The Sachdev-Ye-Kitaev model, scrambling and chaos

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Contents

- The Sachdev-Ye-Kitaev model
  - Large-*N* solvability: conformal symmetry and maximal chaos
  - Experimental proposal 1606.02454 (and realization)
  - Random matrices (RM) and spectral form factor 1611.04650
  - Deformation and suppression of maximal chaos 1707.02197
- Onset of RM behavior in scrambling systems 1803.08050
  - *k*-local and local systems
  - Random circuits
- Characterization of chaos in random systems 1702.06935
  - Quantum Lyapunov spectrum 1809.01671
  - Singular value statistics of two-point correlators 1902.11086

Other related works: 1801.03204, 1812.04770

Collaborators (in SYK-related papers) and references

- Jordan Saul Cotler[a], Guy Gur-Ari[a] (→Google), Masanori Hanada (YITP→Boulder→Southampton)
- Joseph Polchinski[b], Phil Saad[a], Stephen H. Shenker[a], Douglas Stanford[a], Alexandre Streicher[b]
- Ippei Danshita (YITP→Kindai), Hidehiko Shimada (OIST), Hrant Gharibyan[a], Brian Swingle (Maryland)
- Antonio M. García-García (SJTU), Bruno Loureiro (Cambridge), Aurelio Romero-Bermúdez (Leiden)
- Pak Hang Chris Lau (MIT→NTHU), Chen-Te Ma (SCNU & Cape Town), Jeff Murugan (Cape Town & KITP)

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Danshita, Hanada, and MT, PTEP 2017, 083I01 (arXiv:1606.02454)

Cotler, Gur-Ari, Hanada, Polchinski, Saad, Shenker, Stanford, Streicher, and MT, JHEP 1705, 118 (2017) (arXiv:1611.04650)

Hanada, Shimada, and MT, Phys. Rev. E **97**, 022224 (2018) (arXiv:1702.06935)

García-García, Loureiro, Romero-Bermudez, and MT, PRL **120**, 241603 (2018) (arXiv:**1707.02197**)

García-García and MT, Phys. Rev. B **99**, 054202 (2019) (arXiv:**1801.03204**)

Gharibyan, Hanada, Shenker, and MT, JHEP 1807, 124 (2018) (arXiv:1803.08050)

Gharibyan, Hanada, Swingle, and MT, JHEP 1904, 082 (2019) (arXiv:**1809.01671**), submitted (arXiv:**1902.11086**)

Lau, Ma, Murugan, and MT, Phys. Lett. B **795**, 230 (10 August 2019) (arXiv:1812.04770)

The Sachdev-Ye-Kitaev model

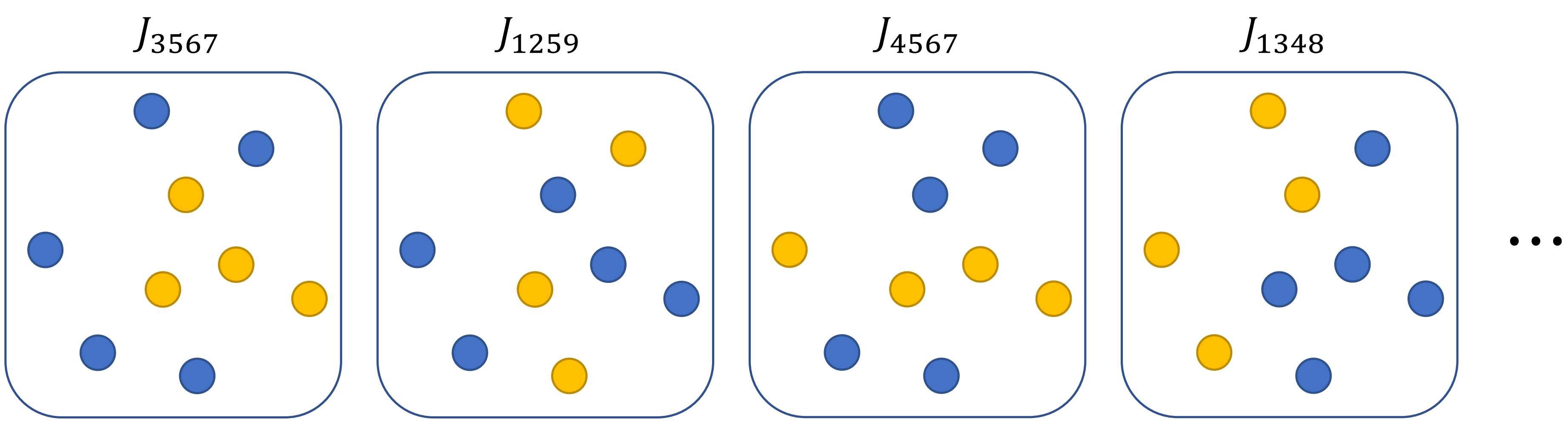
$$\hat{H} = \frac{\sqrt{3!}}{N^{3/2}} \sum_{1\leq a<b<c<d\leq N} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d$$

cf. Sachdev-Ye model (1993)

[A. Kitaev, talks at KITP (2015)]

$\hat{\chi}_{a=1,2,\ldots,N}$: N Majorana fermions ($\{\hat{\chi}_a, \hat{\chi}_b\} = \delta_{ab}$)

J_{abcd} : Gaussian random couplings ($\langle {J_{abcd}}^2 \rangle = J^2 = 1$)



Two versions of the SYK model

N Majorana- or Dirac- fermions randomly coupled to each other

[Majorana version]

$$\hat{H} = \frac{\sqrt{3!}}{N^{3/2}} \sum_{1 \le a < b < c < d \le N} J_{abcd} \hat{\chi}_a \hat{\chi}_b \hat{\chi}_c \hat{\chi}_d$$

[A. Kitaev: talks at KITP
(Feb 12, Apr 7 and May 27, 2015)]

[Dirac version]

$$\hat{H} = \frac{1}{(2N)^{3/2}} \sum_{ij;kl} J_{ij;kl} \hat{c}_i{}^{\dagger} \hat{c}_j{}^{\dagger} \hat{c}_k \hat{c}_l$$

[A. Kitaev's talk]
[S. Sachdev: PRX **5**, 041025 (2015)]

(The first paper by A. Kitaev on the SYK model:
Alexei Kitaev and S. Josephine Suh, arXiv:1711.08467 (JHEP**05**(2018)183);
First papers by J. Ye on the SYK model: arXiv:1809.06667 and arXiv:1809.07577)

Note on the Dirac SYK model

[Dirac version]

$$\hat{H} = \frac{1}{(2N)^{3/2}} \sum_{ij;kl} J_{ij;kl} \hat{c}_i{}^\dagger \hat{c}_j{}^\dagger \hat{c}_k \hat{c}_l$$

Studied for long time in the nuclear theory context

- [J. B. French and S. S. M. Wong, Phys. Lett. B **33**, 449 (1970)]
- [O. Bohigas and J. Flores, Phys. Lett. B **34**, 261 (1971)]

"Two-body Random Ensemble"

Why solvable in the $N \gg 1$ limit?

(after sample average $\langle \cdots \rangle_{\{J\}}$)

Free two-point function $G_0(t)\delta_{ij} = -\langle \mathrm{T}\psi_i(t)\psi_j(0) \rangle = -\frac{1}{2}\mathrm{sgn}(t)\delta_{ij}$

Perturbation expansion by interaction term

$$\hat{H} = \frac{\sqrt{3!}}{N^{3/2}} \sum_{1 \le a<b<c<d \le N} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d$$

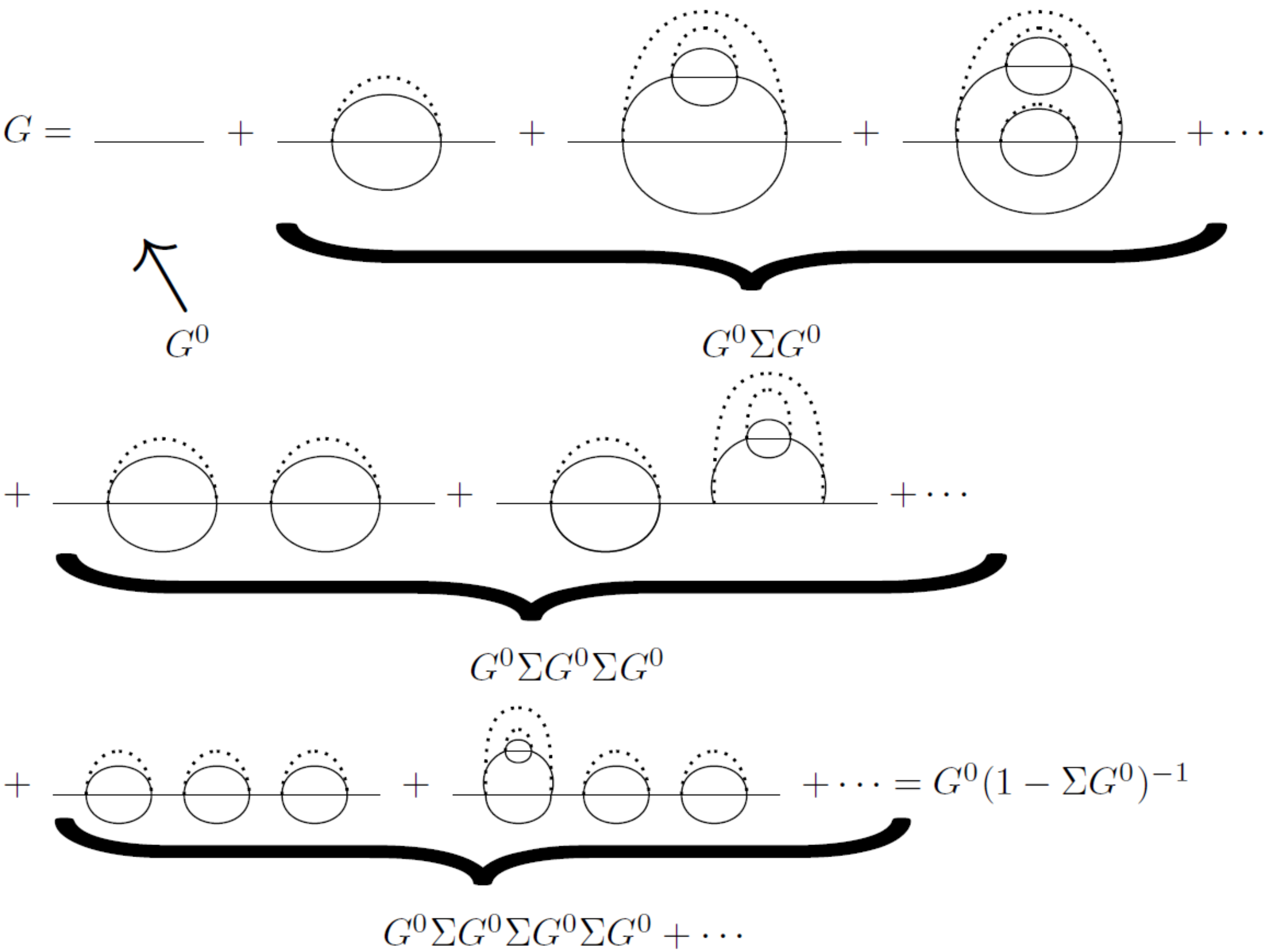
$\langle {J_{abcd}}^2 \rangle_{\{J\}} = J^2$, independent Gaussian distribution

$\langle J_{abcd} J_{abce} \rangle_{\{J\}} = 0$ if $d \neq e$ ➔ Most diagrams average to zero

<u>“Melon-type” diagrams dominate in large *N*</u>

“Melon” diagrams dominate in the $N \gg 1$ limit

Dotted line connects same couplings




•強は異なり
•シグマハイパー核はどこまでわかったか？
•蒸発と乾燥の物理学
•銅酸化物高温超伝導体における擬ギャップと磁場誘起電荷密度波

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BUTSURI
日本物理学会誌

最近の研究から

Sachdev-Ye-Kitaev 模型，ブラックホール，冷却気体系

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Figure from [I. Danshita, M. Tezuka, and M. Hanada: Butsuri **73**(8), 569 (2018)]

Feynman diagrams

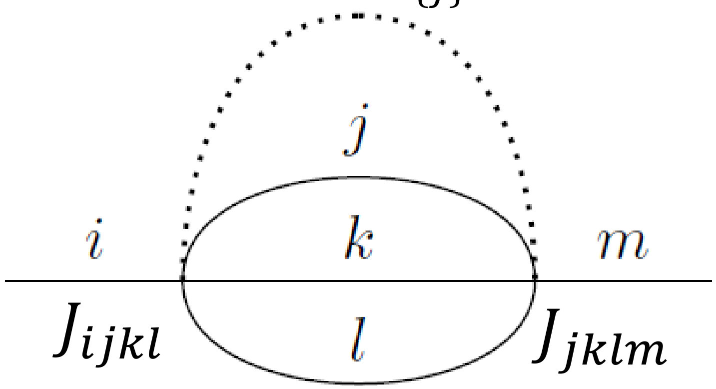
[J. Polchinski and V. Rosenhaus, JHEP 1604 (2016) 001]
[J. Maldacena and D. Stanford, PRD **94**, 106002 (2016)]

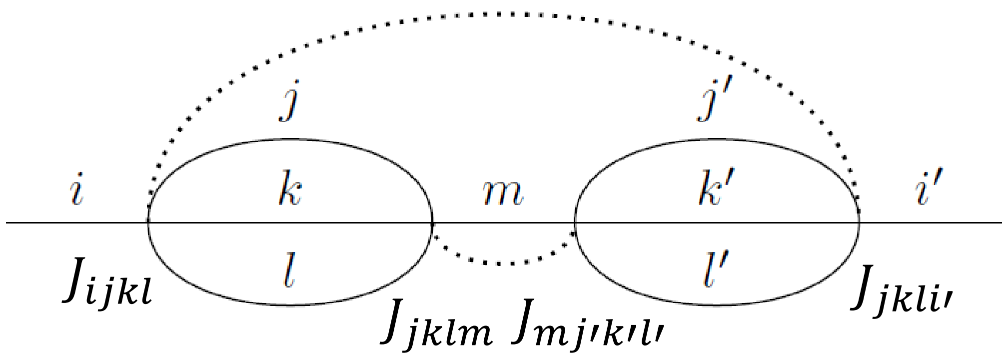
$$\hat{H} = \frac{\sqrt{3!}}{N^{3/2}} \sum_{1\le a<b<c<d\le N} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d$$

$$\{\hat{\chi}_a, \hat{\chi}_b\} = \delta_{ab}$$

$$\langle J_{abcd}{}^2\rangle = J^2 = 1$$

Sample average $\langle\cdots\rangle_{\{J\}}$





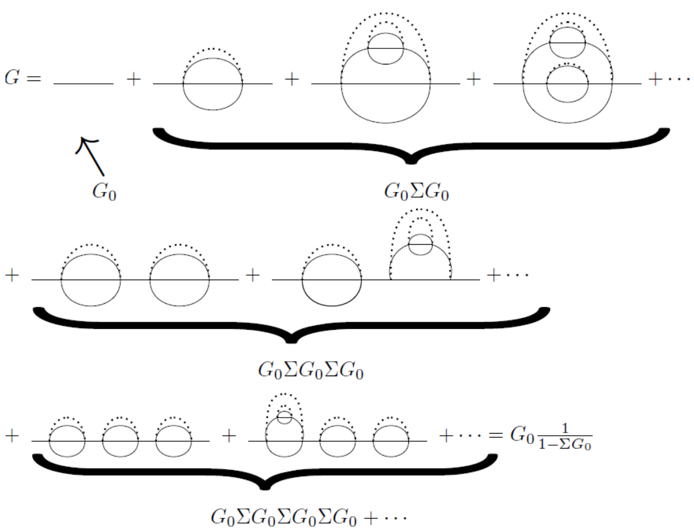
$$\sum_{jkl} \langle J_{ijkl} J_{jklm}\rangle_{\{J\}} = \frac{N^3}{3!}\delta_{im}$$

$$\sum_{m\neq i}\sum_{jklj'k'l'} \langle J_{ijkl}J_{jklm}J_{mj'k'l'}J_{j'k'l'i'}\rangle_{\{J\}} \propto N^4\delta_{ii'}$$

➡ $O(N^0)$ contribution

➡ $O(N^{-2})$ contribution

Reparametrization symmetry



$$G(1-\Sigma G_0) = G_0$$

$$G^{-1} = G_0{}^{-1} - \Sigma$$

$$G(i\omega)^{-1} = \boxed{i\omega} - \Sigma(i\omega) \qquad \Sigma = J^2 G^3$$

Low energy ($\omega, T \ll J$): ignore $i\omega$ and we have

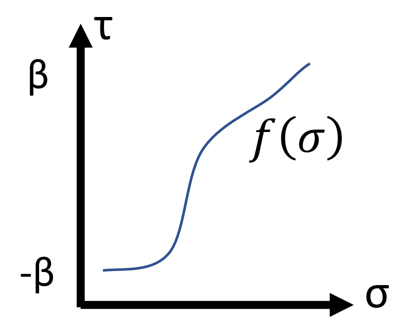
$$\int dt G(t_1, t)\Sigma(t, t_2) = -\delta(t_1, t_2)$$

Invariant under imaginary time reparametrization

$$\tau = f(\sigma),$$

$$G(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-1/4} \frac{g(\sigma_1)}{g(\sigma_2)} G(\sigma_1, \sigma_2),$$

$$\tilde{\Sigma}(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-3/4} \frac{g(\sigma_1)}{g(\sigma_2)} \tilde{\Sigma}(\sigma_1, \sigma_2),$$



Large-N saddle point solution

$$\int dt G(t_1, t) \Sigma(t, t_2) = -\delta(t_1, t_2)$$

(Derived in replica formalism; assume replica symmetry)

$$G_s(\tau_1 - \tau_2) \sim (\tau_1 - \tau_2)^{-1/2} \quad , \quad \Sigma_s(\tau_1 - \tau_2) \sim (\tau_1 - \tau_2)^{-3/2}$$

Not invariant under arbitrary reparametrization,
but invariant under

$$f(\tau) = \frac{a\tau + b}{c\tau + d} \quad , \quad ad - bc = 1.$$

Symmetry broken to $SL(2, R)$.
cf. isometry group of AdS_2
[see e.g. A. Strominger, hep-th/9809027]

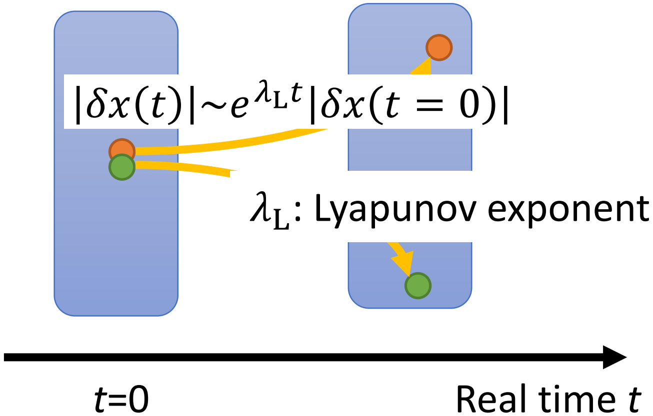
SYK: Nearly CFT_1 "$NCFT_1$"
emergent conformal gauge invariance
[Sachdev, PRX **5**, 041025 (2015)]

[J. Maldacena and D. Stanford, Phys. Rev. D **94**, 106002 (2016)]
Study of the Goldstone modes: *e.g.* [D. Bagrets, A. Altland, and A. Kamenev, Nucl. Phys. B **911**, 191 (2016)]

Definition of Lyapunov exponent using out-of-time-order correlators

$$F(t) = \langle \hat{W}^\dagger(t)\hat{V}^\dagger(0)\hat{W}(t)\hat{V}(0)\rangle \qquad W(t) = \mathrm{e}^{iHt}W\mathrm{e}^{-iHt}$$

Classical:
Infinitesimally different initial states



$$\{x(t), p(0)\}_{\mathrm{PB}}{}^2 = \left(\frac{\partial x(t)}{\partial x(0)}\right)^2 \to e^{2\lambda_{\mathrm{L}} t}$$

Consider operators V and W,

$$C(t) = \langle |[W(t), V(t=0)]|^2\rangle$$
$$= 2\big(1 - \mathrm{Re}\, F(t)\big)$$

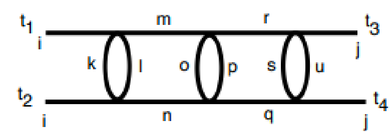
quantifies strength of quantum scrambling

