{N} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d}^{\text{SYK}_4} + i \overbrace{\sum_{1\le a<b}^{N} K_{ab}\hat{\chi}_a\hat{\chi}_b}^{\text{SYK}_2}$$

Gaussian random couplings $J_{abcd}$: average 0, standard deviation $\frac{\sqrt{6}J}{N^{3/2}}$

$K_{ab}$: average 0, standard deviation $\frac{K}{\sqrt{N}}$

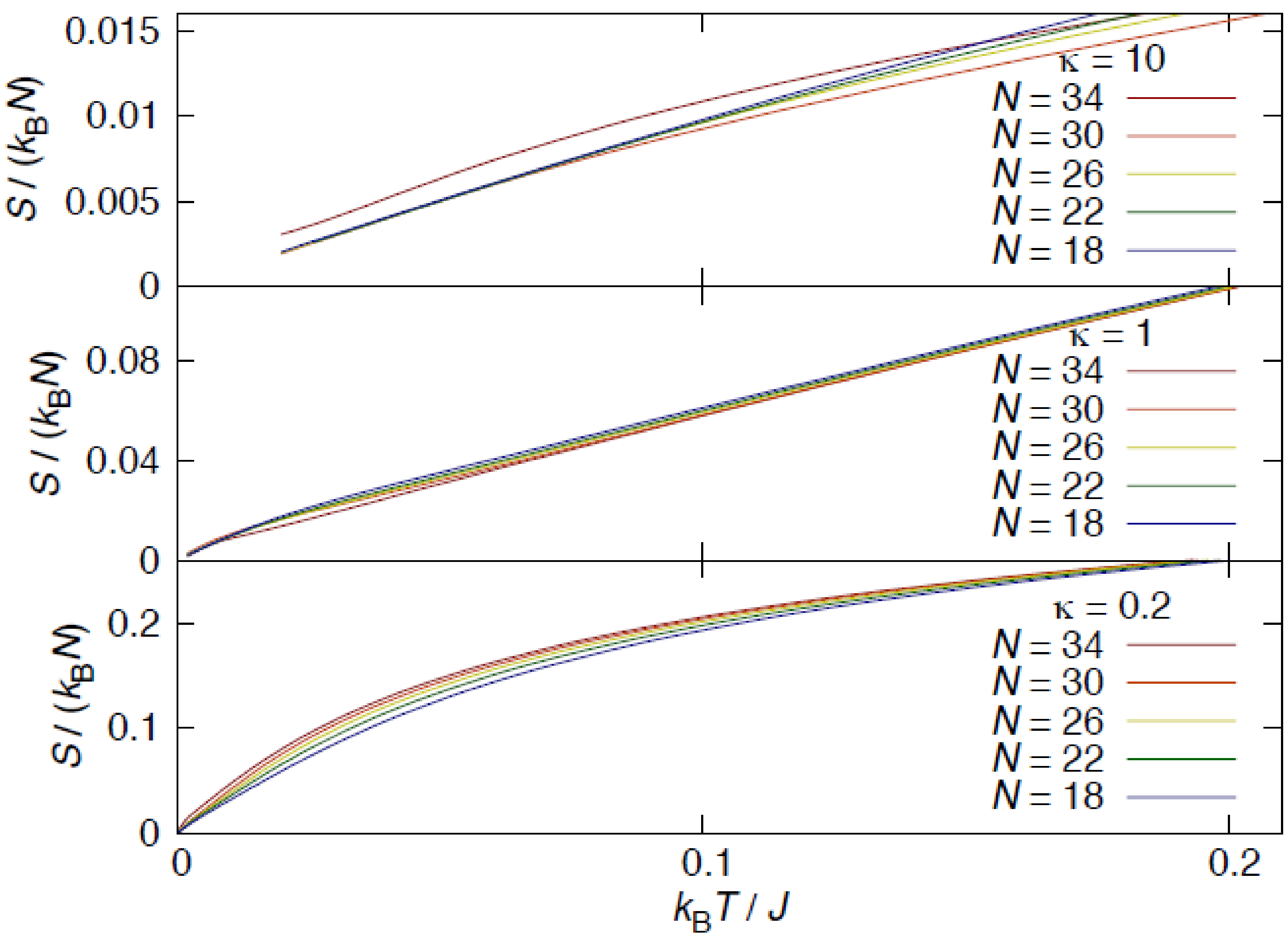
$J = 1$: unit of energy

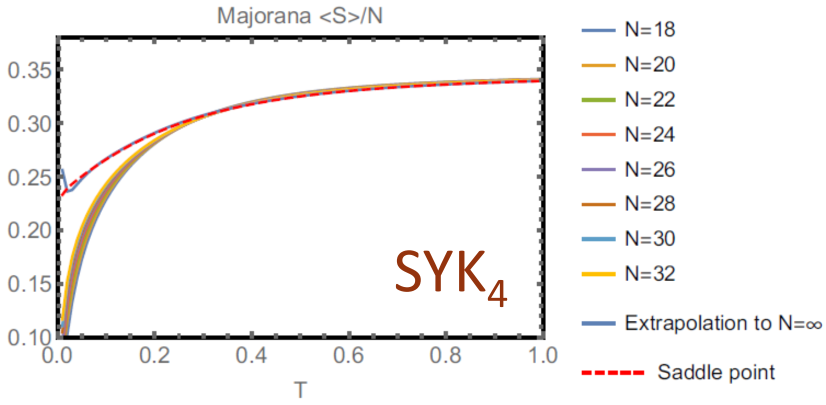
$\text{SYK}_4$ as unperturbed Hamiltonian,
$K$ controls the strength of $\text{SYK}_2$ (one-body random term, solvable)

Both terms respect charge parity in complex fermion description
➔ Numerical exact diagonalization (ED) of $2^{N/2-1}$-dimensional matrix, $N \le 34$ possible

# Entropy per fermion

In SYK$_4$, entropy $S$ per fermion → 0.2324 in large-$N$ limit




[Cotler, …, and MT, JHEP05(2017)118]

Large $K$ (weak SYK$_4$):
$S$ / $N$ almost
independent of $N$,
and vanishing as $T$ → 0

$$\hat{H} = \sum_{1\le a<b<c<d}^{N} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d + i\sum_{1\le a<b}^{N} K_{ab}\hat{\chi}_a\hat{\chi}_b$$