"Black holes are fastest quantum scramblers"
[P. Hayden and J. Preskill 2007] [Y. Sekino and L. Susskind 2008] [Shenker and Stanford 2014]

Chaos bound $\lambda_{\mathrm{L}} = 2\pi k_{\mathrm{B}} T/\hbar$ [J. Maldacena, S. H. Shenker, and D. Stanford, JHEP08(2016)106] saturated by large-N SYK model [Maldacena and Stanford, PRD **94**, 106002 (2016)]

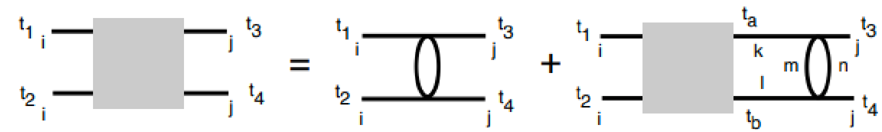
Out-of-time-ordered correlators (OTOCs)

$$\left\langle \hat{\chi}_i(t_1)\hat{\chi}_i(t_2)\hat{\chi}_j(t_3)\hat{\chi}_j(t_4)\right\rangle$$

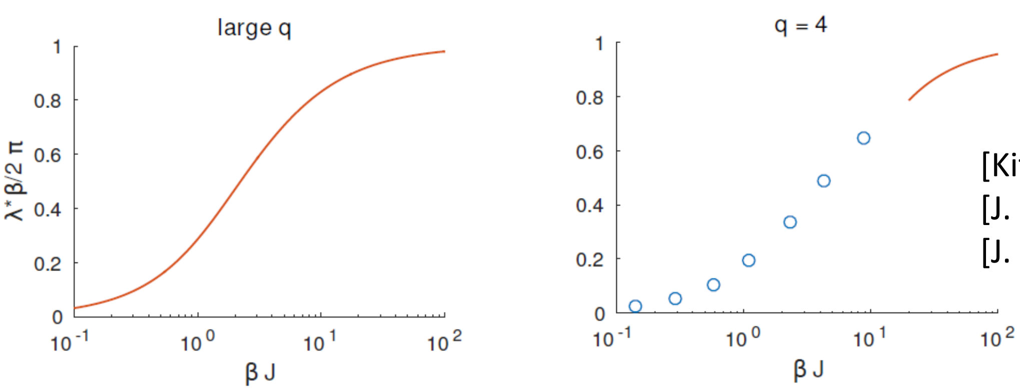


(a)

Regularized OTOC can be calculated for large-*N* SYK model, satisfies the chaos bound $\lambda_{\mathrm{L}} = 2\pi k_{\mathrm{B}}T/\hbar$ at low *T* limit



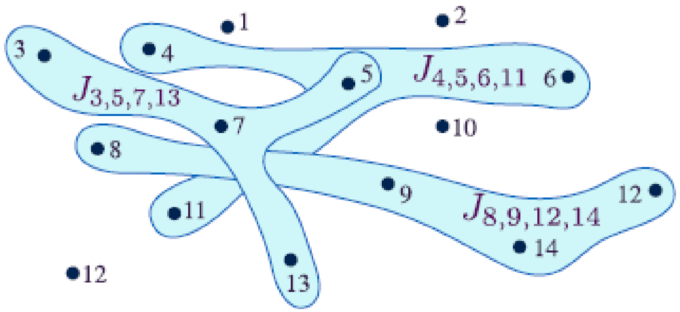
$$\Gamma(t_1,t_2,t_3,t_4) = \Gamma_0(t_1,t_2,t_3,t_4) + \int dt_a dt_b\, \Gamma(t_1,t_2,t_a,t_b)K(t_a,t_b,t_3,t_4)$$



[Kitaev's talks]
[J. Polchinski and V. Rosenhaus, JHEP 1604 (2016) 001]
[J. Maldacena and D. Stanford, Phys. Rev. D **94**, 106002 (2016)]

Holographic connection to gravity

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$



$$\mathcal{Q} = \frac{1}{N} \sum_i \left\langle c_i^\dagger c_i \right\rangle .$$

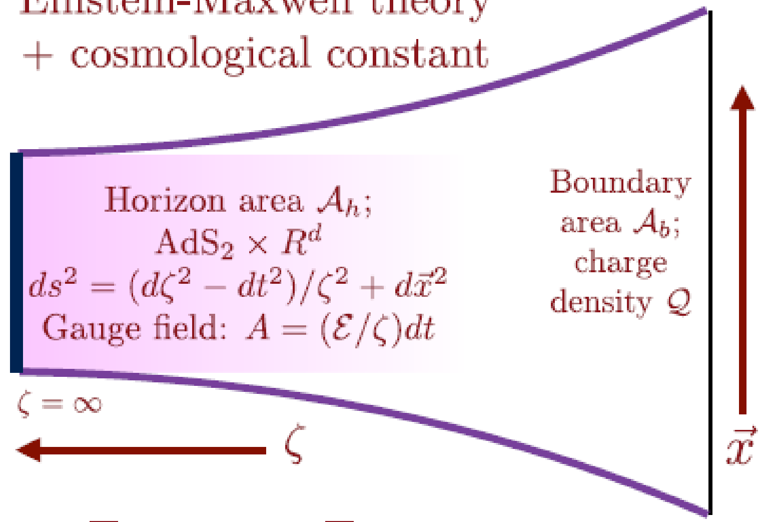
$$-\left\langle c_i(\tau) c_i^\dagger(0) \right\rangle \sim \begin{cases} -\tau^{-1/2} & , \ \tau > 0 \\ e^{-2\pi\mathcal{E}}\, |\tau|^{-1/2} & , \ \tau < 0. \end{cases}$$

Known "equation of state" determines $\mathcal{E}$ as a function of $\mathcal{Q}$

Microscopic zero temperature entropy density $\mathcal{S}$ obeys

$$\frac{\partial \mathcal{S}}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$$

Einstein-Maxwell theory + cosmological constant



$$\mathcal{L} = \overline{\psi}\Gamma^\alpha D_\alpha \psi + m\overline{\psi}\psi$$

$$-\left\langle \psi(\tau)\overline{\psi}(0) \right\rangle \sim \begin{cases} -\tau^{-1/2} & , \ \tau > 0 \\ e^{-2\pi\mathcal{E}}\, |\tau|^{-1/2} & , \ \tau < 0. \end{cases}$$

"Equation of state" relating $\mathcal{E}$ and $\mathcal{Q}$ depends upon the geometry of spacetime far from the AdS_2

Black hole thermodynamics (classical general relativity) yields

$$\frac{\partial \mathcal{S}_{\rm BH}}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$$

[S. Sachdev, Phys. Rev. X **5**, 041025 (2015)]

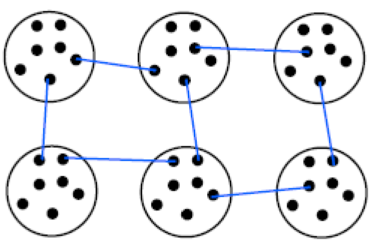
Different models with similar solutions

$$S_{\mathrm{Gurau-Witten}} = \int dt \left(\frac{i}{2}\psi_A^{abc}\partial_t\psi_A^{abc} + g\psi_0^{abc}\psi_1^{ade}\psi_2^{fbe}\psi_3^{fdc}\right)$$

"An SYK-Like Model Without Disorder"
E. Witten, arXiv:1610.09758

$$I = \int \mathrm{d}t \left(\frac{\mathrm{i}}{2}\sum_i \psi_i \frac{\mathrm{d}}{\mathrm{d}t}\psi_i - \mathrm{i}^{q/2} j\psi_0\psi_1\dots\psi_D\right)$$

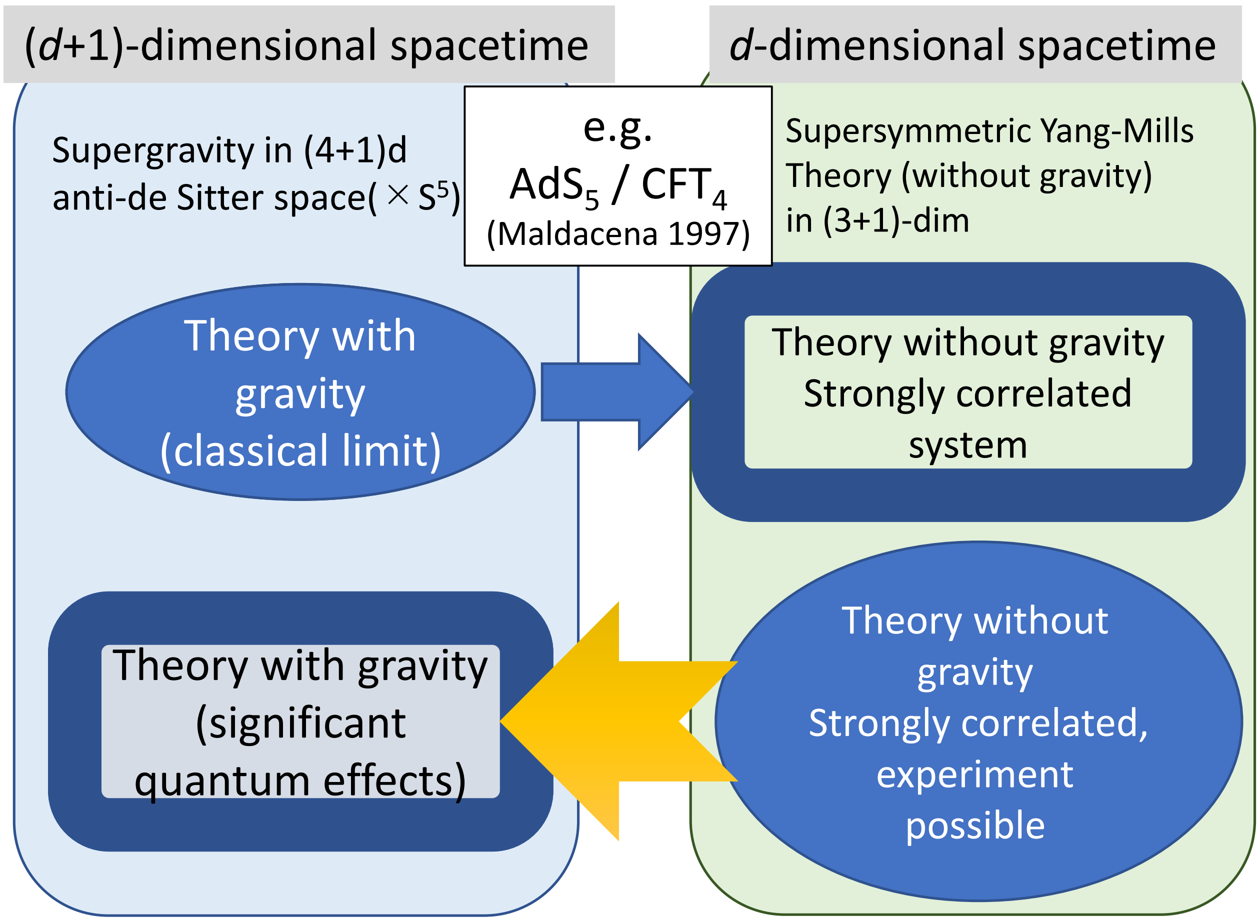
"Uncolored Random Tensors, Melon Diagrams, and the SYK Models"
I. R. Klebanov and G. Tarnopolsky, PRD **95**, 046004 (2017).



Y. Gu, X.-L. Qi, and D. Stanford, "Local criticality, diusion and chaos in generalized Sachdev-Ye-Kitaev models," JHEP05 (2017) 125;
X.-Y. Song, C.-M. Jian, and L. Balents, "Strongly Correlated Metal Built from Sachdev-Ye-Kitaev Models," PRL **119**, 216601 (2017).

See review: V. Rosenhaus: "An introduction to the SYK model" arXiv:1807.03334
cf. K. Okuyama: "Replica symmetry breaking in random matrix model: a toy model of wormhole networks" arXiv:1903.11776

Proposal for experiment



◎ Not limited to classical limit

➔Several supporting evidences

e.g. check of the leading gravity correction for the black hole mass [M. Hanada, Y. Hyakutake, G. ishiki, and J. Nishimura, Science **344**, 882 (2013)]

Many “AdS/CMT” applications

This work:
approach quantum gravity by realizing corresponding non-gravity models in cold gases

Our proposal: coupled atom-molecule model [arXiv:1606.02454]

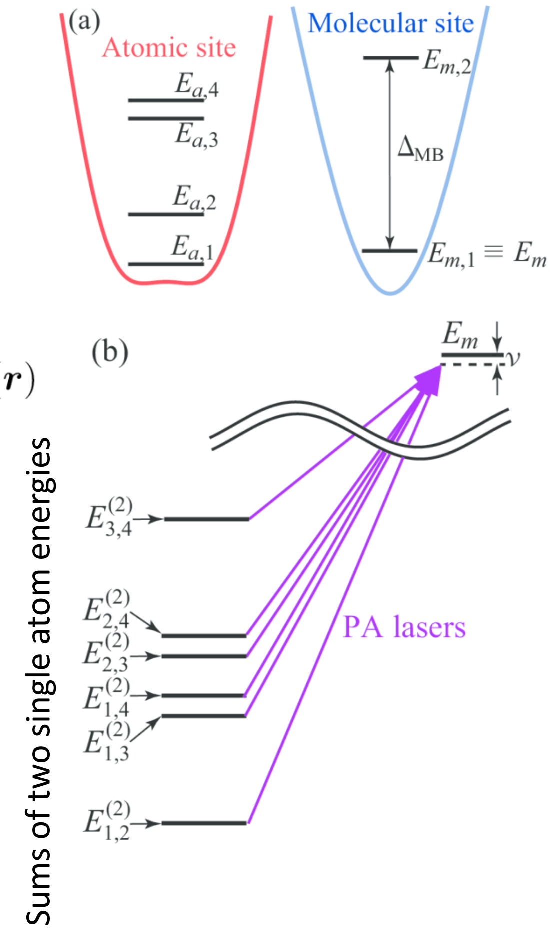
Consider atomic levels *i,j*,… = 1, 2, …, *N* coupled to a molecule state m_1

$$\hat{H}_{\mathrm{m1}} = \nu \hat{m}^\dagger \hat{m} + \sum_{i,j} g_{ij} (\hat{m}^\dagger \hat{c}_j \hat{c}_i + h.c.)$$

$$g_{ij} = \frac{1}{2}\mathrm{sgn}(j-i) \int d\boldsymbol{r}\, \Omega_{i,j}(\mathbf{r}) w_m(\boldsymbol{r}) w_{a,i}(\boldsymbol{r})\, w_{a,j}(\boldsymbol{r})$$

Detuning ν: controlled by laser energy

$\Omega_{i,j}$: space-dependent photoassociation laser
w_m: molecular site wavefunction
$w_{a,i(j)}$: atomic site wavefunction



$s = 1, 2, \ldots, n_s$

Consider multiple molecular states; assume they are short-lived
➔ integrate them out and obtain the effective model for atoms

$$\hat{H}_{\mathrm{eff}} = \sum_{s,i,j,k,l} \frac{g_{s,ij} g_{s,kl}}{\nu_s} \hat{c}_i^\dagger \hat{c}_j^\dagger \hat{c}_k \hat{c}_l$$

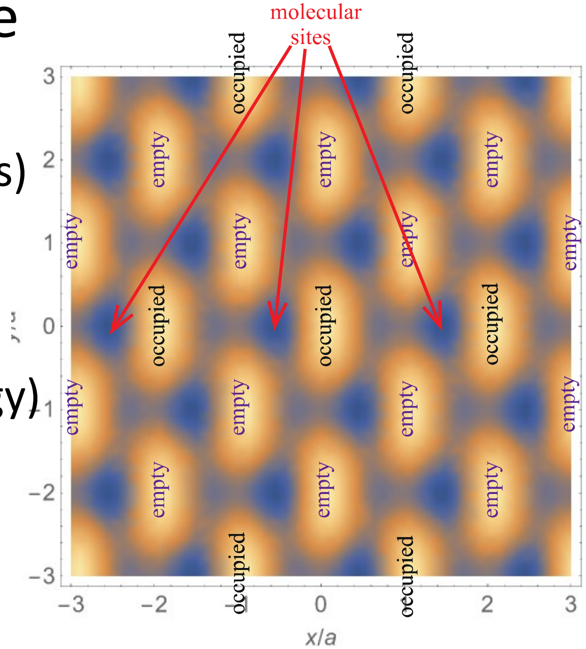
Optical lattice setup in our proposal

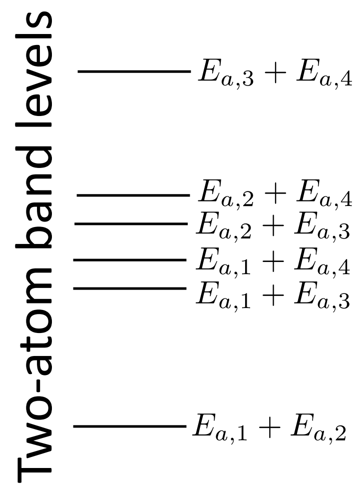
A double-well optical lattice

(no degeneracy in the band levels)

with ^{6}Li

(large recoil energy)





Possible to satisfy required conditions

$$\max(t_i) \lesssim \hbar/\tau_{\exp} \ll J,$$
$$\max(\hbar\Gamma_{\rm PA}, \hbar\Gamma_{{\rm ms},s}) \ll |\nu_s| \ll \Delta_{\min}, \text{ for all } s,$$
$$\Delta_{\max} < \Delta_{\rm MB} < \tilde{\Delta},$$
$$|\nu_s| \ll |U_{s,s'}|, \text{ for all } s \text{ and } s',$$
$$|U_{s,s'}| < \Delta_{\min} \text{ or } \Delta_{\max} < |U_{s,s'}|, \text{ for all } s \text{ and } s'.$$

Realizing real Dirac SYK model

$$\hat{H}_{\text{eff}} = \sum_{s,i,j,k,l} \frac{g_{s,ij} g_{s,kl}}{\nu_s} \hat{c}_i^\dagger \hat{c}_j^\dagger \hat{c}_k \hat{c}_l$$

$s = 1, 2, \ldots, n_s$

(For simplicity we take $\nu_s = (-1)^s \sqrt{n_s}\sigma_s$)

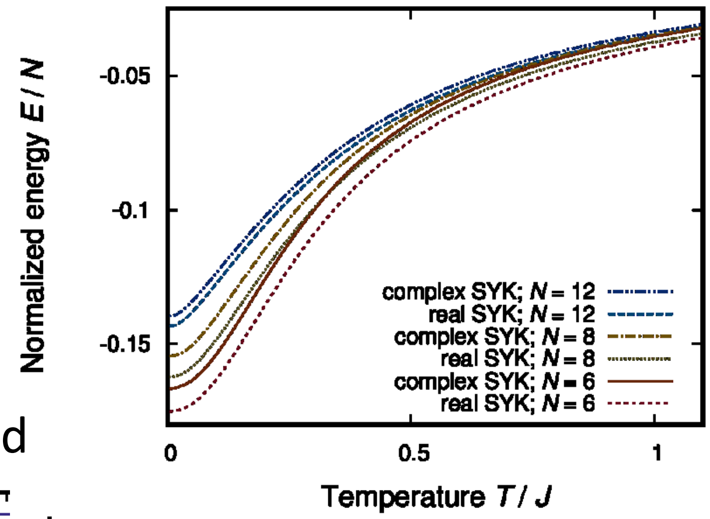
Can be shown to approach the real Dirac version of the SYK model as $n_s \to \infty$.

$$\hat{H} = \frac{1}{(2N)^{3/2}} \sum_{ijkl} J_{ij;kl} \hat{c}_i^\dagger \hat{c}_j^\dagger \hat{c}_k \hat{c}_l,$$

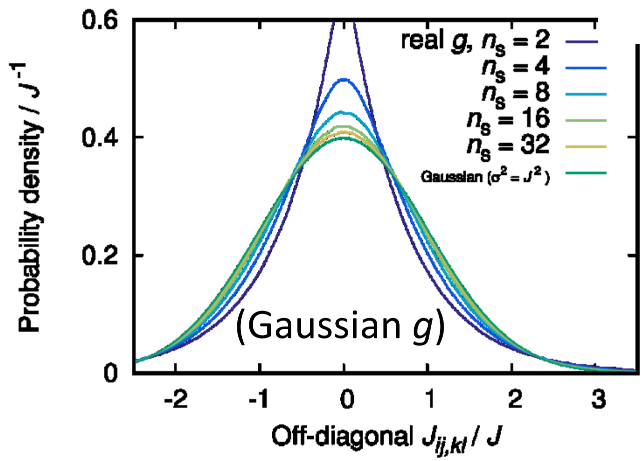
$$J_{ij;kl} = -J_{ji;kl} = -J_{ij;lk},$$

$$J_{ij;kl} = J_{kl,ij}$$

$$\overline{|J_{ij;kl}|^2} = \begin{cases} J^2 & (\{i,j\} \neq \{k,l\}) \\ 2J^2 & (\{i,j\} = \{k,l\}) \end{cases}$$



Gaussian J reproduced

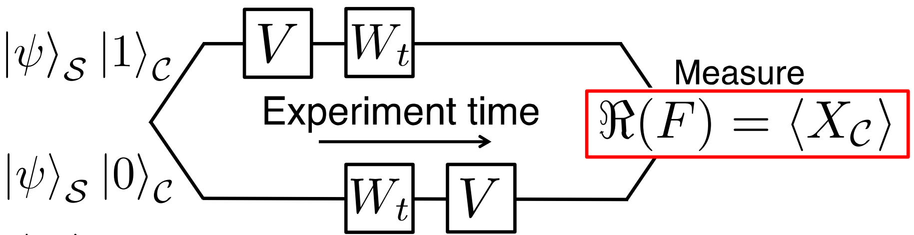


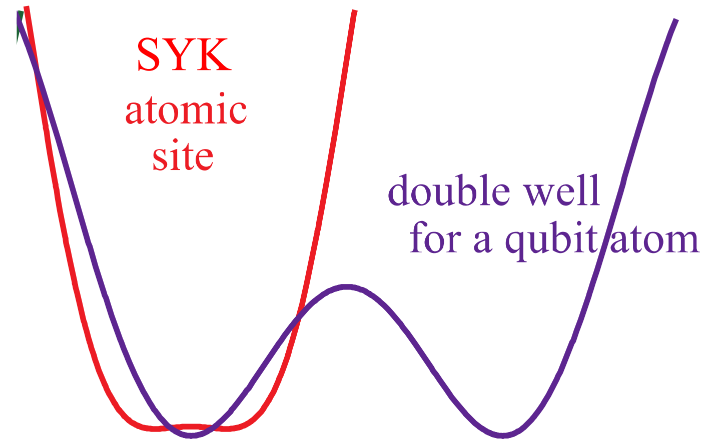
Real SYK:
Physical quantities coincide with those for complex SYK in $N \to \infty$ limit

Out-of-time-order correlation measurement

Interferometric protocol proposed in
B. Swingle *et al.*: PRB **94**, 040302 (2016)

$$F(t) = \langle \hat{W}^\dagger(t)\hat{V}^\dagger(0)\hat{W}(t)\hat{V}(0)\rangle.$$





$|\psi\rangle_S$: Initial state of the probed system

$|0\rangle_C, |1\rangle_C$: states of the control qubit

$$\widehat{W}(t) = \boxed{e^{iHt}}\,\widehat{W}e^{-iHt}$$

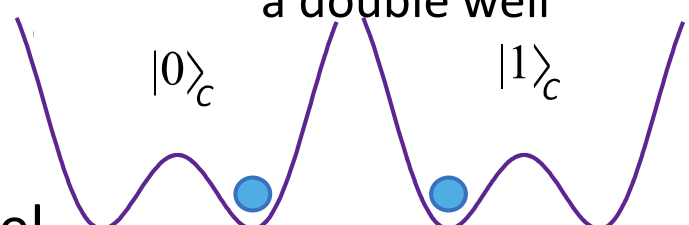
Time evolution with
$H' = -H$ ($\nu' = -\nu$)

Create the cat state

$$|\Psi\rangle = \widehat{W}(t)\hat{V}|\psi\rangle_S|1\rangle_C + \hat{V}\widehat{W}(t)|\psi\rangle_S|0\rangle_C$$

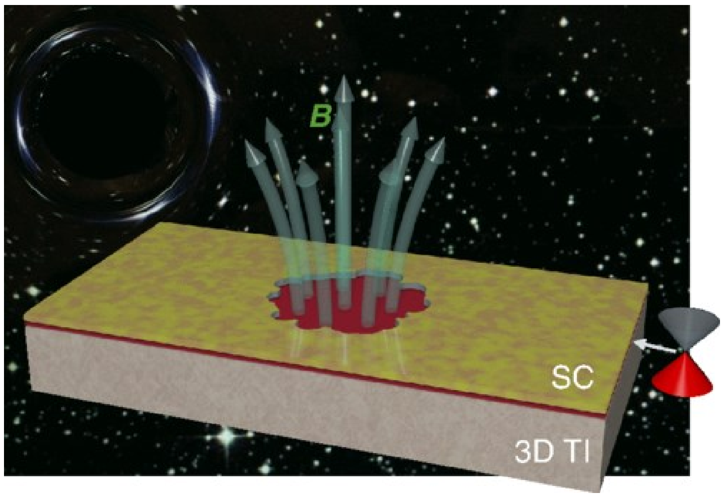
by applying
$\widehat{I_S} \otimes |0\rangle\langle 0|_C + \hat{V} \otimes |1\rangle\langle 1|_C$, $\widehat{W}(t) \otimes \widehat{I_C}$,
and $\hat{V} \otimes |0\rangle\langle 0|_C + \widehat{I_S} \otimes |1\rangle\langle 1|_C$ in this order, then
measure the qubit to find $\mathrm{Re}\,F(t)$ and $\mathrm{Im}\,F(t)$.

Our qubit *C*:
A single particle in
a double well



➔Implementation of this protocol in our model
using a qubit on additional optical double well [1606.02454]

Proposals for experimental realization

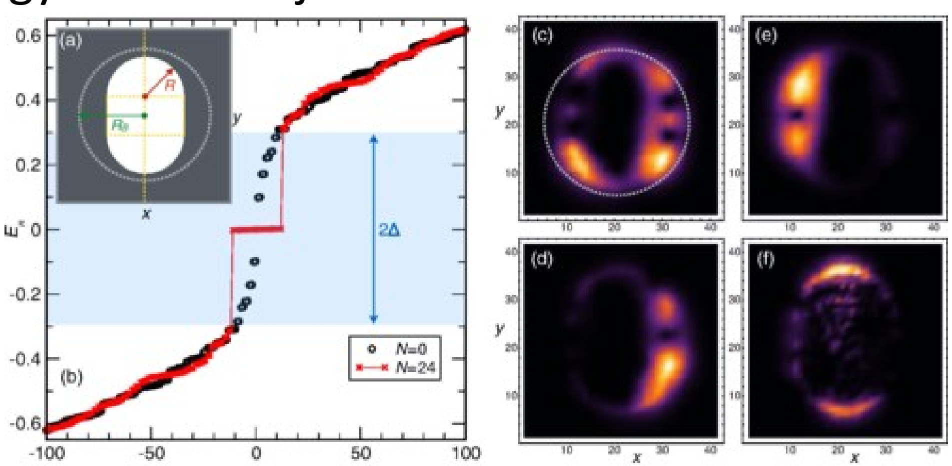


arXiv:1702.04426

N quanta of magnetic flux through a nanoscale hole

Inhomogeneous wave functions due to the irregular shape of the hole

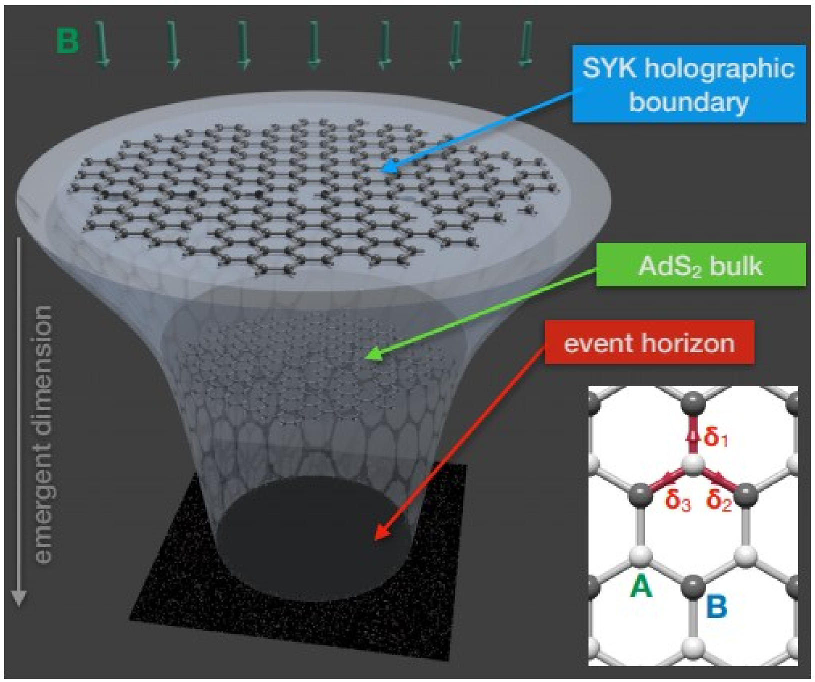
Zero energy states: Majorana fermions



D. I. Pikulin and M. Franz, "Black Hole on a Chip: Proposal for a Physical Realization of the Sachdev-Ye-Kitaev model in a Solid-State System", PRX **7**, 031006 (2017)

Proposals for experimental realization

arXiv:1802.00802



Anffany Chen, R. Ilan, F. de Juan, D.I. Pikulin, M. Franz,
“Quantum holography in a graphene flake with an irregular boundary”,
arXiv:1802.00802 [PRL **121**, 036403 (2018)]

Review Article | Published: 29 November 2018

Mimicking black hole event horizons in atomic and solid-state systems

Marcel Franz & Moshe Rozali

Nature Reviews Materials **3**, 491–501 (2018) | Download Citation

NMR experiment for the SYK model

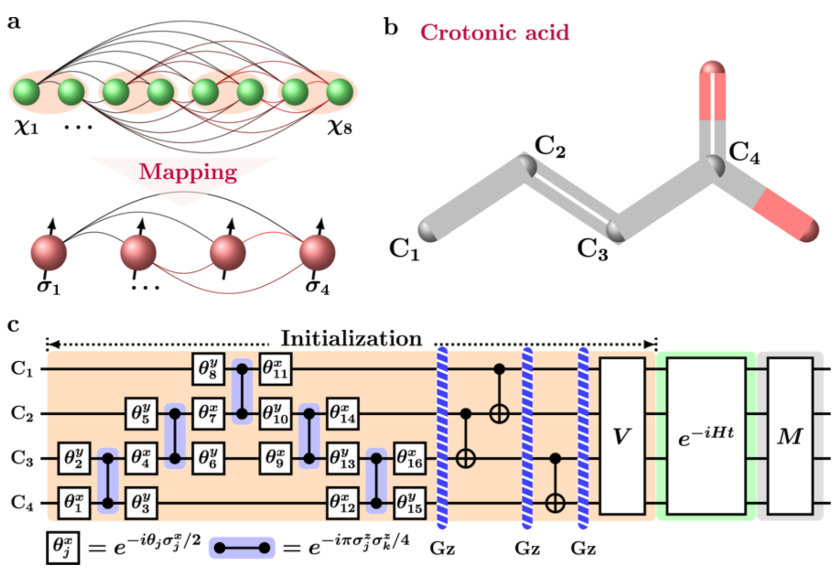
“Quantum simulation of the non-fermi-liquid state of Sachdev-Ye-Kitaev model” Zhihuang Luo, Yi-Zhuang You, Jun Li, Chao-Ming Jian, Dawei Lu, Cenke Xu, Bei Zeng and Raymond Laflamme, npj Quantum Information **5**, 53 (2019)

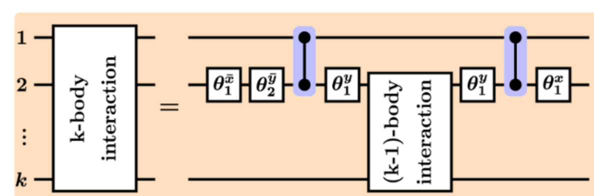
$$H = \frac{J_{ijkl}}{4!}\chi_i\chi_j\chi_k\chi_l + \frac{\mu}{4}C_{ij}C_{kl}\chi_i\chi_j\chi_k\chi_l$$

$$\chi_{2i-1} = \frac{1}{\sqrt{2}}\sigma_x^1\sigma_x^2\cdots\sigma_x^{i-1}\sigma_z^i, \chi_{2i} = \frac{1}{\sqrt{2}}\sigma_x^1\sigma_x^2\cdots\sigma_x^{i-1}\sigma_y^i.$$

$$H = \sum_{s=1}^{70} H_s = \sum_{s=1}^{70} a_{ijkl}^s \sigma_{\alpha_i}^1\sigma_{\alpha_j}^2\sigma_{\alpha_k}^3\sigma_{\alpha_l}^4$$

$$e^{-iH\tau} = \left(\prod_{s=1}^{70} e^{-iH_s\tau/n}\right)^n + \sum_{s<s'}\frac{[H_s, H_{s'}]\tau^2}{2n} + O(|a|^3\tau^3/n^2),$$





The Sachdev-Ye-Kitaev model

N Majorana- or Dirac- fermions randomly coupled to each other

[Majorana version]

$$\hat{H} = \frac{\sqrt{3!}}{N^{3/2}} \sum_{1 \le a < b < c < d \le N} J_{abcd} \hat{\chi}_a \hat{\chi}_b \hat{\chi}_c \hat{\chi}_d$$

[A. Kitaev: talks at KITP (Feb 12, Apr 7 and May 27, 2015)]

[Dirac version]

$$\hat{H} = \frac{1}{(2N)^{3/2}} \sum_{ij;kl} J_{ij;kl} \hat{c}_i^{\dagger} \hat{c}_j^{\dagger} \hat{c}_k \hat{c}_l$$

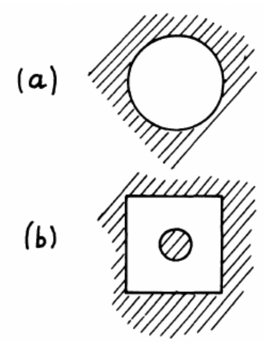
[A. Kitaev's talk]
[S. Sachdev: PRX **5**, 041025 (2015)]

- Solvable in the large *N* limit, Sachdev-Ye "spin liquid" ground state
- Nearly conformal symmetric at low temperature ("emergent …")
- Realizes the Maldacena-Shenker-Stanford chaos bound $\lambda_{\mathrm{L}} = 2\pi k_{\mathrm{B}} T/\hbar$
- Holographically corresponds to a quantum black hole?
- Experimentally realized for small *N*

Generalizations: *q*-fermion interactions "SYK_q", supersymmetric SYK, lattice of SYK lands; etc.

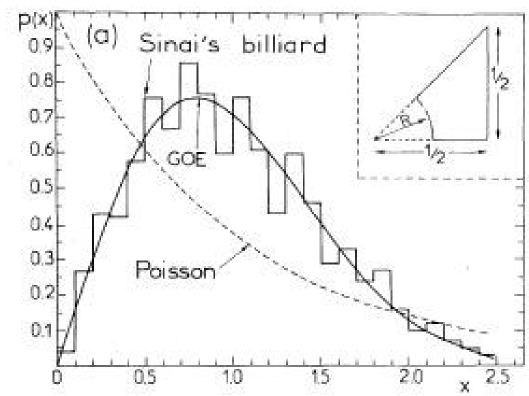
The Bohigas-Giannoni-Schmit conjecture

Assume quantum mechanical systems with a classical limit



circular: integrable

Sinai billiard: chaotic



“Spectral statistics of chaotic systems can be described as a random matrix”

O. Bohigas, M. J. Giannoni, and C. Schmit, Phys. Rev. Lett. 52, 1 (1984);
J. de Phys. Lett. 45, 1015 (1984).

Justifications:

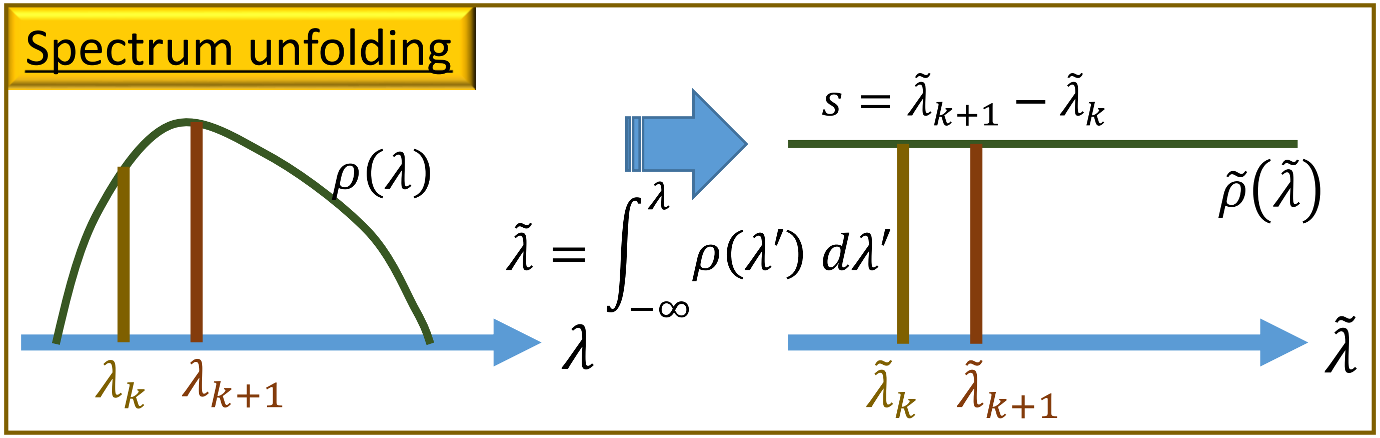
Non-linear sigma-model (Andreev 1993, Altland 2015)

Gutzwiller trace formula in terms of periodic orbits (Berry 1985, Gutzwiller 1990, Sieber, Richter, Braun, Muller, Heusler, …)

Also more examples including systems without clear classical version

Random matrices: level repulsion and spectral rigidity

Assume unfolded spectrum (rescaled so that average distance = 1)



Short range

- $P(s)$: Distribution of normalized level separation s

- $\langle r \rangle$: Average of neighboring gap ratio
$$r = \frac{\min(e_{i+1}-e_i\,,\, e_{i+2}-e_{i+1})}{\max(e_{i+1}-e_i\,,\, e_{i+2}-e_{i+1})}$$

Longer range

- Σ^2 statistics: variance of number of levels in the energy range with M levels on average

- Spectral form factor $g(\beta, t)$: Fourier transform of the density of states

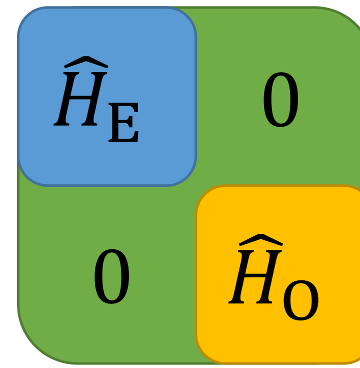
SYK: Exact diagonalization and fermion parity

$$\hat{H} = \frac{\sqrt{3!}}{N^{3/2}} \sum_{1\le a<b<c<d\le N} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d$$

Consider $N_\mathrm{D} = N/2$ complex fermions $\hat{c}_j = \frac{(\chi_{2j-1}+\mathrm{i}\chi_{2j})}{\sqrt{2}}, j = 1, 2, \dots, N_\mathrm{D}$