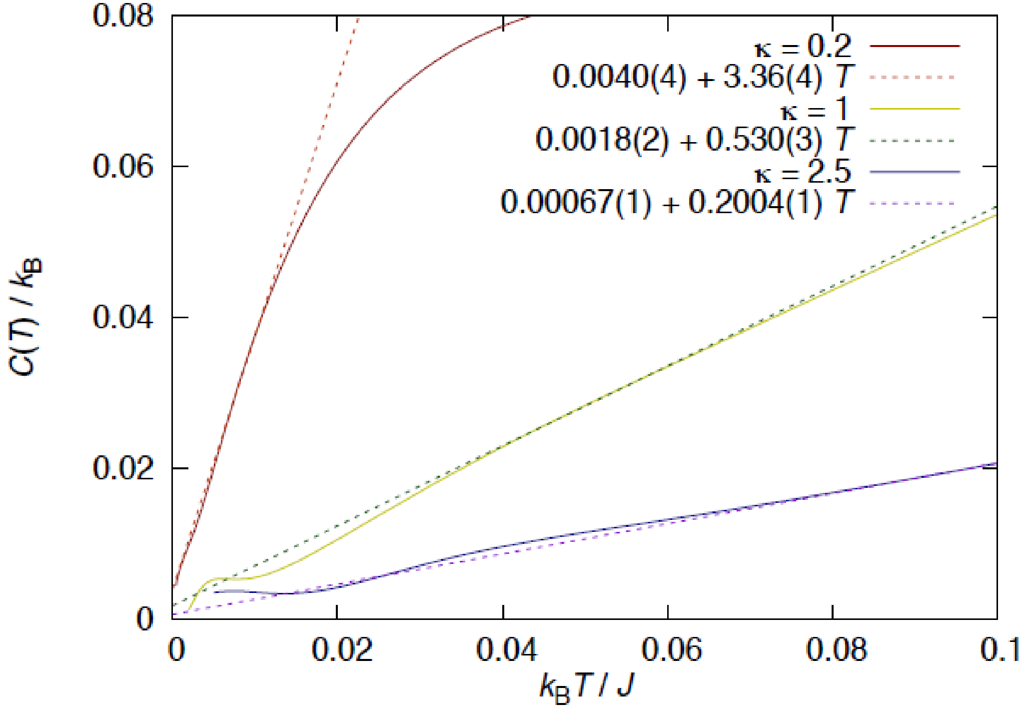
$J_{abcd}$: average 0, std. dev. $\frac{\sqrt{6}}{N^{3/2}}$

$K_{ab}$: average 0, std. dev. $\frac{K}{\sqrt{N}}$

# Specific heat and large-*N* analysis

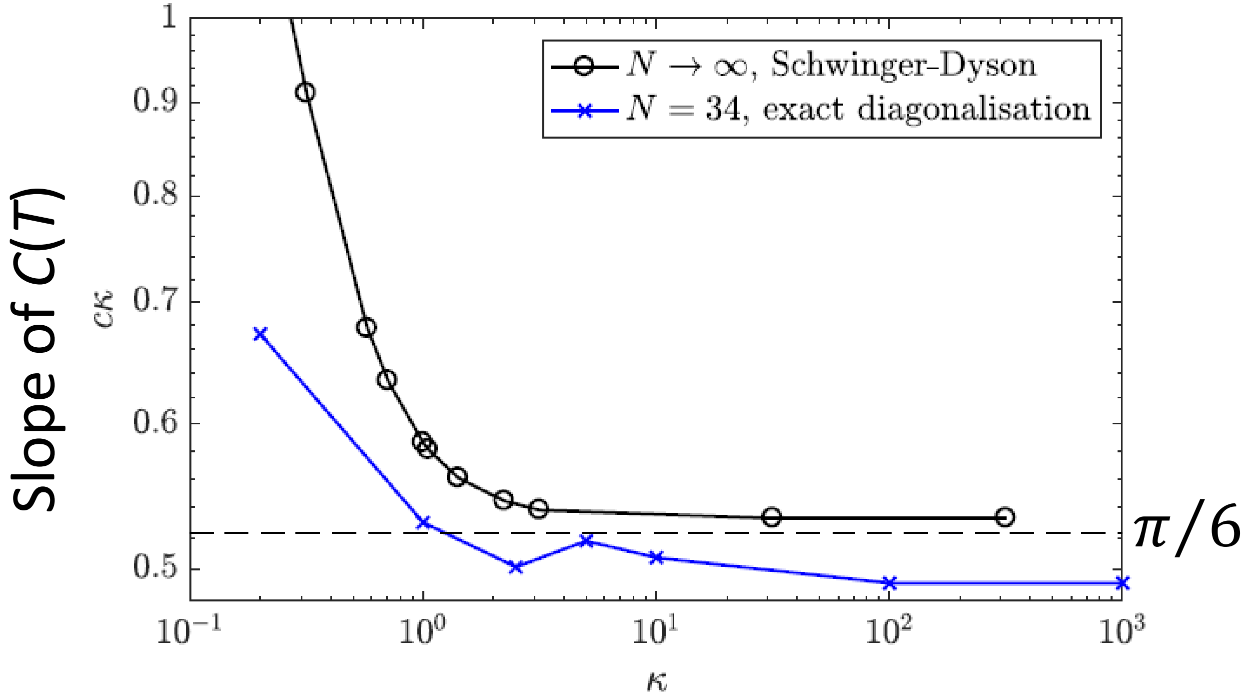
Low *T*: specific heat *C*(*T*) → linear in *T*

*N*=34




$$C(T) = \left\langle \frac{1}{NZ} \sum_k \frac{(E_k - \bar{E})^2}{T^2} e^{-\beta E_k} \right\rangle$$

Large *N*: replica fields + Hubbard-Stratonovich transf.
[cf. Maldacena and Stanford 2016, Sachdev 2015]




← $SYK_4$ limit

$SYK_2$ limit →

# Out-of-time order correlator (OTOC)

$$F(t_1,t_2) \equiv \frac{1}{N^2}\sum_{i,j}^{N} Tr\left[\rho(\beta)^{1/4}\chi_i(t_1)\rho(\beta)^{1/4}\right.$$
$$\left.\times\chi_j(0)\rho(\beta)^{1/4}\chi_i(t_2)\rho(\beta)^{1/4}\chi_j(0)\right]$$
$$\simeq G_R(t_1)G_R(t_2) + \frac{1}{N}\mathcal{F}(t_1,t_2) + \mathcal{O}(N^{-2})$$

$$\rho^{1/4}(\beta) = \left(\frac{e^{-\beta H}}{Z}\right)^{1/4}$$

$$\mathcal{F}(t_1,t_2) = e^{\lambda_L (t_1+t_2)/2} f(t_1 - t_2)$$

$$\mathcal{F}(t_1,t_2) = \int dt_3 dt_4 K_R(t_1,t_2,t_3,t_4)\mathcal{F}(t_3,t_4),$$