$\chi\chi\chi\chi$ preserves parity of complex fermion number

Each (even, odd) sector: $2^{N_\mathrm{D}-1}$($= 65\,536$ for $N = 34$) states

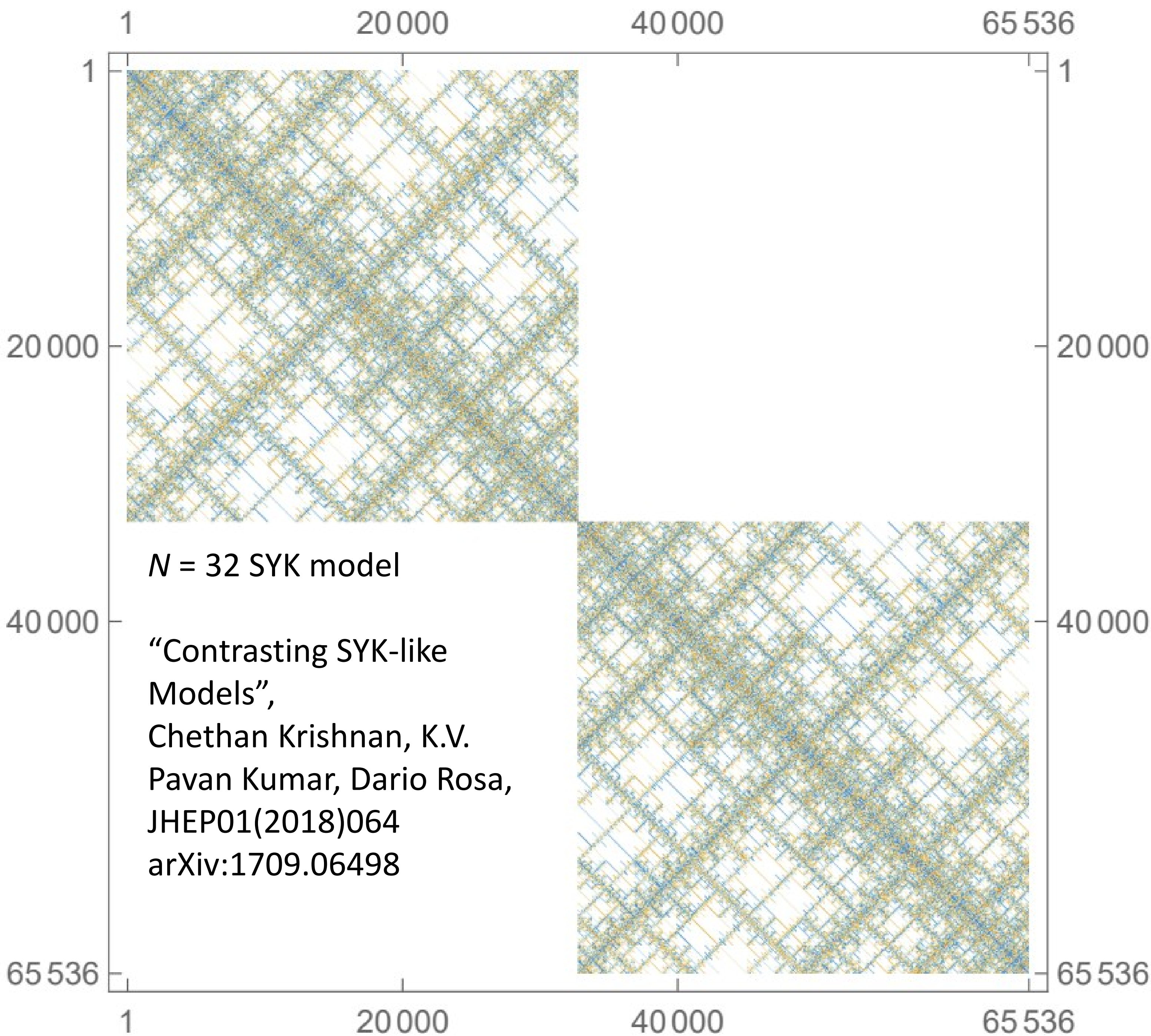


$N = 34$:
$\binom{17}{0} + \binom{17}{2} + \binom{17}{4} = 2517$ (~ 3.8 %) non-zero matrix elements on each row
2^{32} ~ 4 billion complex matrix elements: 64 GiB of memory

➔ Can be fully diagonalized numerically

*(2^{48} ~ 281 trillion) complex number operations, ~ 5 samples / day on a single node
(~ 10 RMB / one $N = 34$ sample or 2^{12} $N = 26$ samples)

cf. Lanczos code for up to $N = 46$ by G. Gur-Ari https://github.com/guygurari/syk
using DMRG-like ideas (see JHEP 1811, 070 (2018))

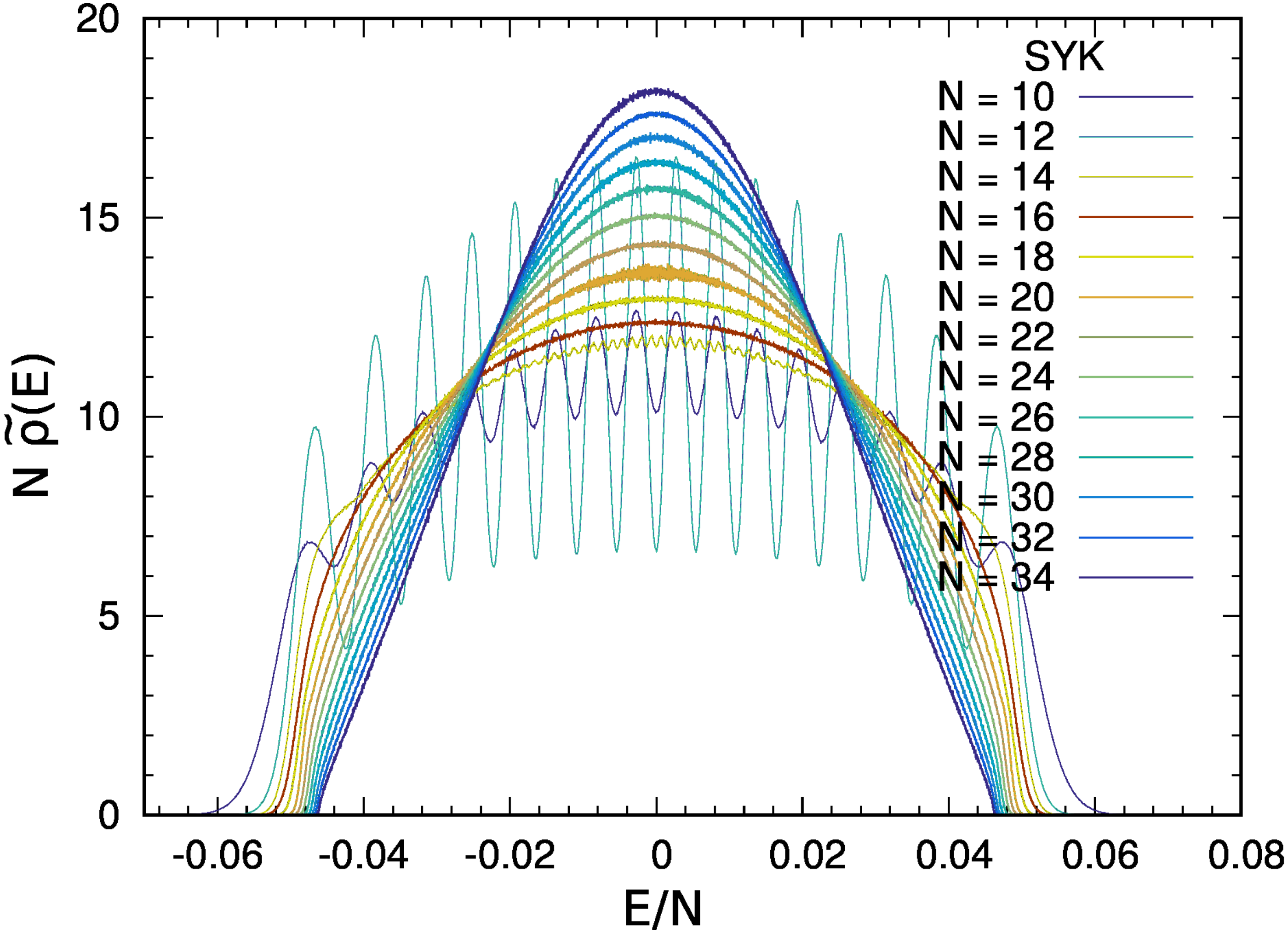


N = 32 SYK model

“Contrasting SYK-like Models”,
Chethan Krishnan, K.V. Pavan Kumar, Dario Rosa,
JHEP01(2018)064
arXiv:1709.06498

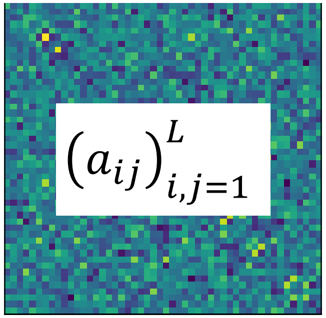
Diagonalization of the Hamiltonian → Eigenvalue spectrum

$$\hat{H} = \sum_{1 \le a < b < c < d \le N_{\mathrm{M}}} J_{abcd} \hat{\chi}_a \hat{\chi}_b \hat{\chi}_c \hat{\chi}_d,$$

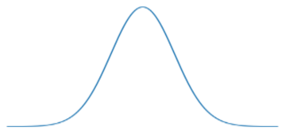


cf. Large *N* [A. M. García-García and J. J. M. Verbaarschot: 1610.03816]

Gaussian random matrices



$a_{ij} = a_{ji}^*$



Gaussian distribution

Density $\propto e^{-\frac{\beta K}{4}\mathrm{Tr}H^2} = \exp\left(-\frac{\beta K}{4}\sum_{i,j}^{K}|a_{ij}|^2\right)$

Real (β=1): Gaussian Orthogonal Ensemble (GOE)

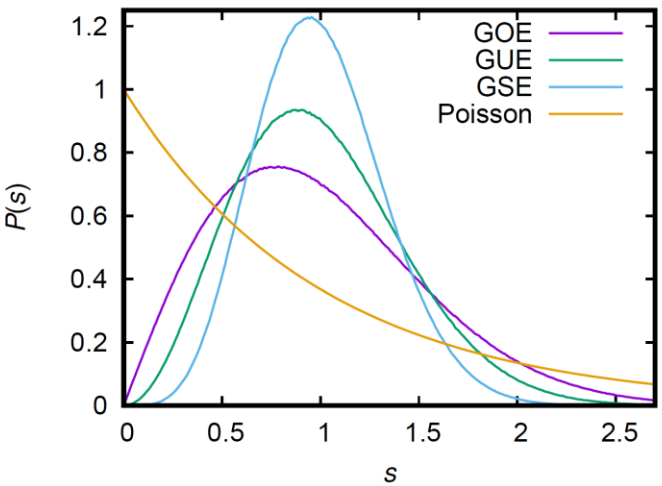
Complex (β=2): G. Unitary E. (GUE)

Quaternion (β=4): G. Symplectic E. (GSE)

Joint distribution function for eigenvalues $\{e_j\}$

Level repulsion

$$p(e_1, e_2, \ldots, e_K) \propto \prod_{1\le i<j\le K} |e_i - e_j|^\beta \prod_{i=1}^{K} e^{-\beta K e_i^2/4}$$



$$r = \frac{\min(e_{i+1} - e_i\,,\ \ e_{i+2} - e_{i+1})}{\max(e_{i+1} - e_i\,,\ \ e_{i+2} - e_{i+1})}$$

- $P(s)$: Distribution of normalized level separation $s_j = \frac{e_{j+1} - e_j}{\Delta(\bar{e})}$

  GOE/GUE/GSE: $P(s) \propto s^\beta$ at small s, has e^{-s^2} tail
  Uncorrelated (Poisson): $P(s) = e^{-s}$

- $\langle r \rangle$: Average of neighboring gap ratio

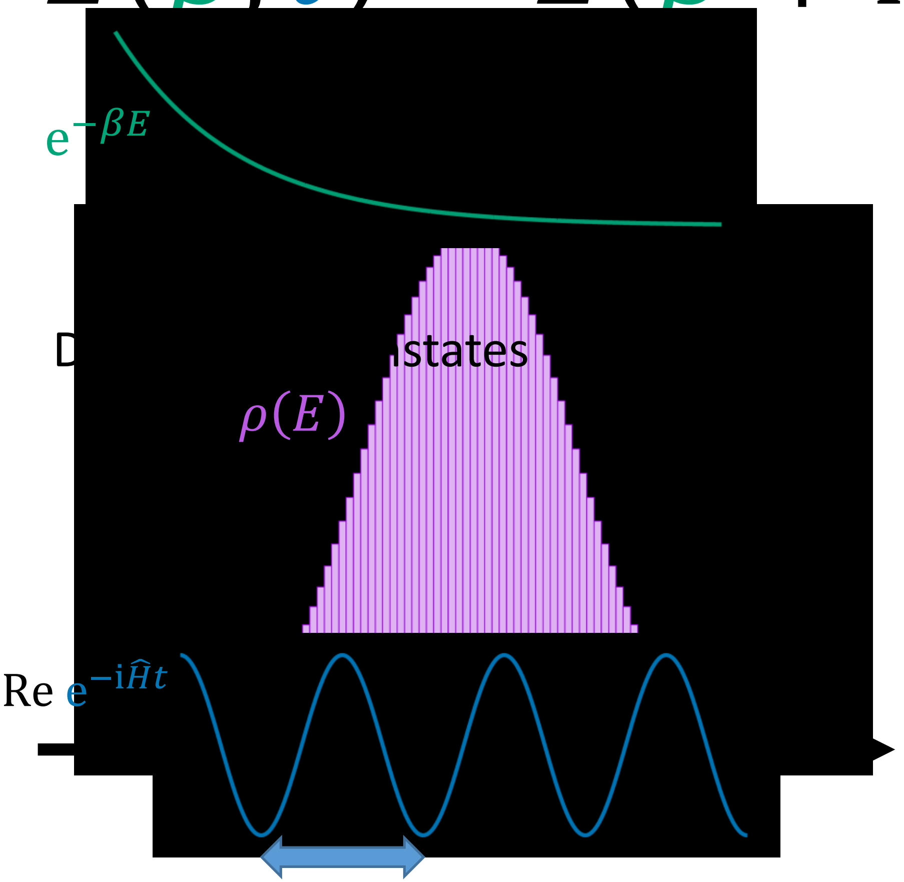
	Uncorrelated	GOE	GUE	GSE
$\langle r \rangle$	2log 2 – 1 = 0.38629…	0.5307(1)	0.5996(1)	0.6744(1)

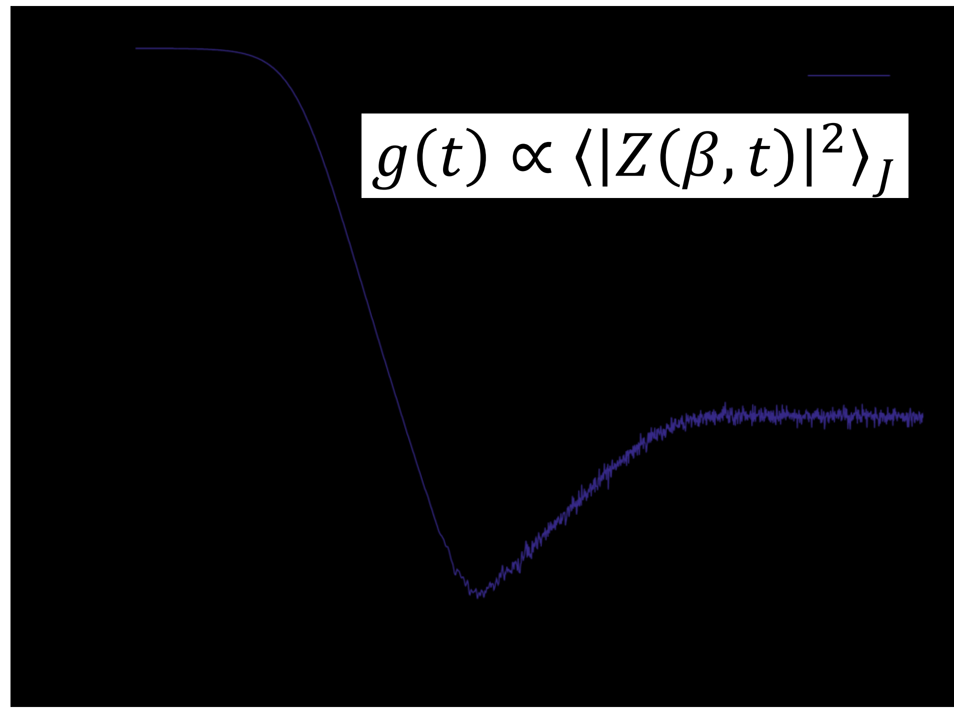
[Y. Y. Atas *et al.* PRL 2013]

Time-dependent partition function and energy scale

$$\mathcal{H}_{\mathrm{M}} = \sum_{a,b,c,d}^{N} J_{abcd}\widehat{\chi_a}\widehat{\chi_b}\widehat{\chi_c}\widehat{\chi_d}$$

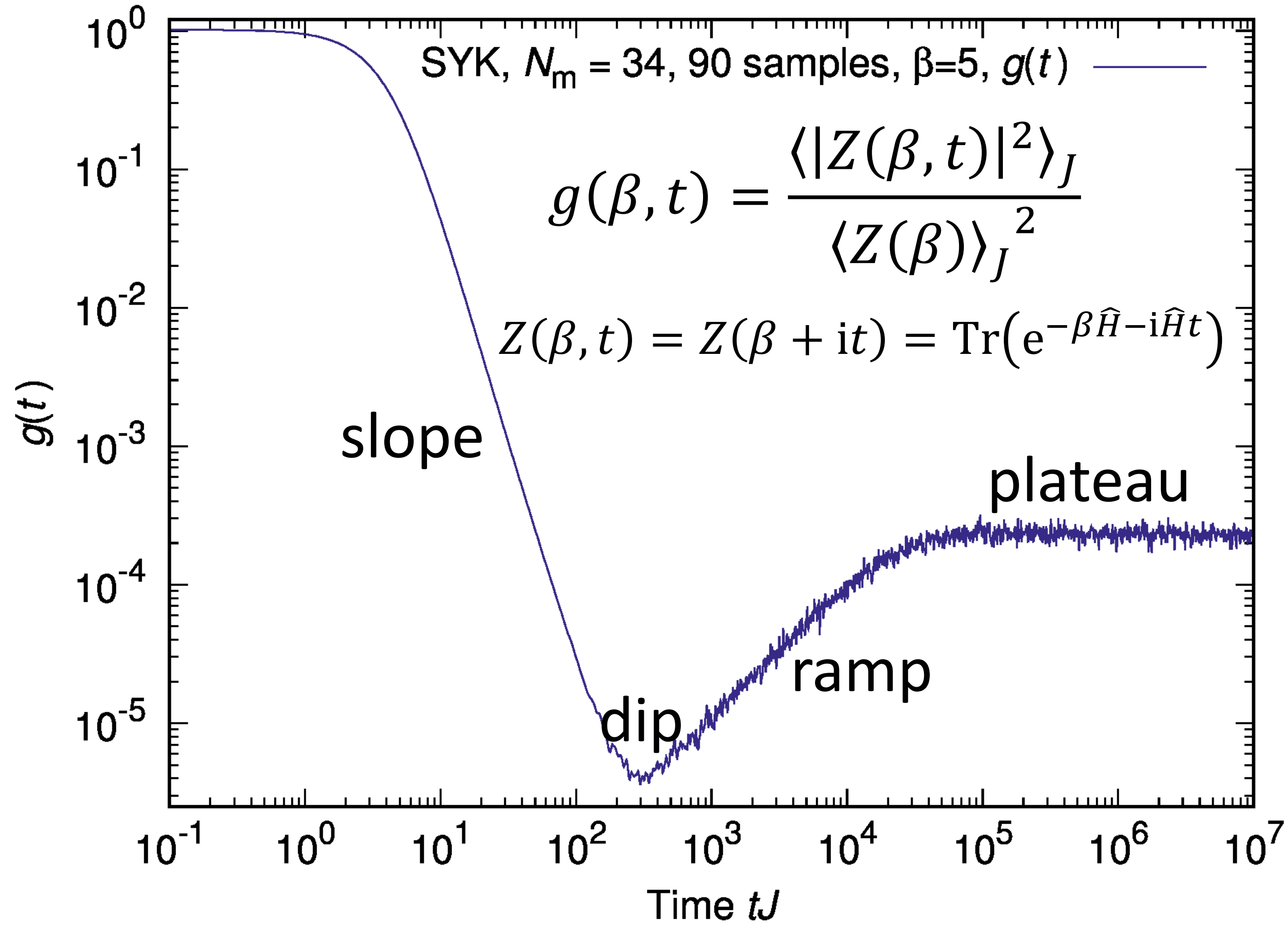
$$Z(\beta, t) = Z(\beta + \mathrm{i}t) = \mathrm{Tr}\left(\mathrm{e}^{-\beta\hat{H} - \mathrm{i}\hat{H}t}\right)$$



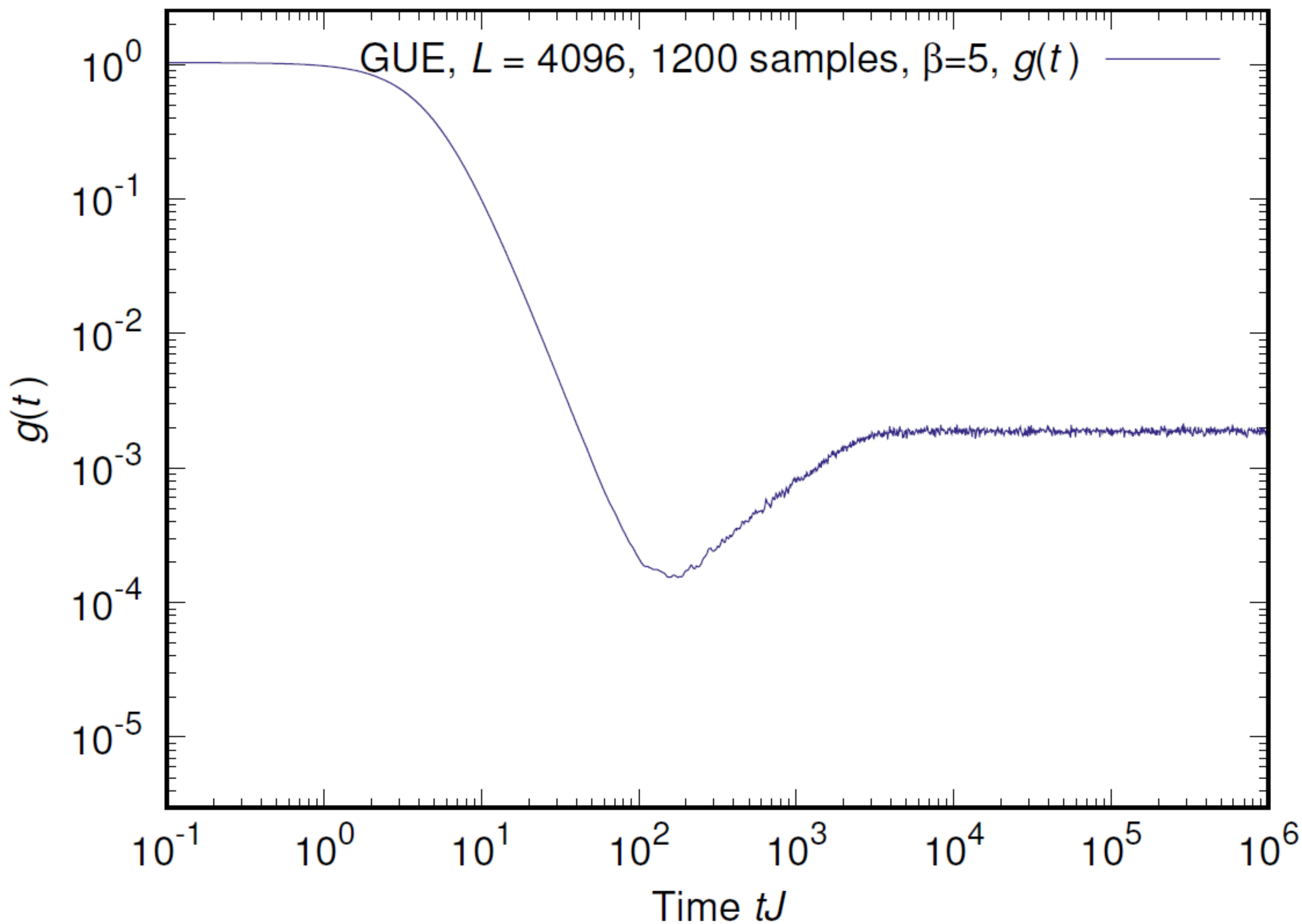


- Analytical continuation of partition function $Z(\beta)$
- Fourier transform of $\rho(E)$ modified by temperature

Spectral form factor

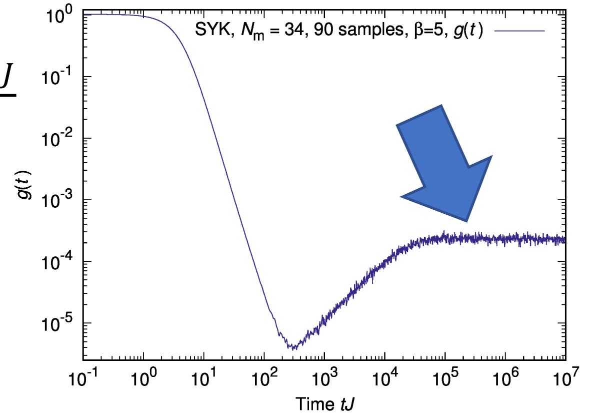


Similar to dense random matrix



Plateau height determined by $Z(\beta)$

$$g(\beta,t) = \frac{\langle |Z(\beta,t)|^2\rangle_J}{\langle Z(\beta)\rangle_J^{\ 2}}$$



For each sample, consider the long time average of

$$|Z(\beta,t)|^2 = \sum_{m,n} \mathrm{e}^{-\beta(E_m+E_n)}\mathrm{e}^{\mathrm{i}(E_m-E_n)t}$$

$$\lim_{T\to\infty}\frac{1}{T}\int_0^T dt\,|Z(\beta,t)|^2 = \sum_E {N_E}^2 \mathrm{e}^{-2\beta E} = N_E Z(2\beta)$$

(if degeneracy of E: N_E is independent of E)

Because $Z \sim \mathrm{e}^{aS}$ $(a>0)$, long-time average $N_E \frac{Z(2\beta)}{Z(\beta)^2}$ will be $\sim \mathrm{e}^{-aS}$ (non-perturbative in 1/N)

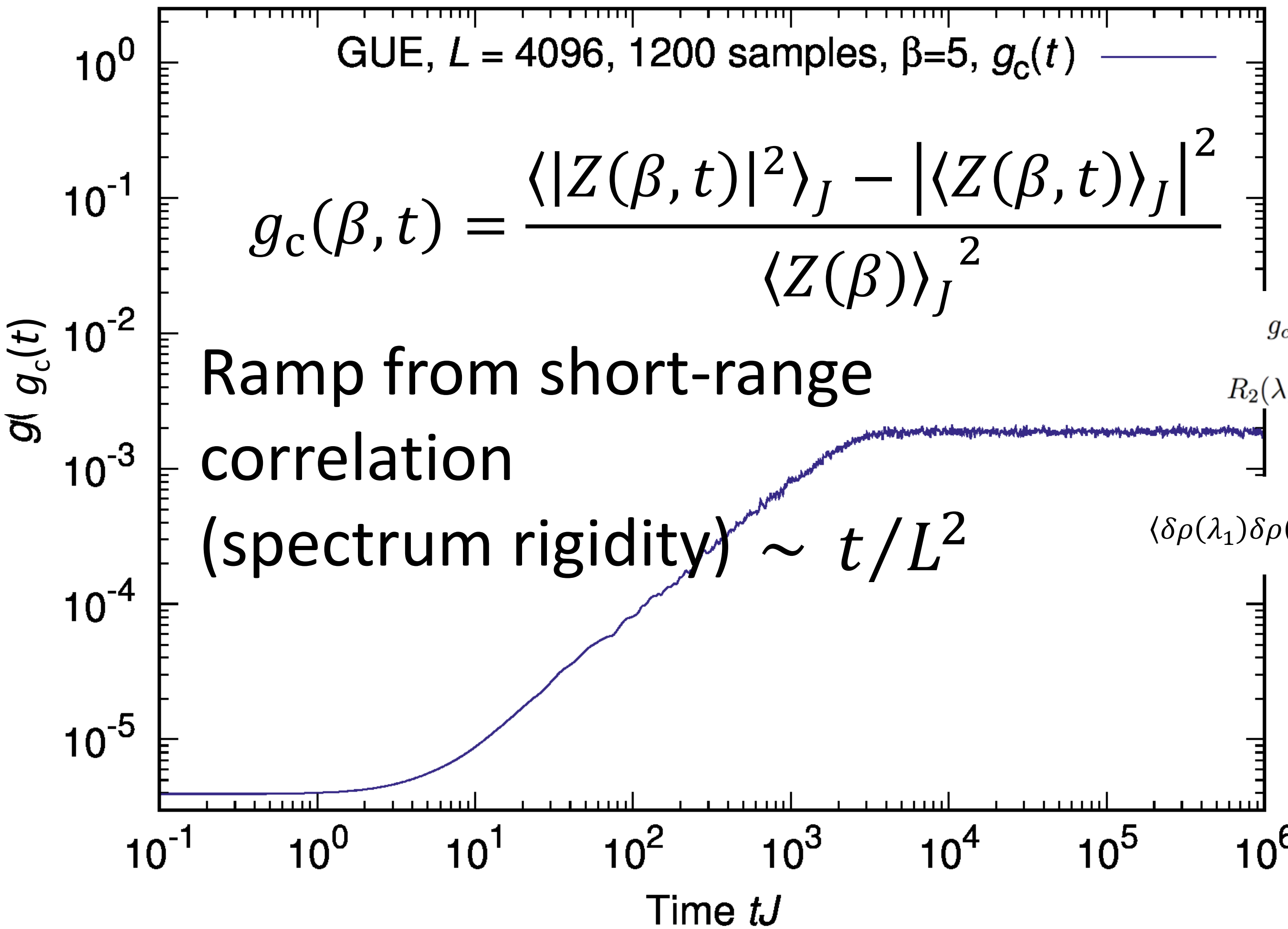
Late time: governed by $g_c(t)$

[You, Ludwig, Xu: arXiv:1602.06964]

BDI class, N_χ Majorana fermions

$N_\chi \pmod 8$	0	1	2	3	4	5	6	7
qdim	1	$\sqrt{2}$	2	$2\sqrt{2}$	2	$2\sqrt{2}$	2	$\sqrt{2}$
lev. stat.	GOE	GOE	GUE	GSE	GSE	GSE	GUE	GOE
$C\ell_{0,N_\chi-1}$	$\mathbb{R}\oplus\mathbb{R}$	$\mathbb{R}$	$\mathbb{C}$	$\mathbb{H}$	$\mathbb{H}\oplus\mathbb{H}$	$\mathbb{H}$	$\mathbb{C}$	$\mathbb{R}$

Dense random matrix reproduces the late-time ramp & plateau behavior



$$g_c(\beta,t) = \frac{\langle |Z(\beta,t)|^2\rangle_J - \left|\langle Z(\beta,t)\rangle_J\right|^2}{\langle Z(\beta)\rangle_J^{\,2}}$$

Ramp from short-range correlation (spectrum rigidity) $\sim t/L^2$

L: matrix dimension

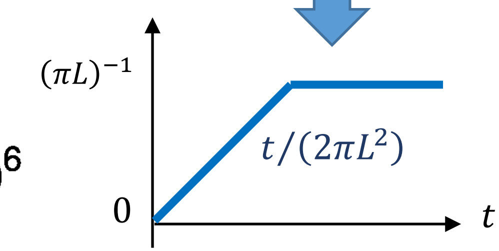
$$g_c(t;0) = \int d\lambda_1 d\lambda_2\, R_2(\lambda_1,\lambda_2) e^{i(\lambda_1-\lambda_2)t},$$

$$R_2(\lambda_1,\lambda_2) \equiv \langle \delta\rho(\lambda_1)\delta\rho(\lambda_2)\rangle_{\mathrm{GUE}}.$$

$$\delta\rho(\lambda) \equiv \rho(\lambda) - \rho_s(\lambda)$$

$$\langle \delta\rho(\lambda_1)\delta\rho(\lambda_1-\lambda)\rangle_{\mathrm{GUE}} = -\frac{\sin^2 L\lambda}{(\pi L\lambda)^2} + \frac{1}{\pi L}\delta(\lambda)$$

F. T.



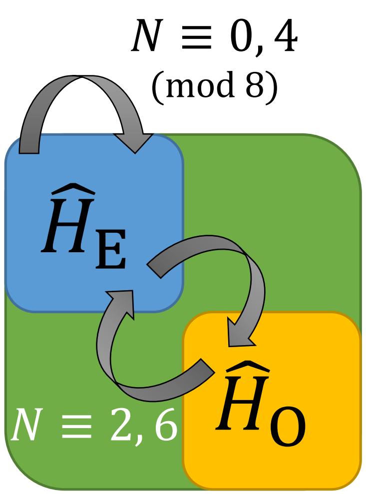
N mod 8 classification of Majorana SYK$_{q=4}$

$$\hat{H} = \frac{\sqrt{3!}}{N^{3/2}} \sum_{1 \le a<b<c<d \le N} J_{abcd} \hat{\chi}_a \hat{\chi}_b \hat{\chi}_c \hat{\chi}_d$$

SPT phase classification for class BDI:
$\mathbb{Z} \to \mathbb{Z}_8$ due to interaction
[L. Fidkowski and A. Kitaev, PRB 2010, PRB 2011]

Introduce $N/2$ complex fermions $\hat{c}_j = \frac{(\hat{\chi}_{2j-1} + i\hat{\chi}_{2j})}{\sqrt{2}}$

$\hat{\chi}_a \hat{\chi}_b \hat{\chi}_c \hat{\chi}_d$ respects the complex fermion parity
Even ($\hat{H}_\mathrm{E}$) and odd ($\hat{H}_\mathrm{O}$) sectors: $L = 2^{N/2-1}$ dimensions



N mod 8	0	2	4	6
η	-1	+1	+1	-1
$\hat{X}^2$	**+1**	+1	**-1**	-1
$\hat{X}$ maps H_E to	H_E	H_O	H_E	H_O
Class	**AI**	**A+A**	**AII**	**A+A**
Gaussian ensemble	**GOE**	**GUE**	**GSE**	**GUE**

$$\hat{X} = \hat{K} \prod_{j=1}^{N/2} (\hat{c}_j^\dagger + \hat{c}_j)$$

$$\hat{X} \hat{c}_j \hat{X} = \eta \hat{c}_j^\dagger \quad [\hat{X}, \hat{H}] = 0$$

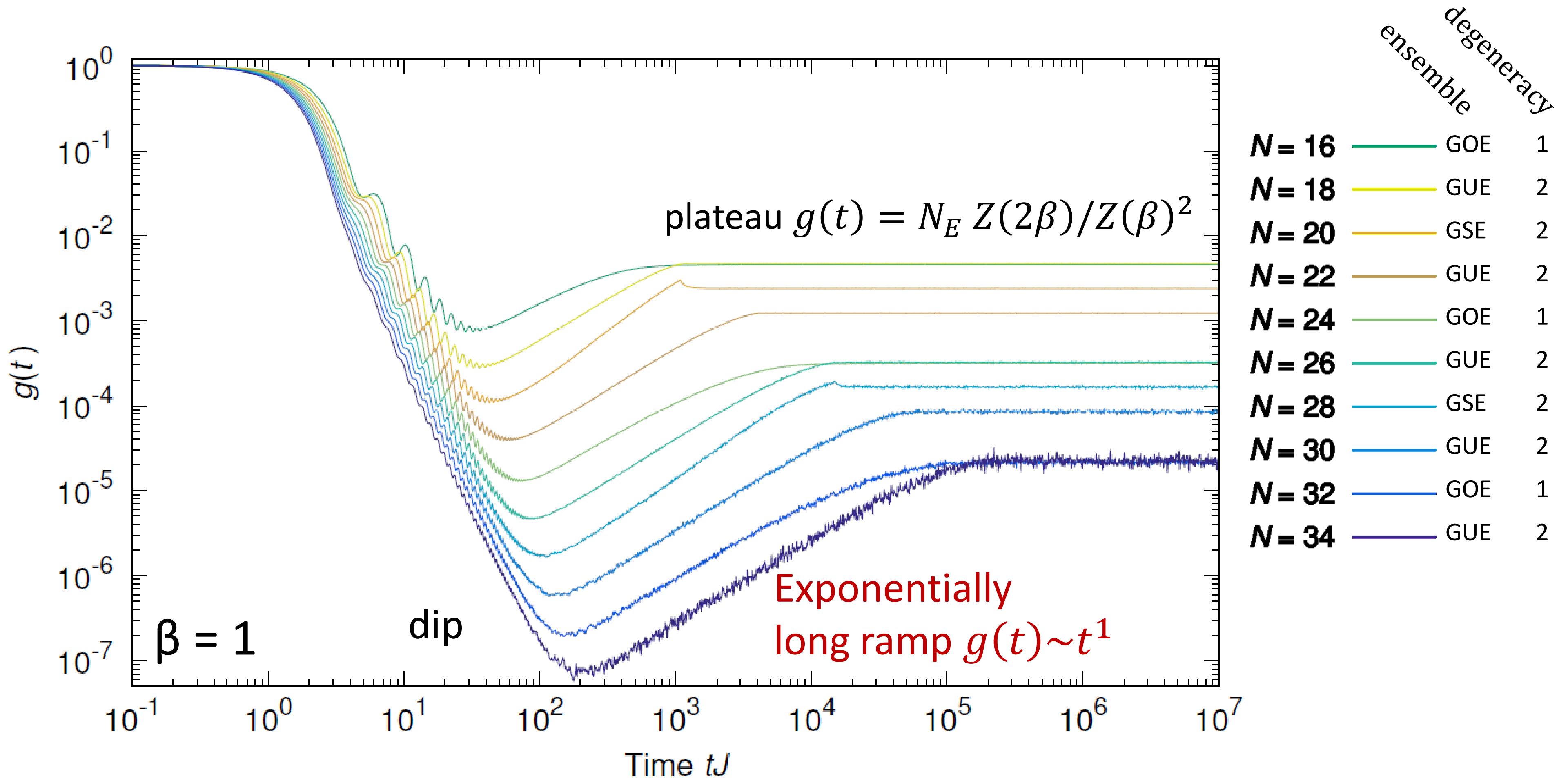
[You, Ludwig, and Xu, PRB 2017]

[Fadi Sun and Jinwu Ye, 1905.07694] for SYK$_q$, supersymmetric SYK

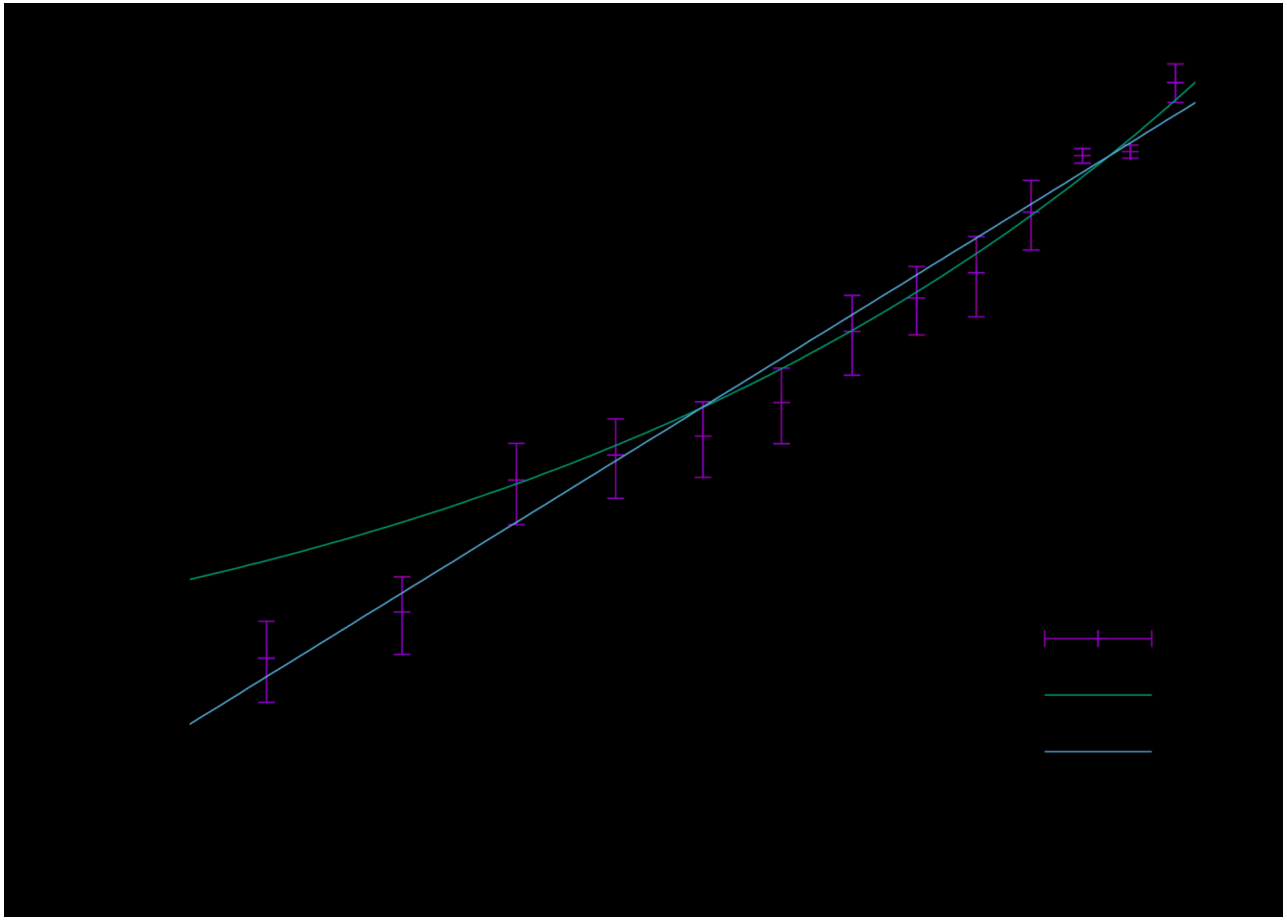
SYK: sparse matrix, but energy spectral statistics strongly resemble that of the corresponding (dense) Gaussian ensemble [Cotler, …, MT, JHEP 2017]