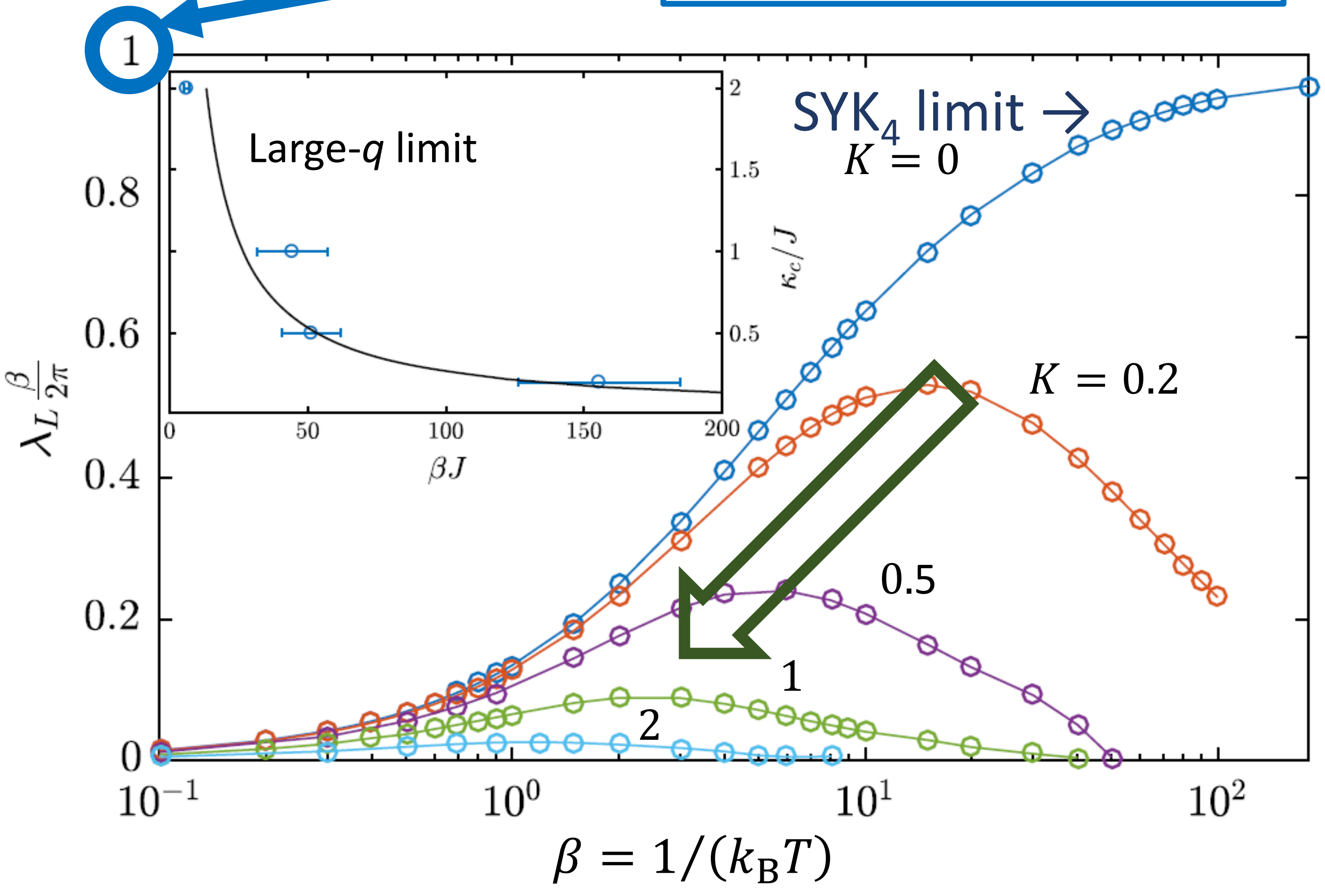
$$K_R(t_1,t_2,t_3,t_4) = G_R(t_1)G_R(t_2)[3J^2G_{lr}^2(t_3-t_4)+\kappa^2],$$

$$G_{lr}(\omega) = \frac{2ie^{-\frac{\beta}{2}\omega}}{1+e^{-\beta\omega}}\,\mathrm{Im}(G_R(\omega))$$

Lyapunov exponent is obtained by solving

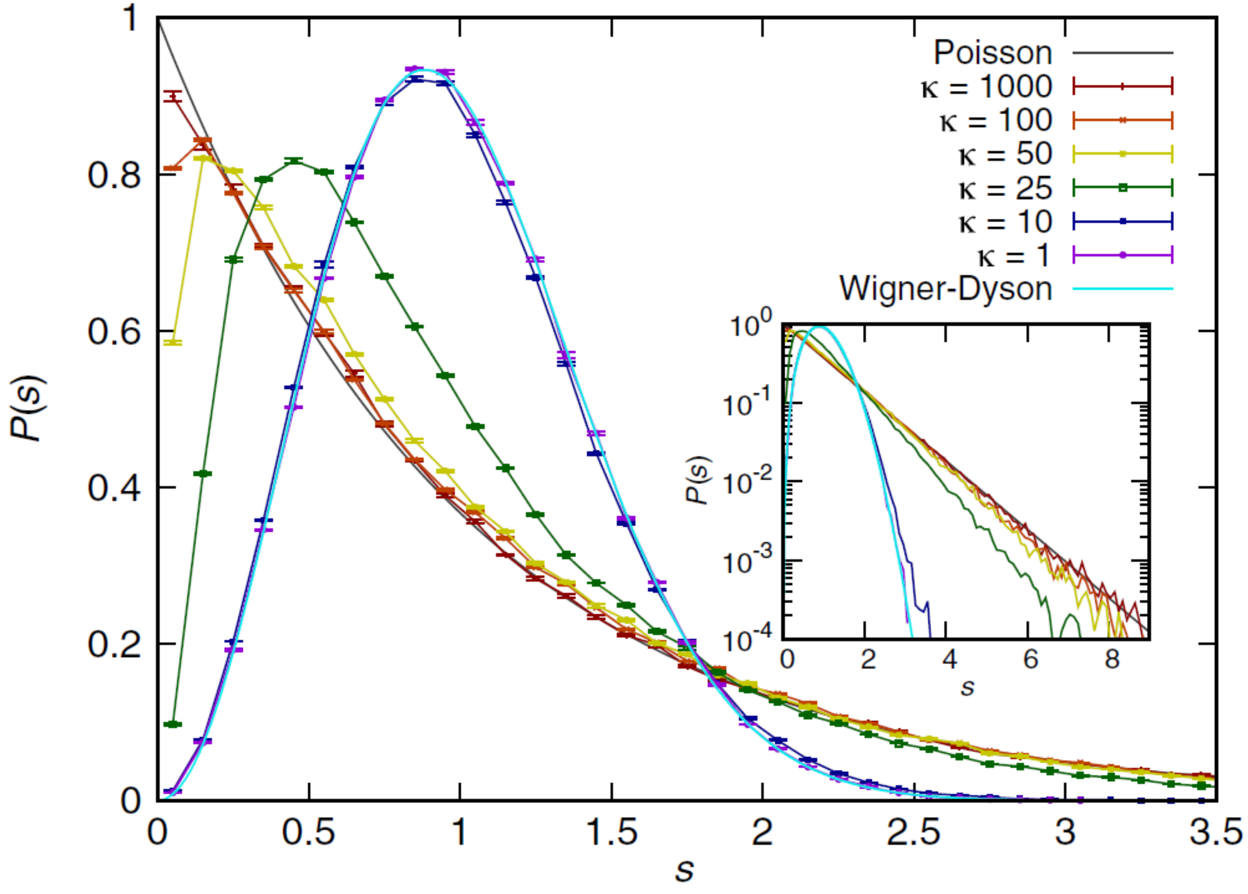
$$f(\omega') = \left|G^R\left(\omega' + i\frac{\lambda_L}{2}\right)\right|^2\left[\kappa^2 f(\omega') + 3\int\frac{d\omega}{2\pi}g_{lr}(\omega'-\omega)f(\omega)\right]$$
$$\omega' = \omega_1 - i\lambda_L/2$$

Chaos bound [Maldacena, Shenker, and Stanford 2016]




Deviation from the chaos bound as SYK$_2$ component is introduced

# Small κ: RMT-like behavior of energy spectrum




Central 10 % of eigenvalues

## $P(s)$ : level spacing distribution

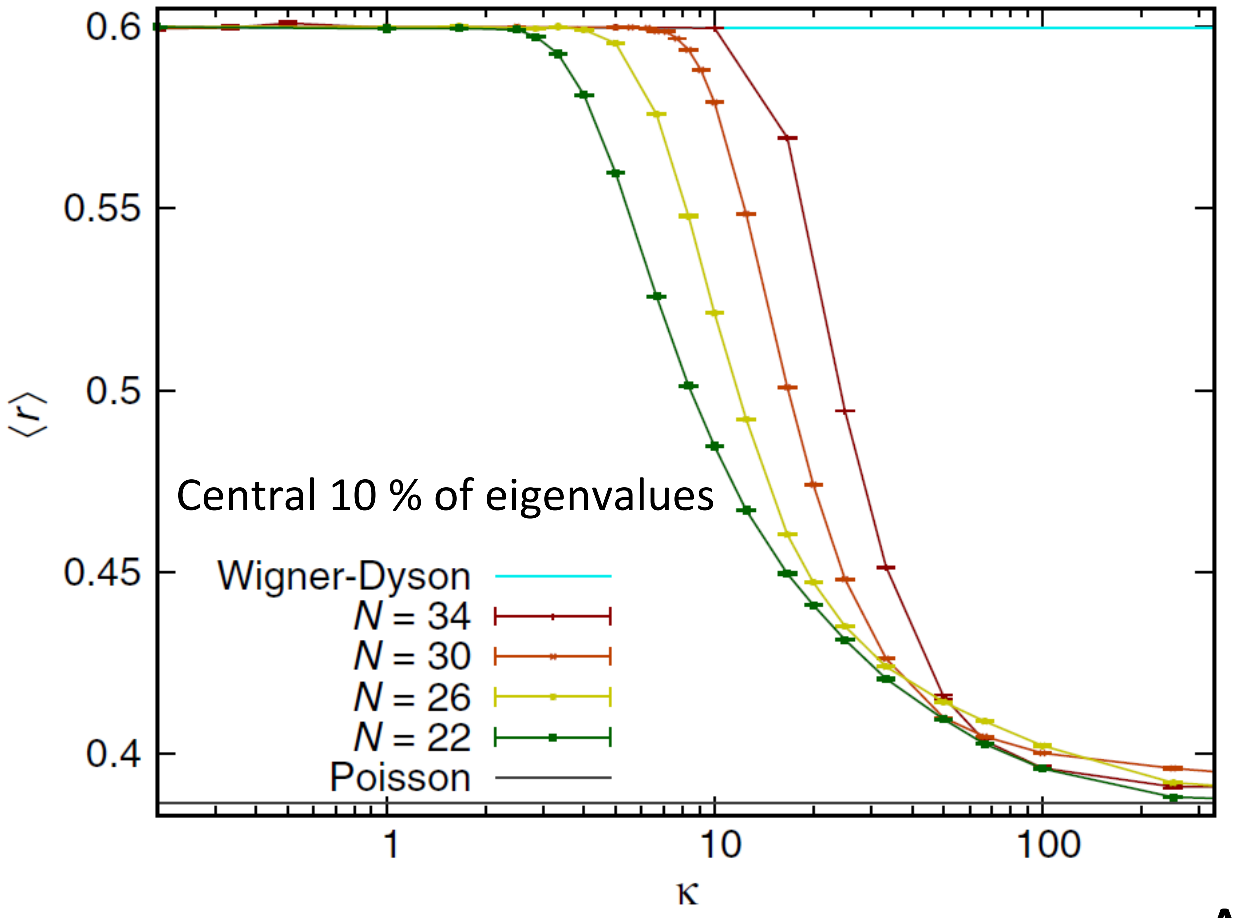
Ratio of consecutive level spacing $E_{i+1} - E_i$ to the local mean level spacing Δ (requires unfolding of the spectrum using a fitting of the spectral density)

$SYK_4$ limit (small $K$): RMT

$SYK_2$ (large $K$): Poisson ($e^{-S}$)

(GUE if $N \equiv 2 \pmod 4$)

# Small κ: RMT-like behavior of energy spectrum




## $\langle r \rangle$ : average adjacent gap ratio

Average of $\frac{\min(E_{i+1}-E_i, E_{i+2}-E_{i+1})}{\max(E_{i+1}-E_i, E_{i+2}-E_{i+1})}$

(does not require unfolding)

$SYK_4$ (κ = 0): random matrix

$SYK_2$ (large κ): Poisson $(2\log 2 - 1 \approx 0.386)$

($\approx 0.599$ for GUE [Y. Y. Atas *et al.* PRL 2013])

Also: spectral form factor $|Z(\beta + i\tau)|^2/Z(\beta)^2$ shows robust ramp reflecting level rigidity

# Time-dependent Lyapunov spectra in quantum chaotic systems

arXiv:1809.01671 “Quantum Lyapunov Spectrum”

Hrant Gharibyan (Stanford),

Masanori Hanada (Boulder → Southampton),

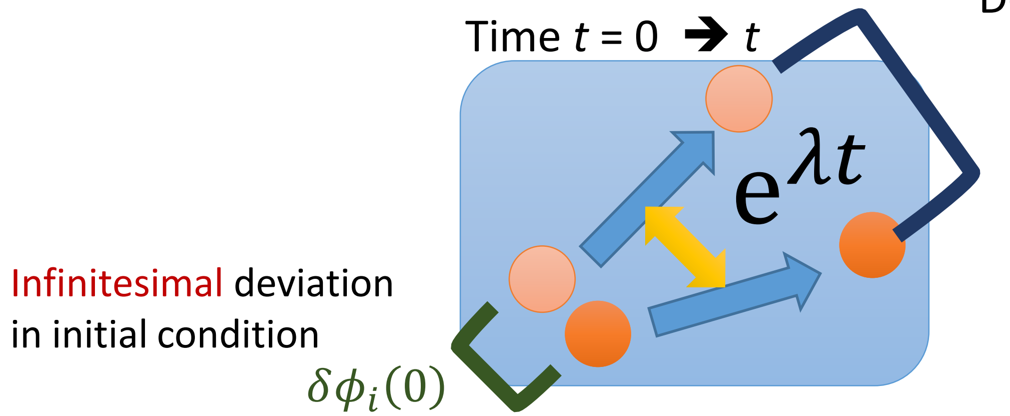
Brian Swingle (Maryland),

and Masaki Tezuka (Kyoto)

# Statistics of classical Lyapunov spectrum

M. Hanada, H. Shimada, and M. Tezuka,
Phys. Rev. E **97**, 022224 (2018)

Classical system with $K$ degrees of freedom




Deviation at $t$ : linear in the initial deviation

$$\delta\phi_i(t) = M_{ij}\delta\phi_j(0)$$

Singular values of $M_{ij}$: $\{s_k(t)\}_{k=1}^{K}$
Time-dependent Lyapunov spectrum

$$\left\{\lambda_k(t) = \frac{\log s_k(t)}{t}\right\}_{k=1,2,\ldots,K}$$

- Often the $t \to \infty$ limit is studied,

while we focus on the finite-time behavior

$\{\lambda_k(t)\}_{k=1}^{K}$ depends on the details of the system
➔ Probability distribution of the unfolded gap approaches that of Gaussian random matrices
(Examples: logistic map, Lorenz attractor, BFSS matrix model & mass-deformed version ➔ GOE)

Similar behavior observed for singular values of random band matrix products

Real matrix → GOE
Complex matrix → GUE

Out-of-Time-Ordered Correlation

# Our definition for quantum systems (cf. OTOC)

$\delta x_i(t) = M_{ij}\delta x_j(0)$

[Norbert Wiener 1938][Larkin & Ovchinnikov 1969]

Canonically conjugate variables $x, p$ at different times

$M_{ij} = \frac{\delta x_i(t)}{\delta x_j(0)}$ Singular values: $\{s_k(t)\} = \{e^{\lambda_k t}\}$

$$\{x(t), p(0)\}_{\mathrm{PB}}{}^2 = \left(\frac{\partial x(t)}{\partial x(0)}\right)^2 \to e^{2\lambda_{\mathrm{L}} t}$$

$$L = \left(\frac{\delta x_i(t)}{\delta x_j(0)}\right)^2$$

Corresponding quantity in quantum systems: for bosonic $V, W$ : $C_T(t) = -\left\langle \left[\hat{V}(t), \widehat{W}(0)\right]^2 \right\rangle$

Out-of-time correlator is included $\langle \hat{V}(t)\widehat{W}(0)\hat{V}(t)\widehat{W}(0)\rangle$ etc.