1611.04650

The SYK spectral form factor: N dependence



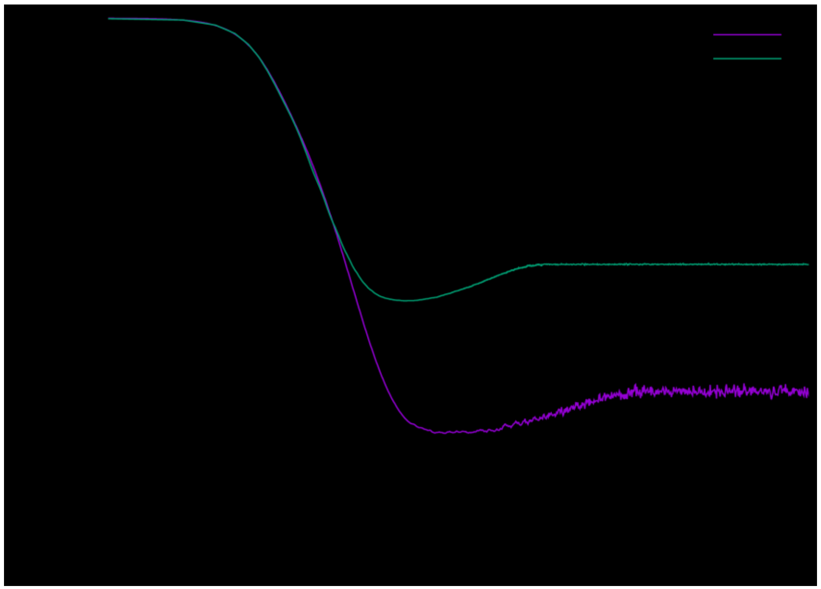
Dip time



Correlation function

$$G(t) = \langle \chi_a(t)\chi_a(0)\rangle$$

Dip-ramp-plateau structure similar to $g(\beta, t)$ for $N \equiv 2 \pmod 8$



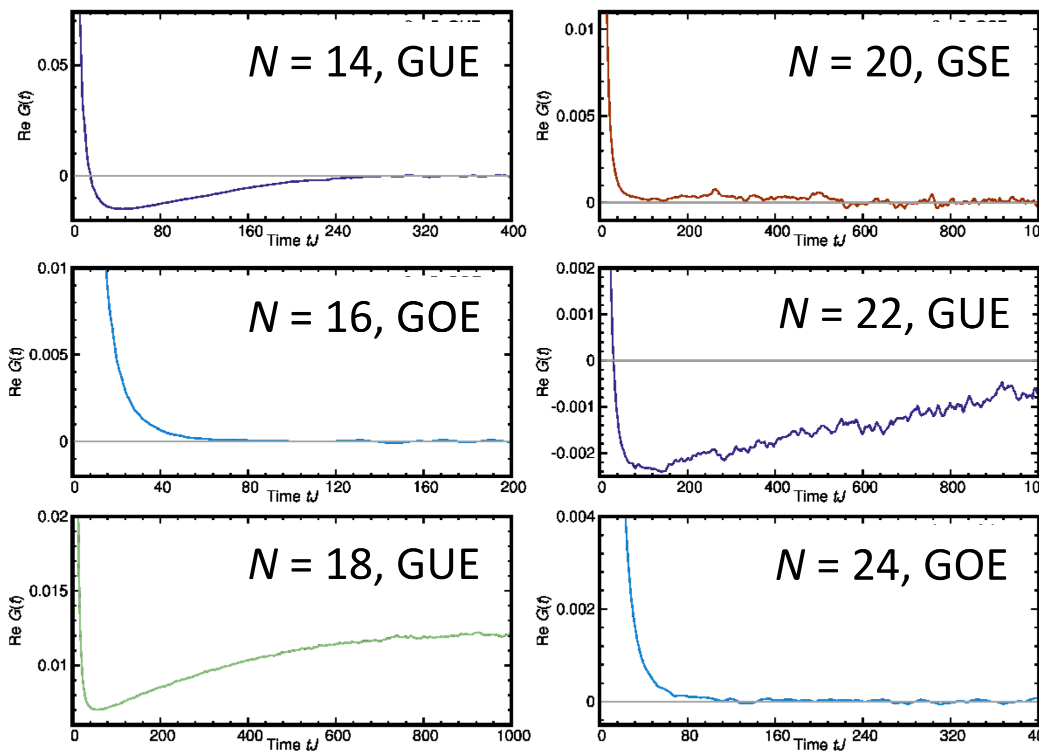
$$\hat{X} = K \prod_{j=1}^{N/2} \left(\hat{c}_j^\dagger + \hat{c}_j\right), \hat{c}_j = \frac{(\chi_{2j-1} + \mathrm{i}\chi_{2j})}{\sqrt{2}}$$

$N \equiv 0 \pmod 8$: $\hat{X}$ maps each charge parity sector to itself and $\hat{X}^2 = 1$ (no protected degeneracy)

$N \equiv 2 \pmod 8$: $\hat{X}$ maps each sector to the other and $\langle \mathrm{even}|\chi|\mathrm{odd}\rangle$ finite

$N \equiv 4 \pmod 8$: $\hat{X}$ maps each charge parity sector to itself and $\hat{X}^2 = -1$ (only internal degeneracy)

$N \equiv 6 \pmod 8$: $\hat{X}$ maps each sector to the other but $\langle \mathrm{even}|\chi|\mathrm{odd}\rangle = 0$



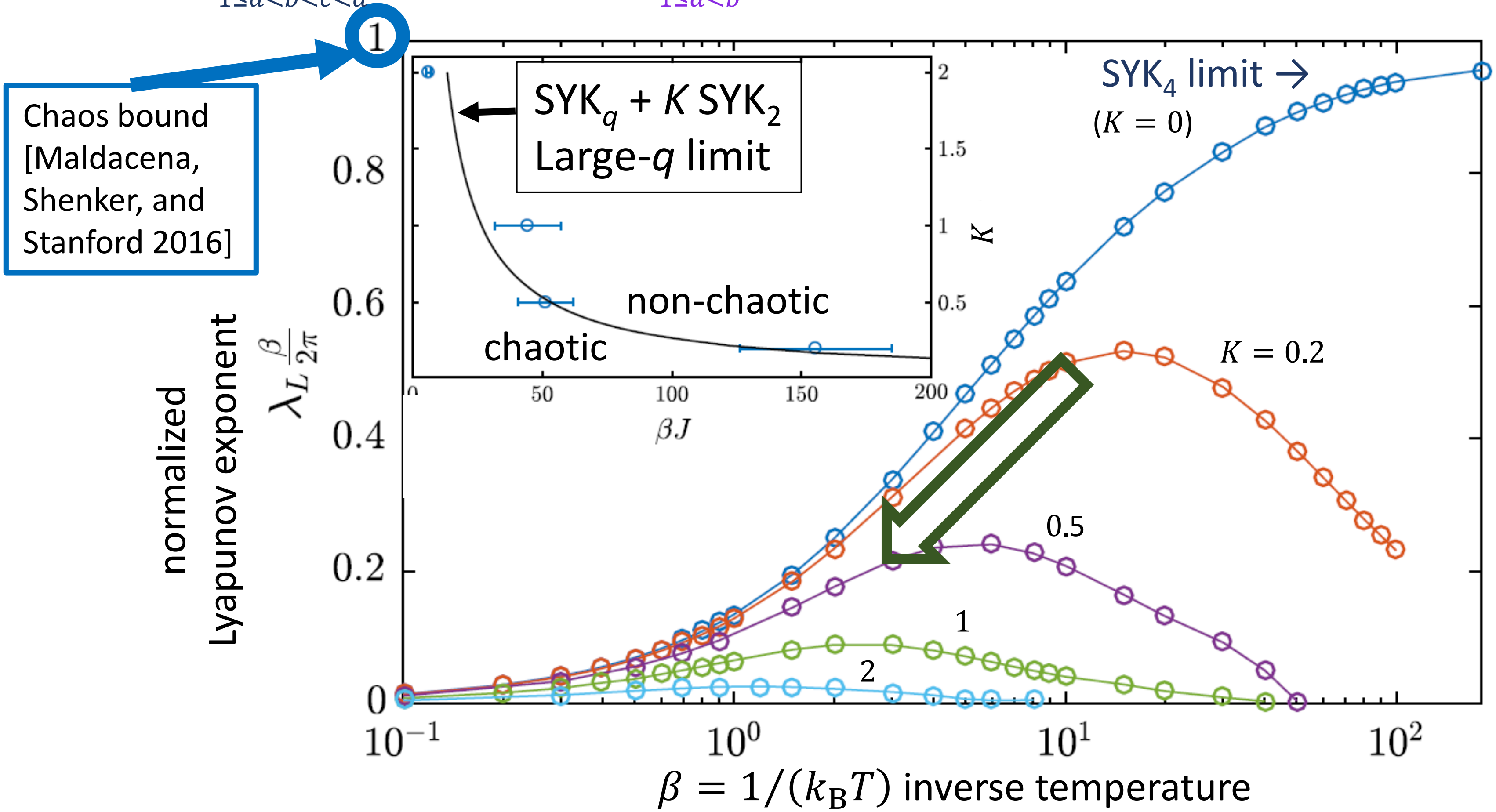
1707.02197

SYK_4 + SYK_2: Large-N calculation for OTOC

$$\hat{H} = \sum_{1\le a<b<c<d}^{N} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d + i\sum_{1\le a<b}^{N} K_{ab}\hat{\chi}_a\hat{\chi}_b$$

SYK_4 SYK_2

K_{ab}: standard deviation $\frac{K}{\sqrt{N}}$



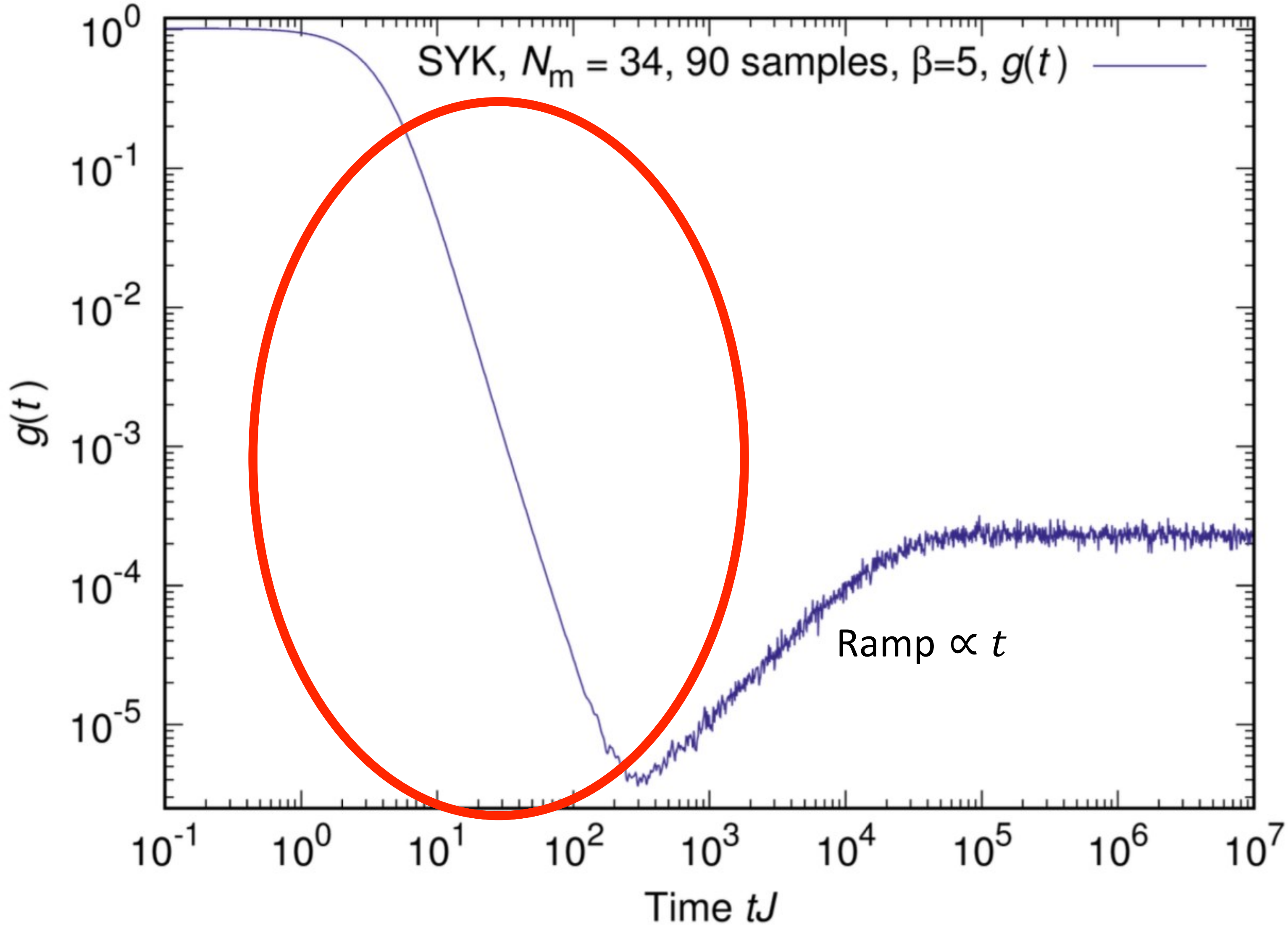
A. M. Garcia-Garcia, B. Loureiro, A. Romero-Bermudez, and MT, PRL **120**, 241603 (2018)

Deviation from the chaos bound as SYK_2 component is introduced

Contents

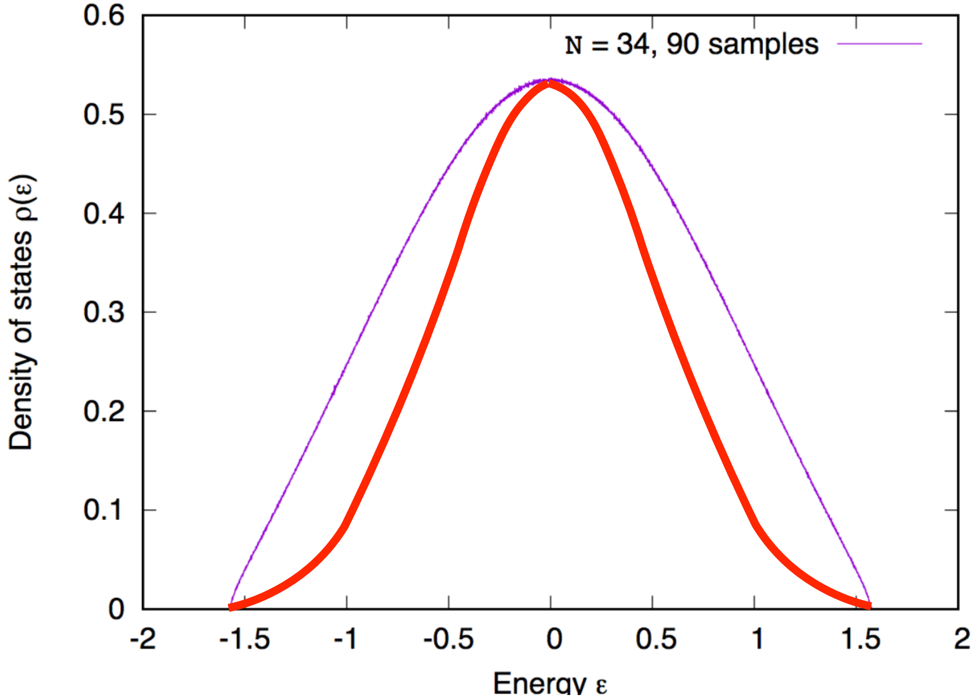
- The Sachdev-Ye-Kitaev model
  - Large-*N* solvability: conformal symmetry and maximal chaos
  - Experimental proposal 1606.02454 (and realization)
  - Random matrices (RM) and spectral form factor 1611.04650
  - Deformation and suppression of maximal chaos 1707.02197
- **Onset of RM behavior in scrambling systems 1809.01671**
  - ***k*-local and local systems**
  - **Random circuits**
- Characterization of chaos in random systems
  - Quantum Lyapunov spectrum 1809.01671
  - Singular value statistics of two-point correlators 1902.11086

Where does the ramp start?



'Slope' hides the beginning of the ramp

Energy spectrum of the SYK model

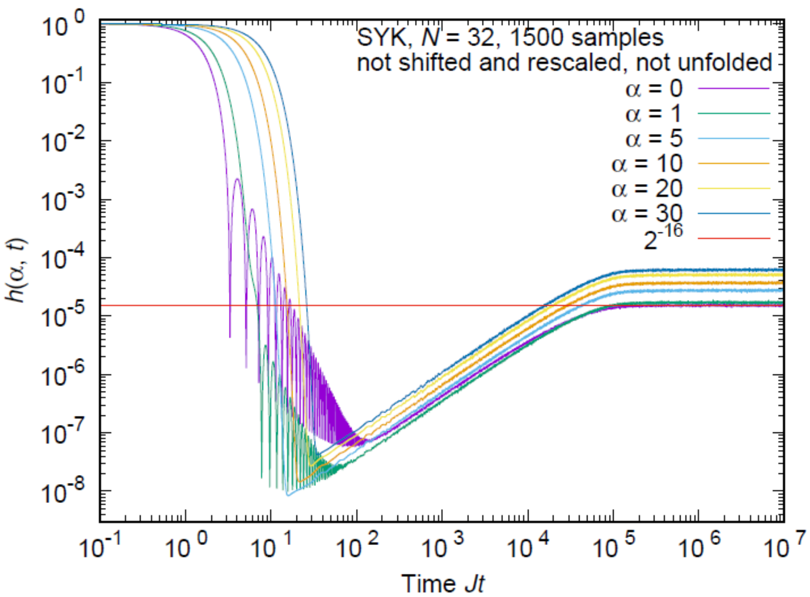


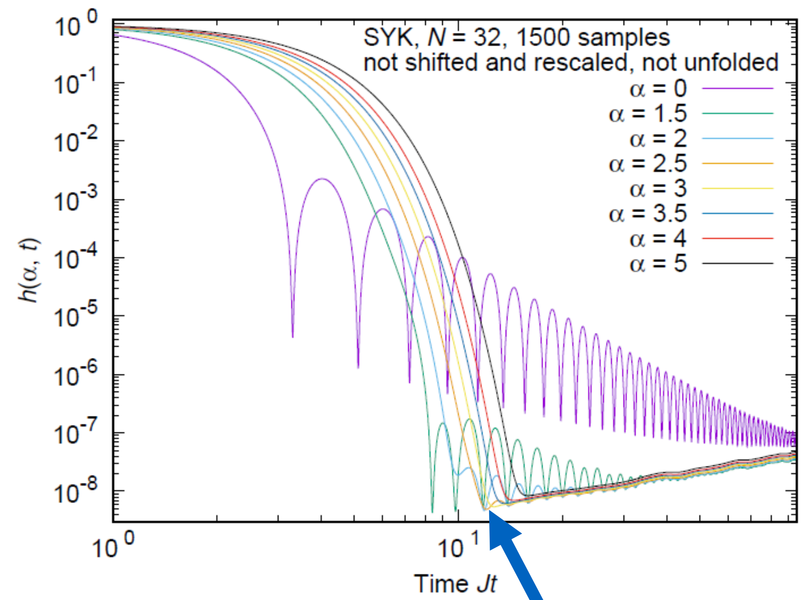
$$Y(\alpha, t)Y^*(\alpha, t) = \sum_{m,n} e^{-\alpha(E_n^2+E_m^2)} e^{+i(E_m-E_n)t}$$