For systems of fermions
e.g. Sachdev-Ye-Kitaev (SYK) model?

$$\widehat{H} = \frac{\sqrt{3!}}{N^{3/2}} \sum_{1\le a<b<c<d\le N} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d$$

$\hat{\chi}_a$: Majorana fermion, $\widehat{M}_{ab}(t=0) = \{\hat{\chi}_a, \hat{\chi}_b\} = \delta_{ab}$

Anticommutator

$$\widehat{M}_{ab}(t) = \{\hat{\chi}_a(t), \hat{\chi}_b(0)\}$$

$$\hat{L}_{ab}(t) = \sum_{j=1}^{N} \widehat{M}_{ja}(t)\widehat{M}_{jb}(t)$$

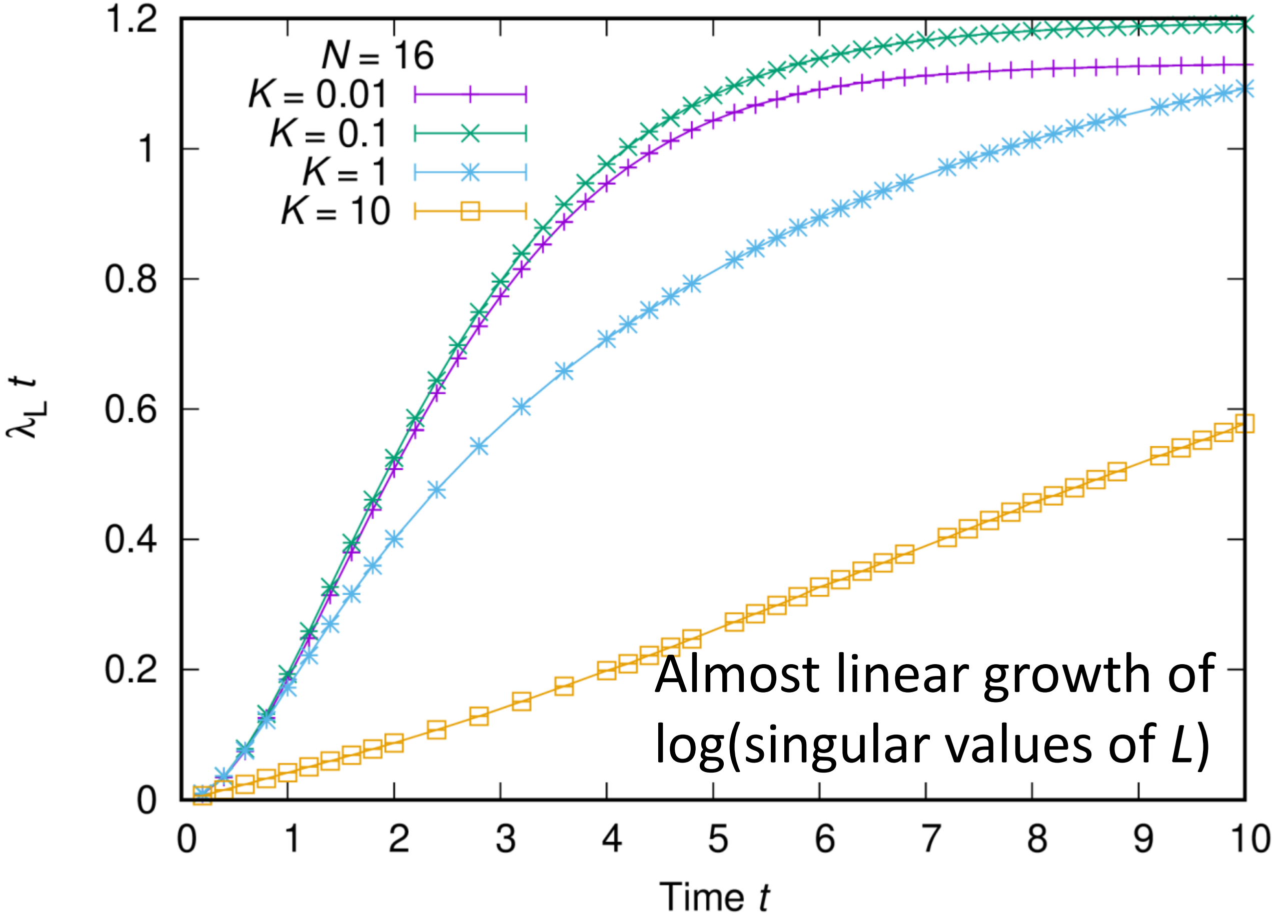
Our definition: for state $\phi$ (e.g. eigenstate)

For matrix $\langle\phi|\hat{L}_{ab}(t)|\phi\rangle$, obtain singular values $\{s_k(t)\}_{k=1}^{N}$

and the Lyapunov spectrum is defined as $\left\{\lambda_k(t) = \frac{\log s_k(t)}{2t}\right\}$.

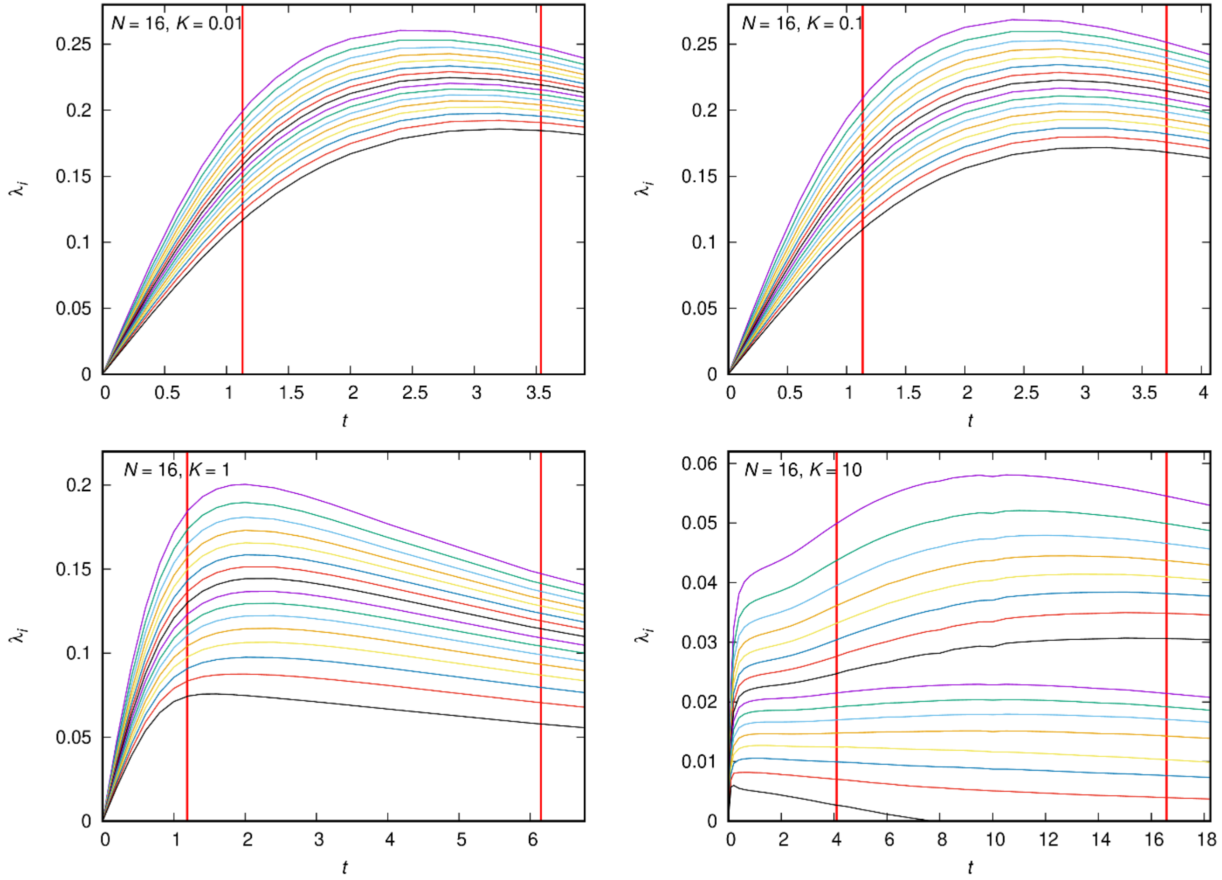
Other possibilities: see Rozenbaum-Ganeshan-Galitski, 1801.10591; Hallam-Morley-Green: 1806.05204