'Slope' depends on the edge of the density of states

Spectral form factor

$$|Y(\alpha,t)|^2 = \left|\sum_i e^{-\alpha E_i^2 - itE_i}\right|^2$$

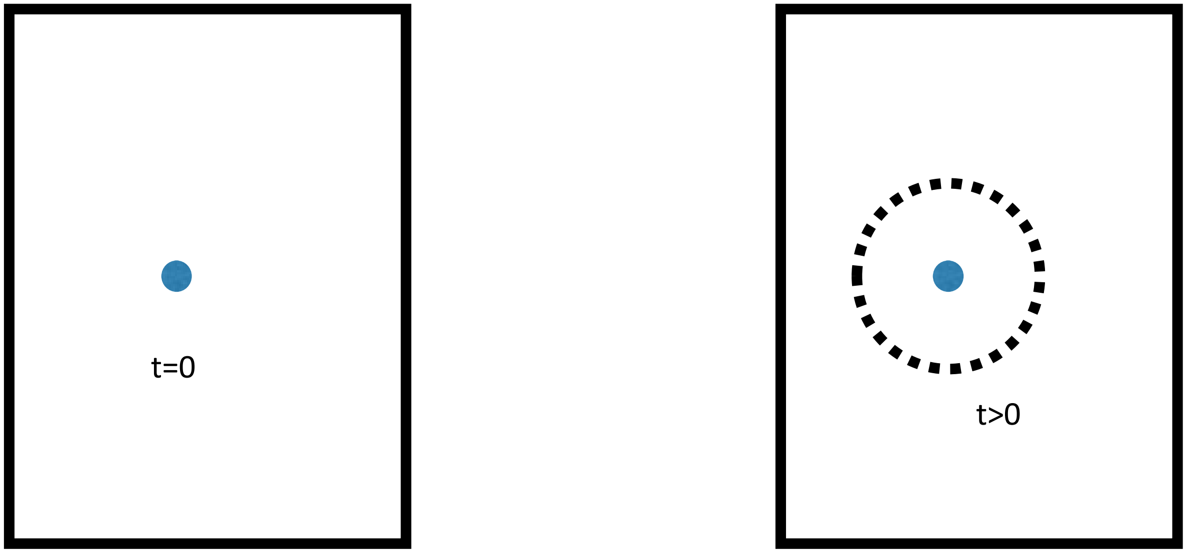




$t_{\min} = 12.5$ for $\alpha = 2.9$

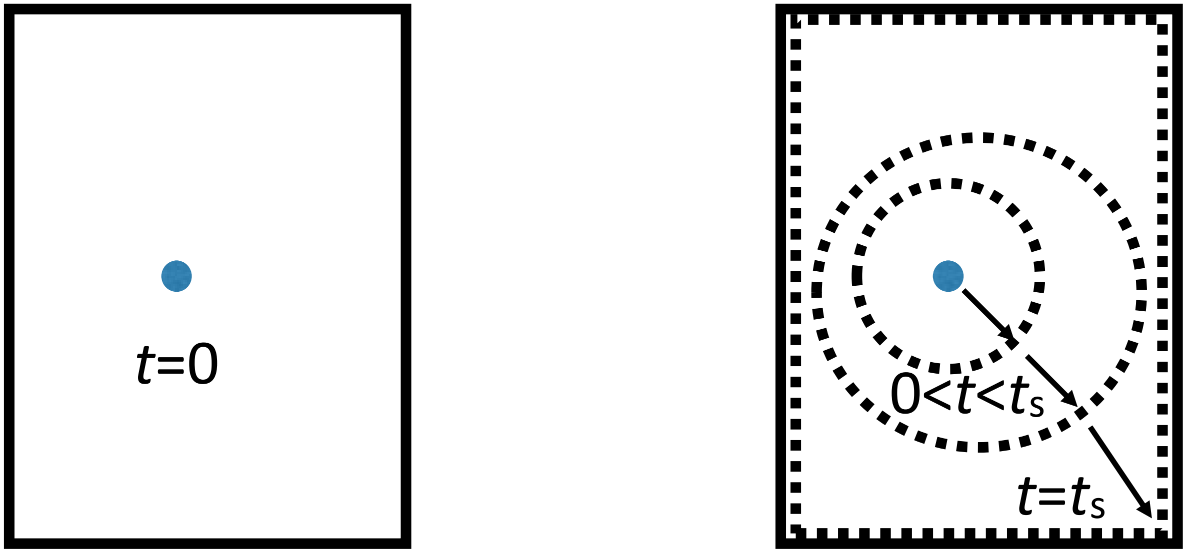
$t_{\min}$: almost constant? ($8 < t_{\min} < 15$ for $10 \leq N \leq 34$)

Scrambling



After some time, non-local measurements are needed for information on the local perturbation at $t = 0$ ("information scrambling")

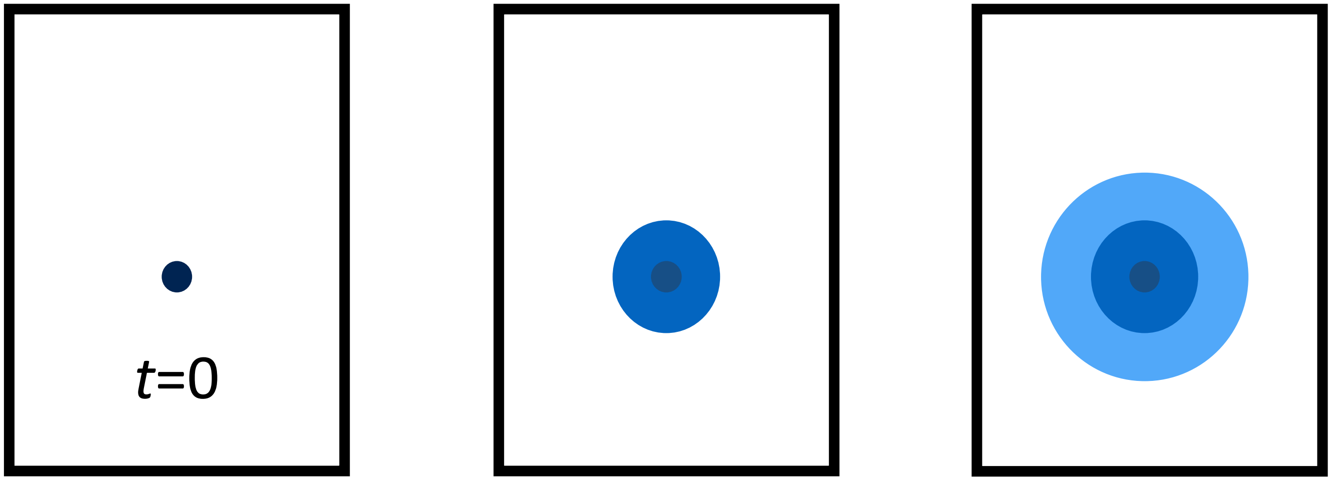
Scrambling



After t=t_s , information has been scrambled with the entire system

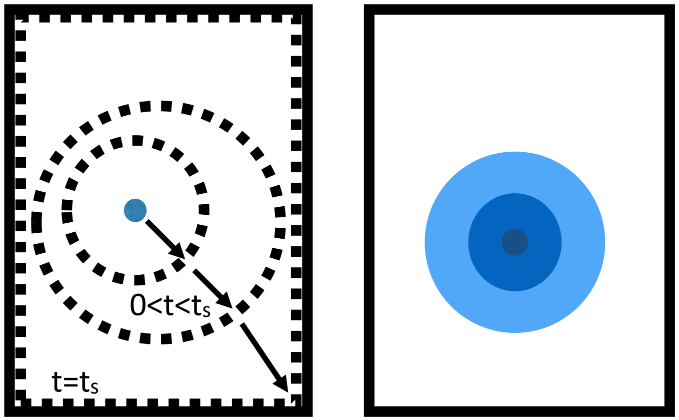
'scrambling time'

Diffusion



Conserved quantity (e.g. charge) diffuses, eventually (after diffusion time t_d , also called the Thouless time) will be uniformly distributed

Scrambling or diffusion?



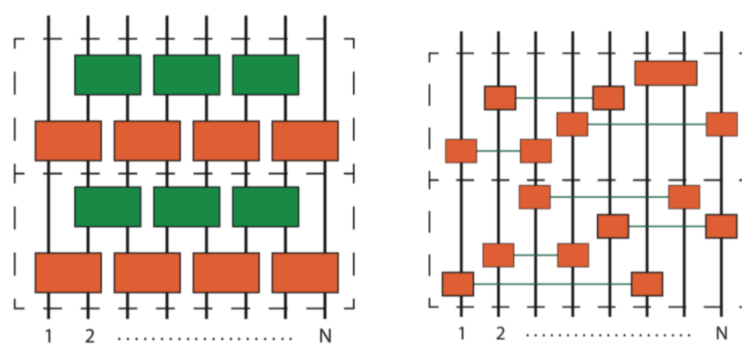
G. Gharibyan, M. Hanada, S. H. Shenker, and MT, JHEP **1807**, 124 (2018) (arXiv:1803.08050)

In this talk, we show the following examples:

- Known case: band matrix (single particle hopping)
- Numerical results on spin systems

see also: Random circuit-based discussion in our paper

- RMT universality observed after 'ramp time' t_{ramp}
- Physical interpretation?
- Relationship to BH information paradox? scrambling? diffusion?
- Our results: ramp time seems to be determined by diffusion, not by scrambling



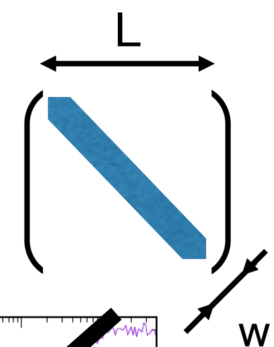
Random band matrix

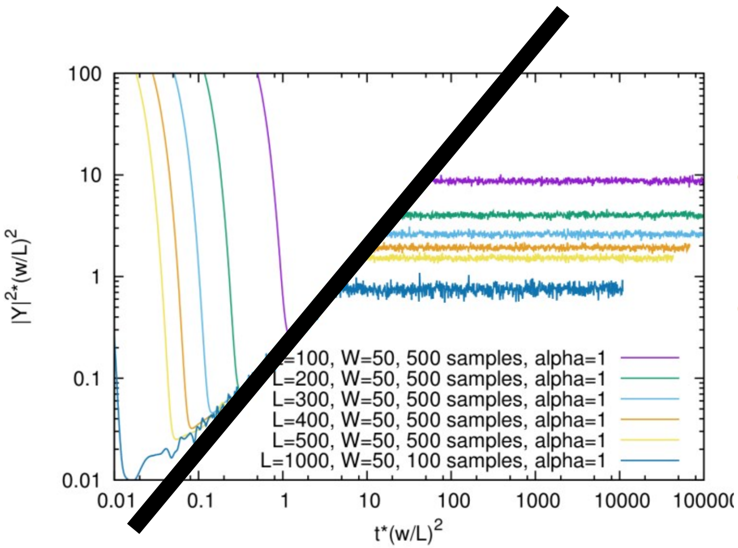
see L. Erdös and A. Knowles, "The Altshuler-Shklovskii Formulas for Random Band Matrices I: the Unimodular Case," Comm. Math. Phys. **333**, 1365 (2015) for derivation of the scaling

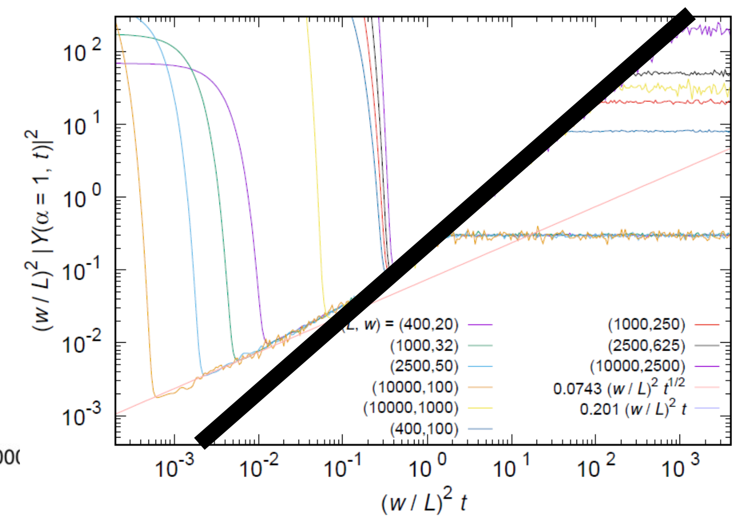
(Single particle hopping: diffusion is defined)

$$|Y(\alpha,t)|^2 = \left|\sum_i e^{-\alpha E_i^2 - itE_i}\right|^2$$

$$M_{ij} = M_{ij}^{(0)} \cdot \frac{e^{-\frac{(i-j)^2}{2w^2}}}{\sqrt{w}}$$







$t_{\text{scrambling}} \sim (L/w)^1$

$t_{\text{diffusion}} \sim (L/w)^2$

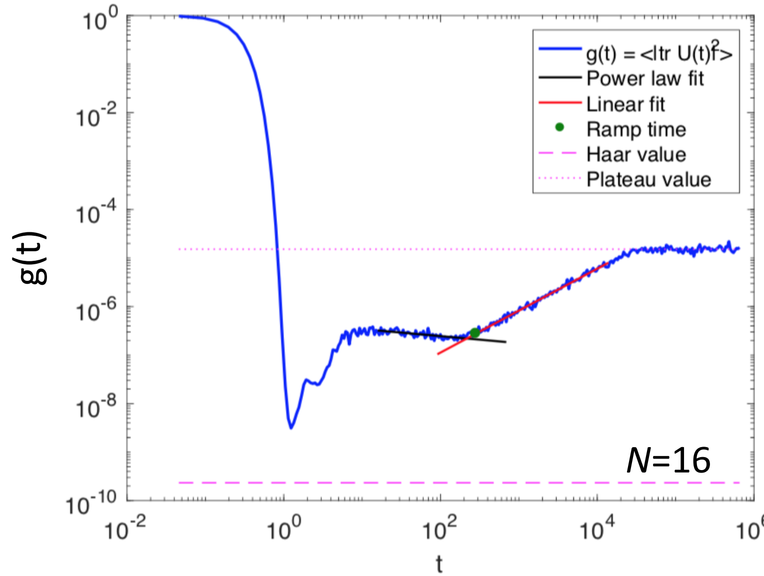
$t_{\text{ramp}} \sim (L/w)^2$

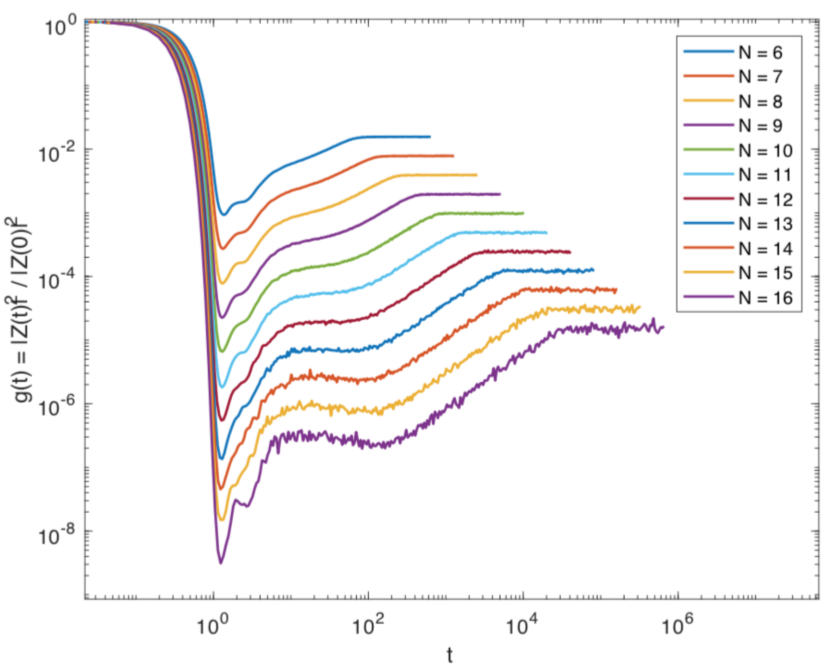
$S = 1/2$ spin chain

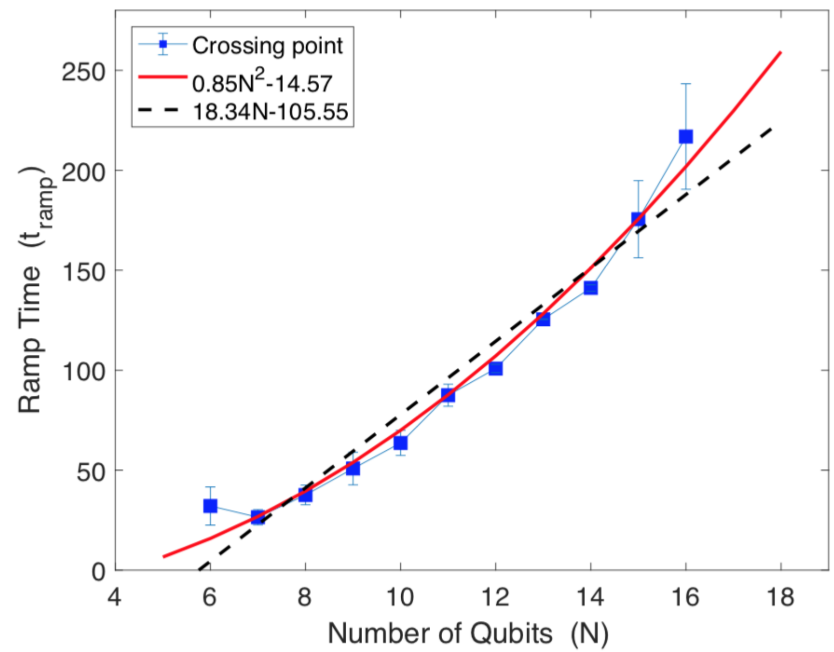
$$H = \frac{1}{4} \sum_{i=1}^{N-1} \sum_{\alpha,\beta=0}^{3} J_i^{\alpha\beta} \sigma_i^\alpha \otimes \sigma_{i+1}^\beta$$

J: normal distribution

$$\{\sigma^0, \sigma^1, \sigma^2, \sigma^3\} = \{I, X, Y, Z\}$$







Better overlap with diffusion time (N^2)

Contents

- The Sachdev-Ye-Kitaev model
  - Large-*N* solvability: conformal symmetry and maximal chaos
  - Experimental proposal 1606.02454 (and realization)
  - Random matrices (RM) and spectral form factor 1611.04650
  - Deformation and suppression of maximal chaos 1707.02197
- Onset of RM behavior in scrambling systems 1803.08050
  - *k*-local and local systems
  - Random circuits
- **Characterization of chaos in random systems 1702.06935**
  - **Quantum Lyapunov spectrum 1809.01671**
  - **Singular value statistics of two-point correlators 1902.11086**

Other related works: 1801.03204, 1812.04770

How to characterize quantum chaos?