# SYK: dependence on the $SYK_2$ coefficient $K$




Almost linear growth of log(singular values of $L$)

Sample- and state-averaged full Lyapunov spectrum: time dependence




Close to constant between red lines
(20 % and 80 % of the saturated value of $\lambda_{\mathrm{L}} t$)

# Classical KS entropy vs entanglement entropy production

Initial state: $|\psi(t=0)\rangle = |\underbrace{000\ldots000}_{N/2}\rangle$ in the complex fermion basis

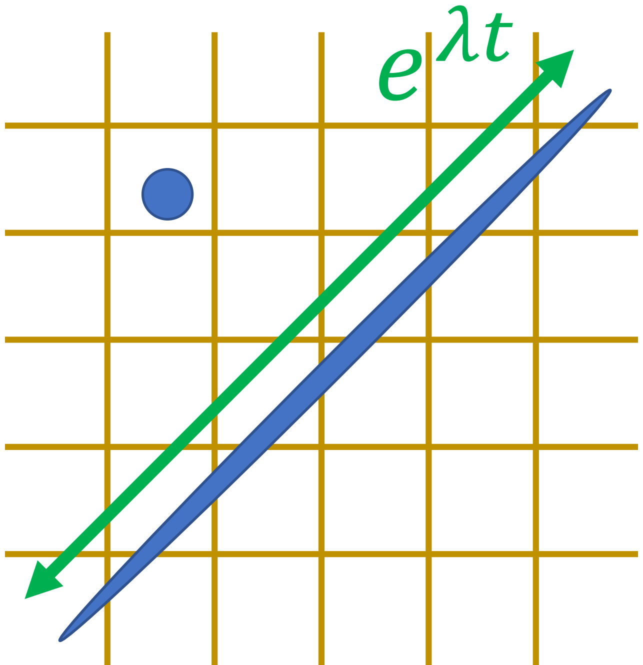

$\hat{c}_j = \frac{(\chi_{2j-1} + \mathrm{i}\chi_{2j})}{\sqrt{2}}$

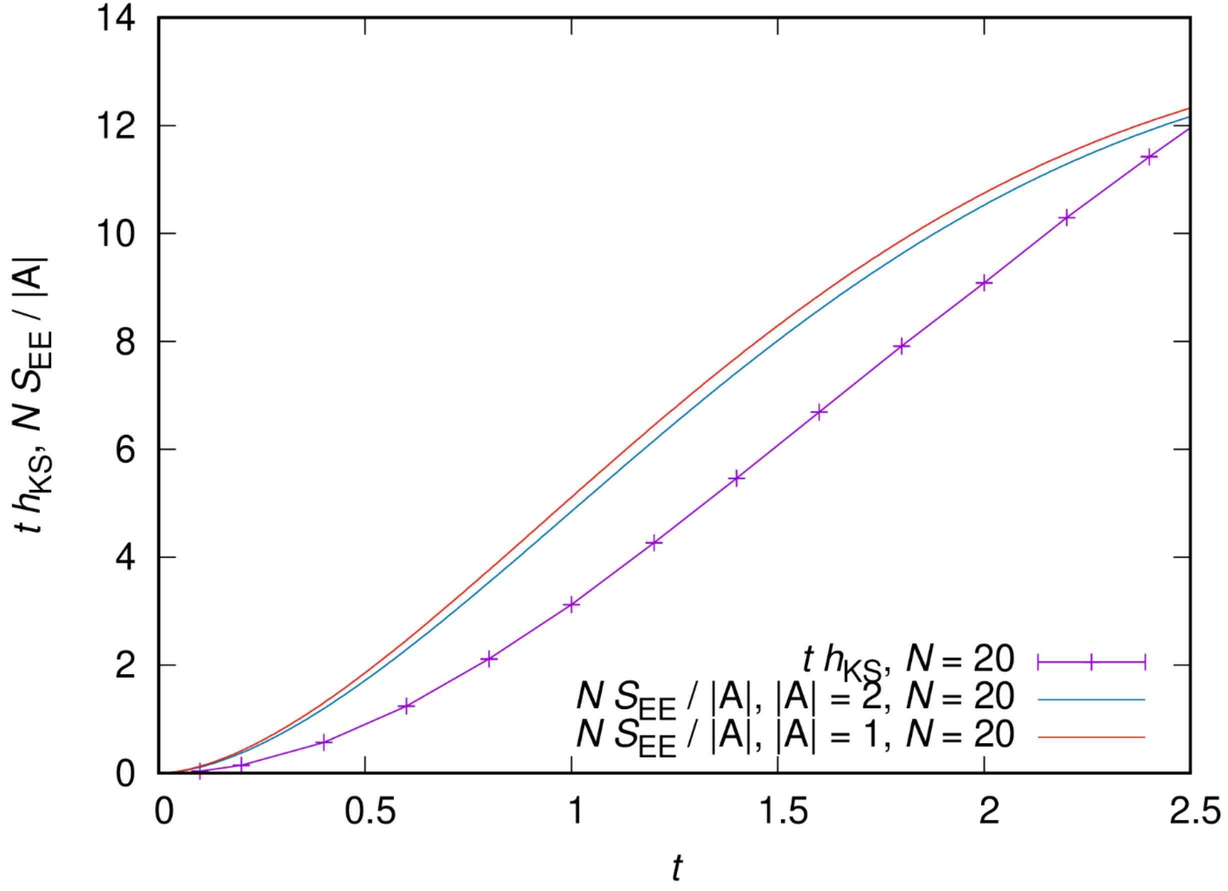
Coarse-grained entropy
= log(# of cells covering the region)
~ (sum of positive $\lambda$ ) $t$

Kolmogorov-Sinai entropy $h_{\mathrm{KS}}$
= (sum of $\lambda$ ) = entropy production rate

$\rho_{\mathrm{A}}(t) = \mathrm{Tr}_{\mathrm{B}}\, \rho(t)$,
$\rho(t) = |\psi(t)\rangle\langle\psi(t)|$

$S_{\mathrm{EE}}(t) = -\mathrm{Tr}\log(\rho_{\mathrm{A}}(t))$



# Classical KS entropy vs entanglement entropy production

Coarse-grained entropy
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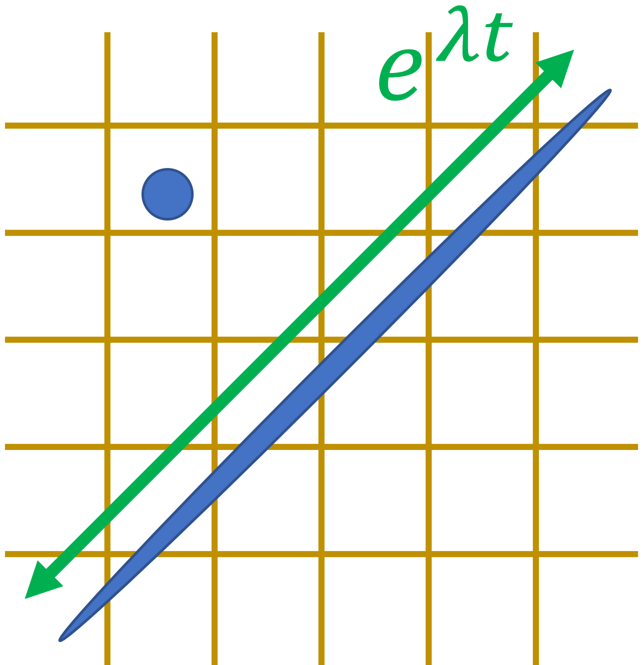
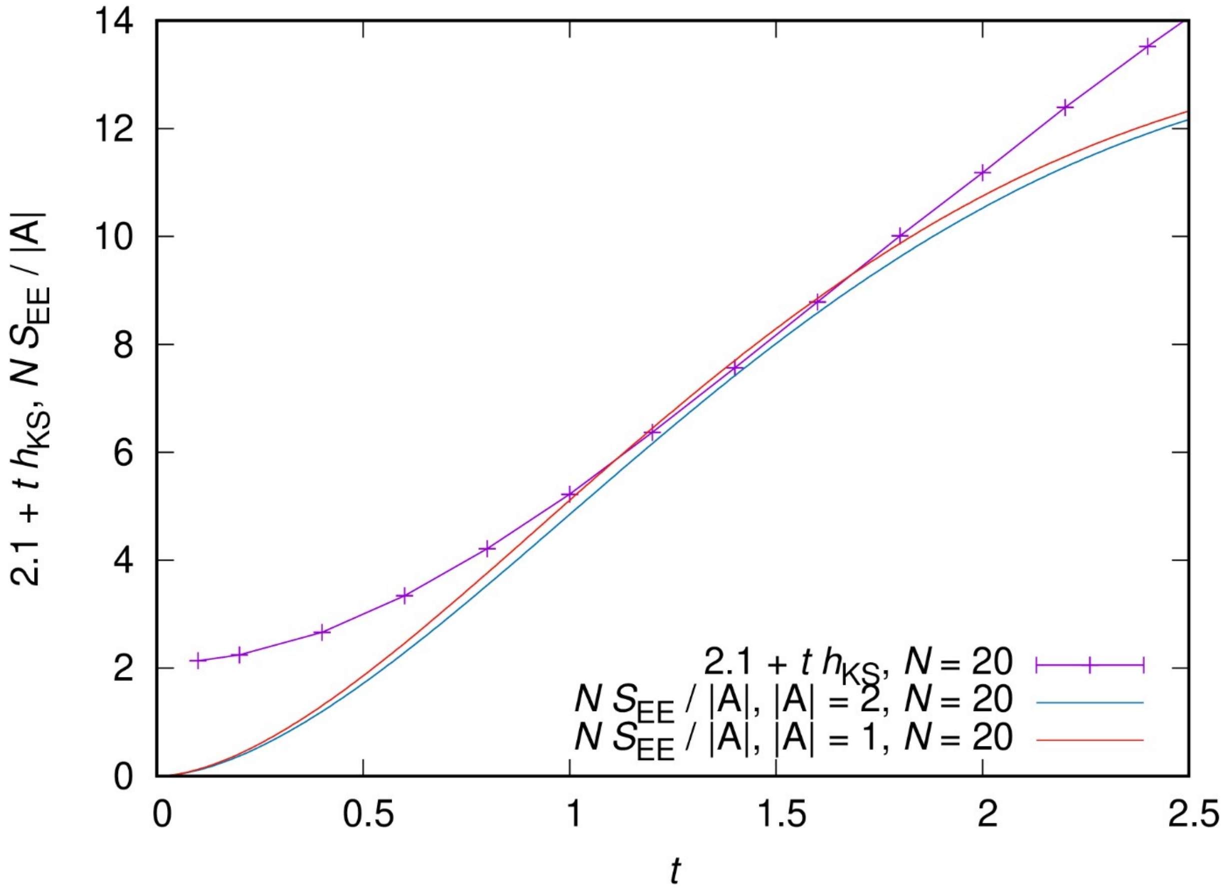
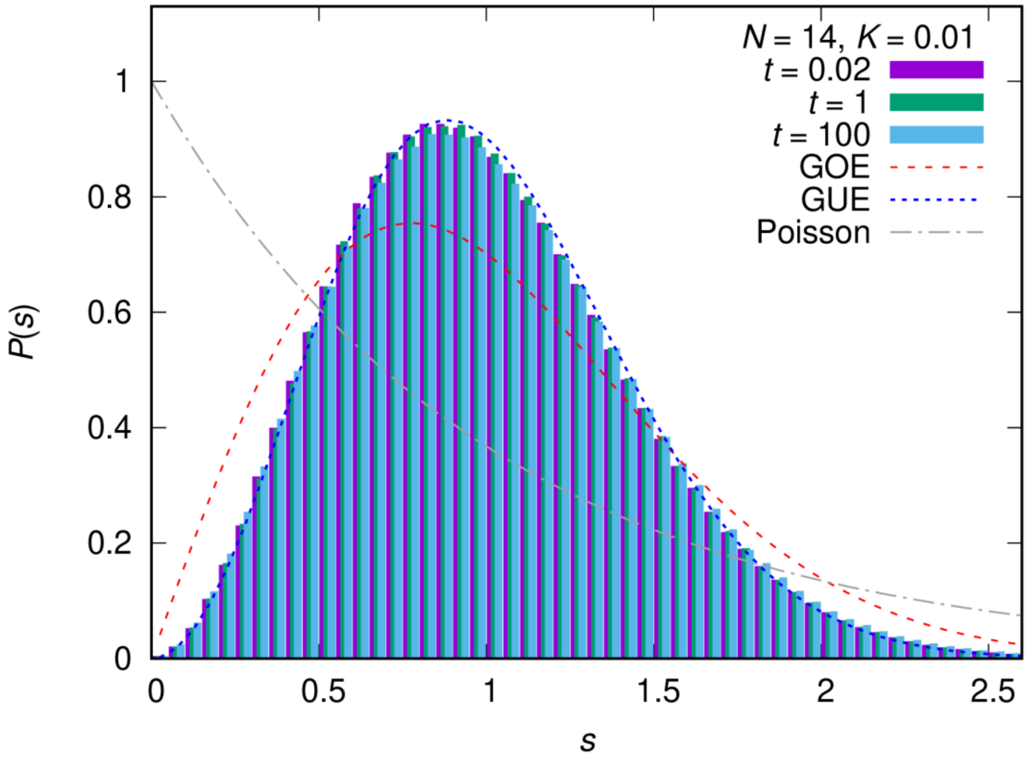


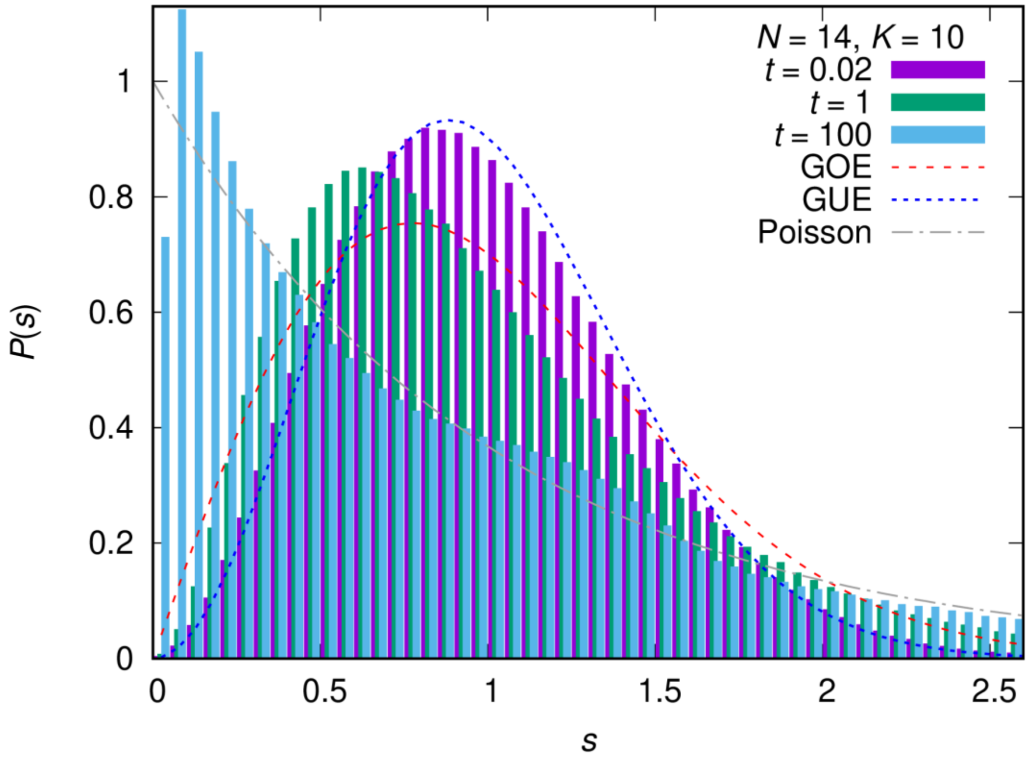

Similar time scale for saturation in SYK model; other models?

# Spectral statistics: SYK

(fixed-*i* unfolding: unfold each gap by its average)




Close to $SYK_4$ :
remains GUE even after long time



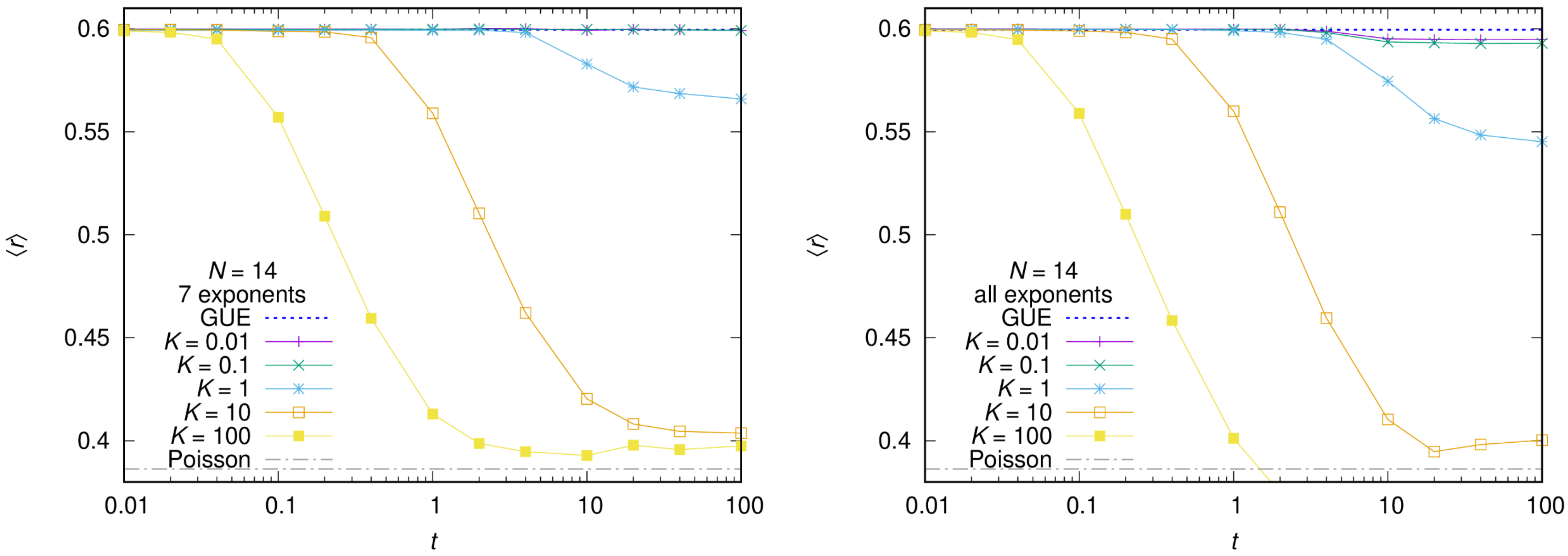

Strong one-body hopping term:
approaches GUE

# Spectral statistics: SYK

$\langle r \rangle$ : average of the adjacent gap ratio $\frac{\min(\epsilon_{i+1}-\epsilon_i\,,\epsilon_{i+2}-\epsilon_{i+1})}{\max(\epsilon_{i+1}-\epsilon_i\,,\epsilon_{i+2}-\epsilon_{i+1})}$

Uncorrelated (Poisson): $2\log 2 - 1 \approx 0.386$

Correlated: larger (GOE: 0.5307, GUE: 0.5996 etc. ) [Atas *et al.*, PRL 2013]



# Summary of this part

- SYK model: an all-to-all random two-body ($SYK_2$) term destroys chaos of $SYK_4$ in the low temperature / long-time limits

- Proposed a definition of the quantum Lyapunov spectrum
- SYK model: linear growth of $\lambda_{\mathrm{L}} t$;
  KS entropy ~ entanglement entropy production rate
- Statistics of Lyapunov spectra: random-matrix like in $SYK_4$ limit, becomes Poisson-like as $SYK_2$ term is introduced (also studied XXZ: MBL)

➔ Do we have to use out-of-time ordered operator products?

# Summary

$$\hat{H}_{\mathrm{SYK}} = \frac{\sqrt{3!}}{N^{3/2}} \sum_{1 \le a < b < c < d \le N} J_{abcd} \hat{\chi}_a \hat{\chi}_b \hat{\chi}_c \hat{\chi}_d$$

black hole

holography

non-gravitational quantum system

cold atom realization

Danshita, Hanada and MT, PTEP 2017 [1606.02454]

maximally chaotic (chaos limit)

$\lambda_{\mathrm{Lyapunov}}$

maximally chaotic (chaos limit)

random matrix behavior of finite-time Lyapunov spectrum

[Classical] Hanada, Shimada and MT, PRE 2018 [1702.02197]

[Quantum] Gharibyan, Hanada, Swingle and MT, 1809.01671

modifications to study chaos / integrable transition, many-body localization

García-García et al., PRL 2018 [1707.02197]

García-García and MT, 1801.03204

numerical analysis; relation to other scrambling systems

Cotler et al., JHEP 2017 [1611.04650]

Gharibyan et al., JHEP 2018 [1803.08050]