$\hat{H}$ = const.

$$i\frac{d}{dt}|\psi\rangle = \hat{H}|\psi\rangle \qquad |\psi(t)\rangle = \hat{\mathrm{T}}\exp\left[-i\int_0^t \hat{H}(t')dt\right]|\psi(t=0)\rangle = \exp\left(-i\hat{H}t\right)|\psi(t=0)\rangle$$

Linear dynamics

Unitary time evolution

- Long time: energy level statistics

Correlation between levels, as in random matrices

Short range: Normalized level separation distribution, gap ratio, …
Longer range: Number variance, spectral form factor, …

cf. Bohigas-Giannoni-Schmit conjecture

- Shorter time: out-of-time correlator

Classically,

$$\{x_i(t), p_j(0)\}_{\mathrm{PB}}{}^2 = \left(\frac{\partial x_i(t)}{\partial x_j(0)}\right)^2 \to e^{2\lambda_{\mathrm{L}}t} \text{ for large } t$$

Quantum version:

OTOC: $C_T(t) = \left\langle \left|\left[\hat{W}(t), \hat{V}(t=0)\right]\right|^2 \right\rangle$

$= \left\langle \hat{W}^\dagger(t)\hat{V}^\dagger(0)\hat{W}(t)\hat{V}(0)\right\rangle + \cdots$

Numerically

➔ limited to small systems

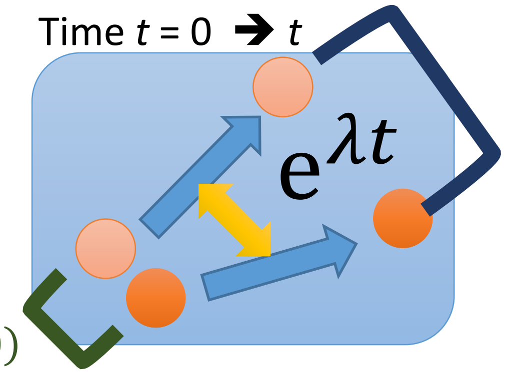
➔ Hard to see exponential time dependence

1702.06935

Finite-time Lyapunov spectrum in classical chaotic systems

Classical systems with K degrees of freedom

Infinitesimal deviation in initial condition



Deviation at t : linear in initial deviation

$$\delta\phi_i(t) = T_{ij}\delta\phi_j(0)$$

Singular values of T_{ij}: $\{a_k(t)\}_{k=1}^{K}$

Time-dependent Lyapunov spectrum

$$\left\{\lambda_k(t) = \frac{\log a_k(t)}{t}\right\}_{k=1,2,\ldots,K}$$

- Usually we consider the $t \to \infty$ limit, chaotic if $\max(\lambda_k) > 0$
- We focus on finite time behavior

Spectrum $\{\lambda_k(t)\}_{k=1}^{K}$ depends on system details

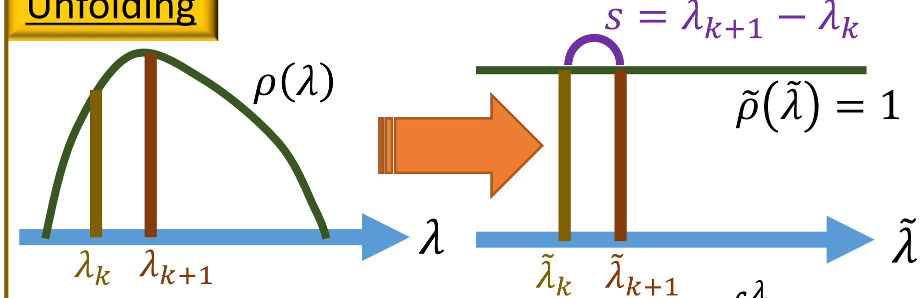
➔ Any universality for chaotic cases?

In many chaotic systems, for large K, the Lyapunov spectrum behaves like that of a Gaussian random matrix at some time scale.

Numerical evidences

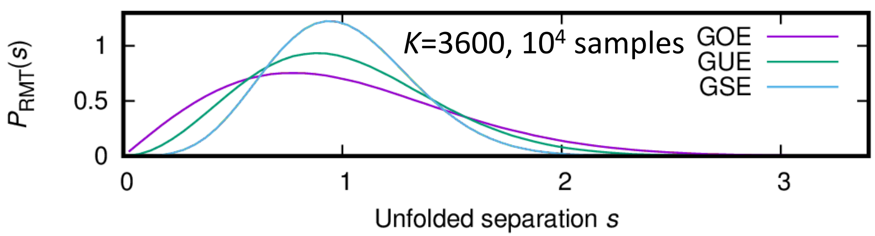
Level separation distribution $P(s)$ for the unfolded Lyapunov spectrum approaches that of random matrix eigenvalues $P_{\mathrm{RMT}}(s)$ at some time scale if the degree of freedom K is large, so does the spectral form factor.

Unfolding



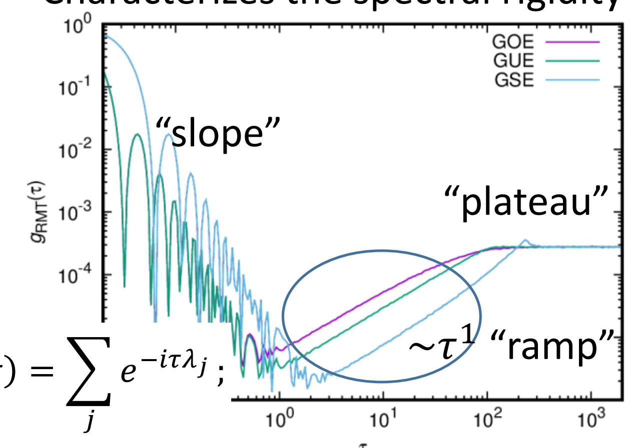
$\rho(\lambda)$: Smoothed density of states

$$\tilde{\lambda} = \int_{-\infty}^{\lambda} \rho(\lambda')\, d\lambda'$$



Spectral form factor

Characterizes the spectral rigidity



$$Z(\tau) = \sum_j e^{-i\tau\lambda_j};$$

$$g(\tau) = \frac{|Z(\tau)|^2}{Z(0)^2} = K^{-2} \sum_{jk} e^{i\tau(\lambda_j - \lambda_k)}$$

Models: logistic map, Lorenz attractor,
D0 brane matrix model (without fermions) and its mass deformation, random band matrix products

cf. Lyapunov spectrum for random coupling
[S. K. Patra and A. Ghosh]

- Kuramoto model [PRE **93**, 032208 (2016)],
- Map networks [EPL **117**, 60002 (2017)]

Strong coupling ⇔ GOE
Weak coupling ⇔ Poisson

Quantum Lyapunov spectrum

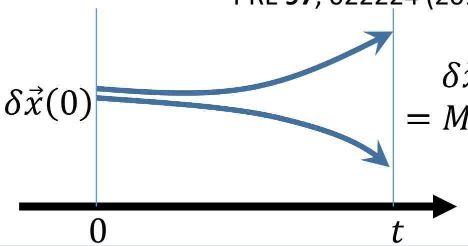
1809.01671

Finite-time **classical Lyapunov spectrum**: obeys RMT statistics for chaos

[Hanada, Shimada, and MT: PRE **97**, 022224 (2018)]

Singular values of $M_{ij} = \left(\frac{\partial x_i(t)}{\partial x_j(0)}\right)$ **at finite *t***: $\{s_k(t)\} = \{e^{\lambda_k t}\}$

$L = \{x_i(t), p_j(0)\}_{\mathrm{PB}}{}^2 = \left(\frac{\partial x_i(t)}{\partial x_j(0)}\right)^2 \to e^{2\lambda_{\mathrm{L}} t}$ for large t



OTOC: $C_T(t) = \left\langle \left|\left[\widehat{W}(t), \widehat{V}(t=0)\right]\right|^2 \right\rangle = \langle \widehat{W}^\dagger(t)\widehat{V}^\dagger(0)\widehat{W}(t)\widehat{V}(0)\rangle + \cdots$

Quantum Lyapunov spectrum: Define $\widehat{M}_{ab}(t)$ as (anti)commutator of $\hat{O}_a(t)$ and $\hat{O}_b(0)$

$$\hat{L}_{ab}(t) = \left[\widehat{M}(t)^\dagger \widehat{M}(t)\right]_{ab} = \sum_{j=1}^{N} \widehat{M}_{ja}(t)^\dagger \widehat{M}_{jb}(t)$$

For $N \times N$ matrix $\langle\phi|\hat{L}_{ab}(t)|\phi\rangle$, obtain singular values $\{s_k(t)\}_{k=1}^{N}$.

The Lyapunov spectrum is defined as $\left\{\lambda_k(t) = \frac{\log s_k(t)}{2t}\right\}$.

Quantum Lyapunov spectrum for SYK model + modification

$$\hat{H} = \sum_{1\le a<b<c<d}^{N} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d + i\sum_{1\le a<b}^{N} K_{ab}\hat{\chi}_a\hat{\chi}_b$$

J_{abcd}: s. d. $= \frac{\sqrt{6}}{N^{3/2}}$

K_{ab}: s. d. $= \frac{K}{\sqrt{N}}$

- Define $\hat{L}_{ab}(t) = \sum_{j=1}^{N} \hat{M}_{ja}(t)\hat{M}_{jb}(t)$ for time-dependent anticommutator $\hat{M}_{ab}(t) = \{\hat{\chi}_a(t), \hat{\chi}_b(0)\}$.

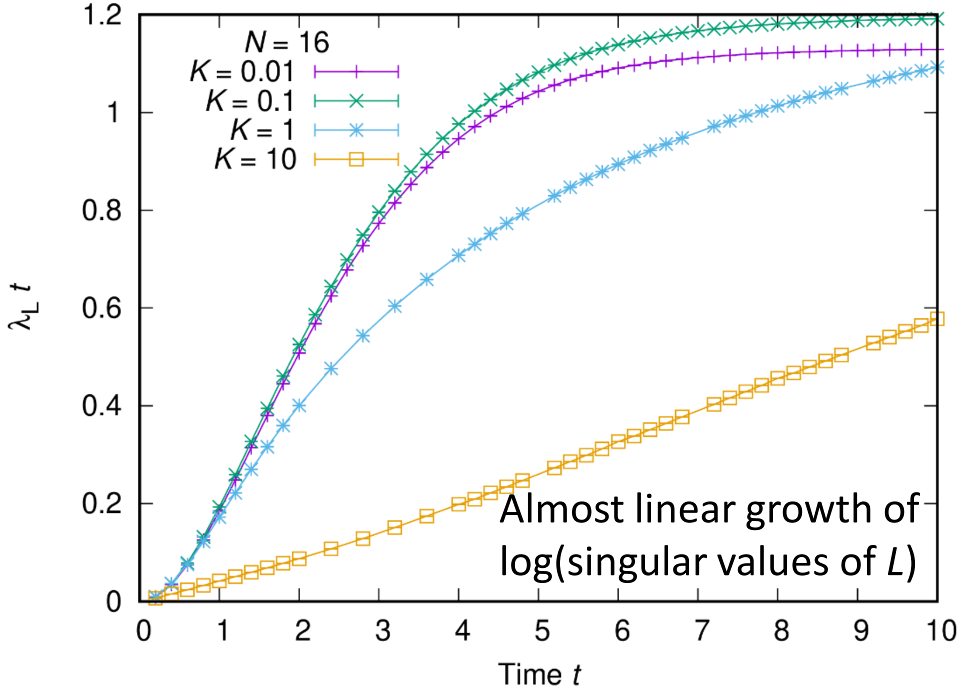
- Obtain the singular values $\{a_k(t)\}_{k=1}^{K}$ of $\langle\phi|\hat{L}_{ab}(t)|\phi\rangle$

- Quantum Lyapunov spectrum: $\left\{\lambda_k(t) = \frac{\log a_k(t)}{2t}\right\}_{k=1,2,\dots,K}$ (also dependent on state ϕ)

Other possibilities: see Rozenbaum-Ganeshan-Galitski, 1801.10591; Hallam-Morley-Green: 1806.05204

1809.01671

Growth of (largest Lyapunov exponent)*time



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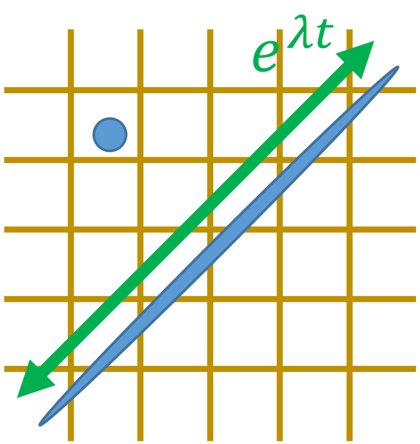
Kolmogorov-Sinai entropy vs entanglement entropy（纠缠熵）production

Coarse-grained entropy
= log(# of cells covering the region)
~ (sum of positive λ) t



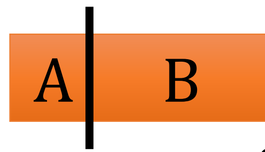
Kolmogorov-Sinai entropy h_{KS}
= (sum of positive λ)
= entropy production rate

Initial state with $S_{\mathrm{EE}} = 0$:

$$|\psi(t=0)\rangle = |000 \dots 000\rangle$$

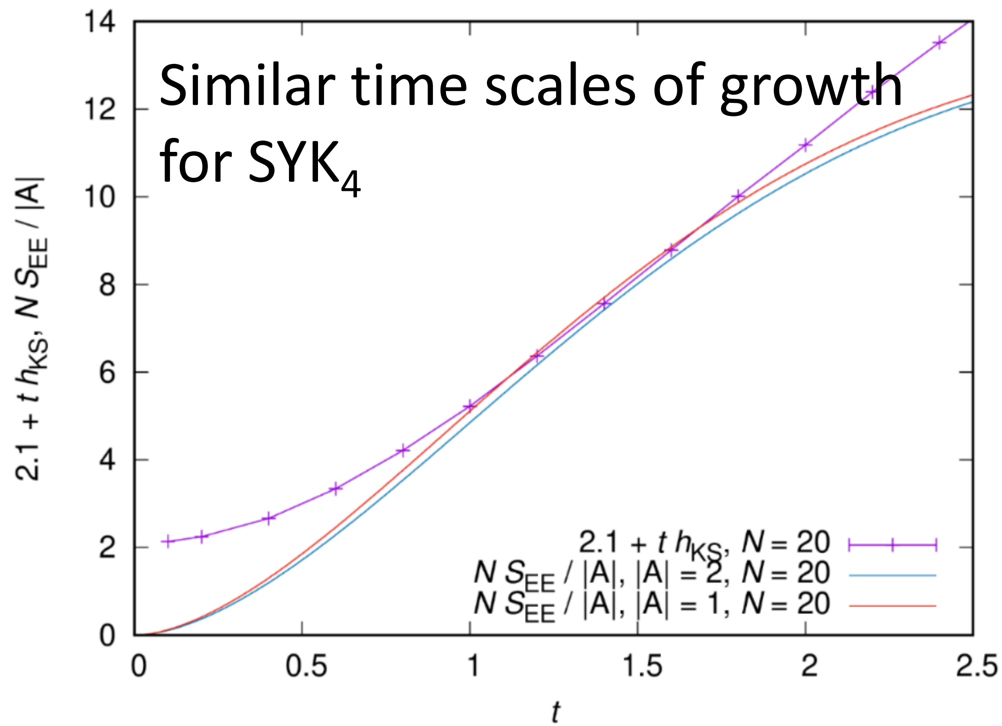
in the complex fermion basis

$$\hat{c}_j = \frac{(\chi_{2j-1} + \mathrm{i}\chi_{2j})}{\sqrt{2}}$$



$$\rho_{\mathrm{A}}(t) = \mathrm{Tr}_{\mathrm{B}}\,\rho(t),\ \rho(t) = |\psi(t)\rangle\langle\psi(t)|$$

$$S_{\mathrm{EE}}(t) = -\mathrm{Tr}\,\rho_{\mathrm{A}}(t)\log\bigl(\rho_{\mathrm{A}}(t)\bigr)$$



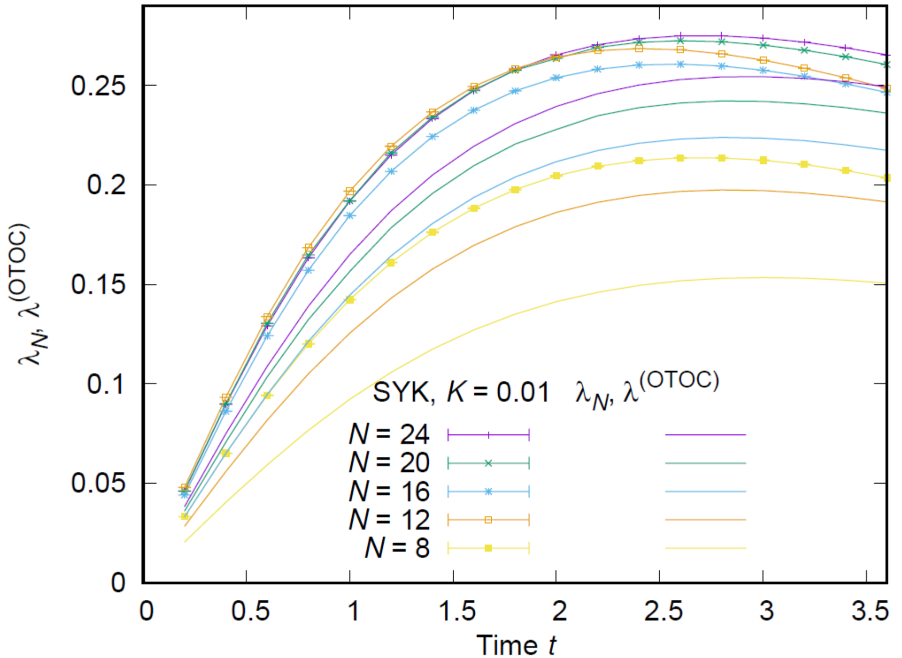
1809.01671

Fastest entropy production?

SYK$_4$ limit

- λ_N and $\lambda_{\rm OTOC} = \frac{1}{2t}\log\left(\frac{1}{N}\sum_{i=1}^{N} e^{2\lambda_i t}\right)$ approach each other; difference decreases as $1/N$
- Same for λ_N and λ_1:

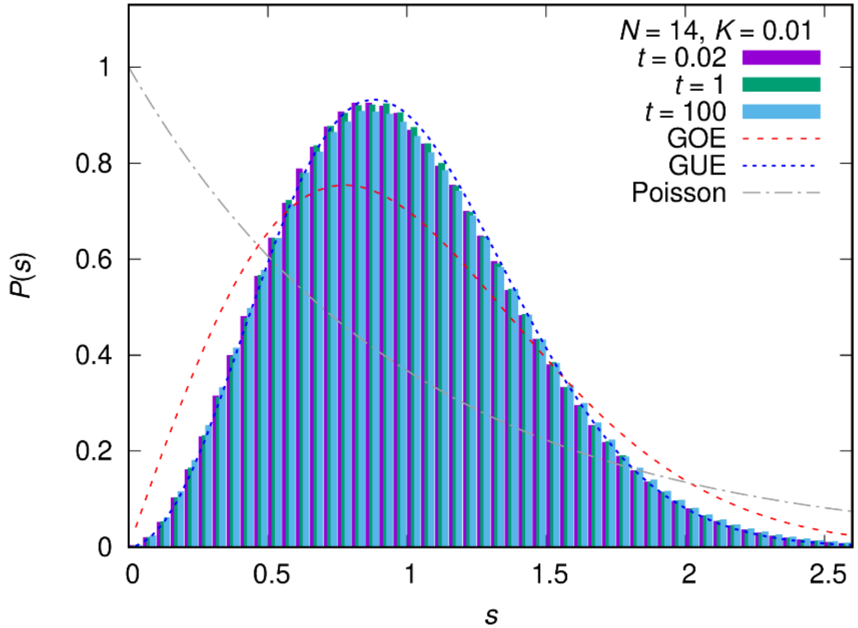
  all exponent → single peak
- All saturate the MSS bound at strong coupling (low *T*) limit
- Growth rate of entanglement entropy $\sim h_{\rm KS}$ = sum of positive (all) λ_i



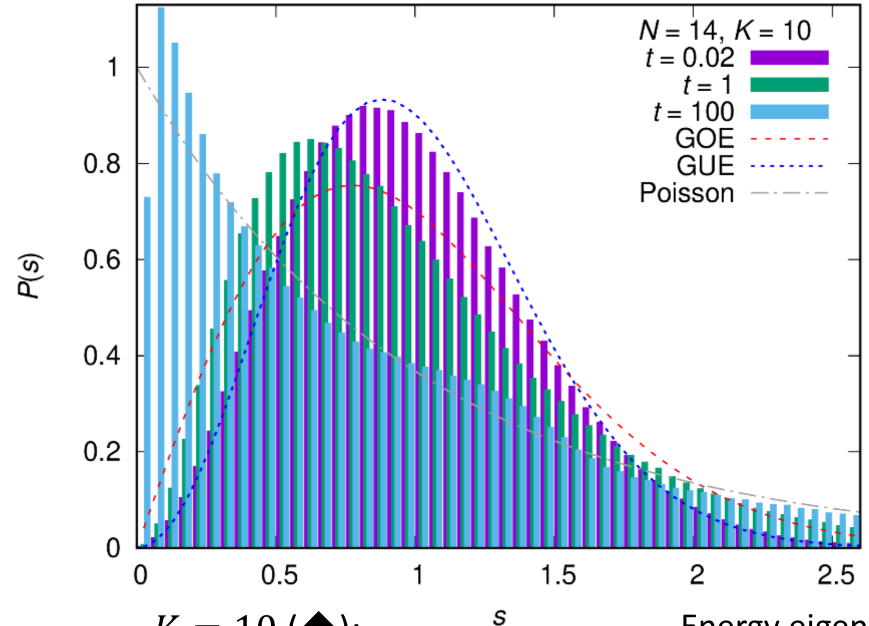
➔ [conjecture] SYK model: not only the fastest scramblers, but also fastest entropy generators

Spectral statistics of quantum Lyapunov spectrum: SYK

1809.01671

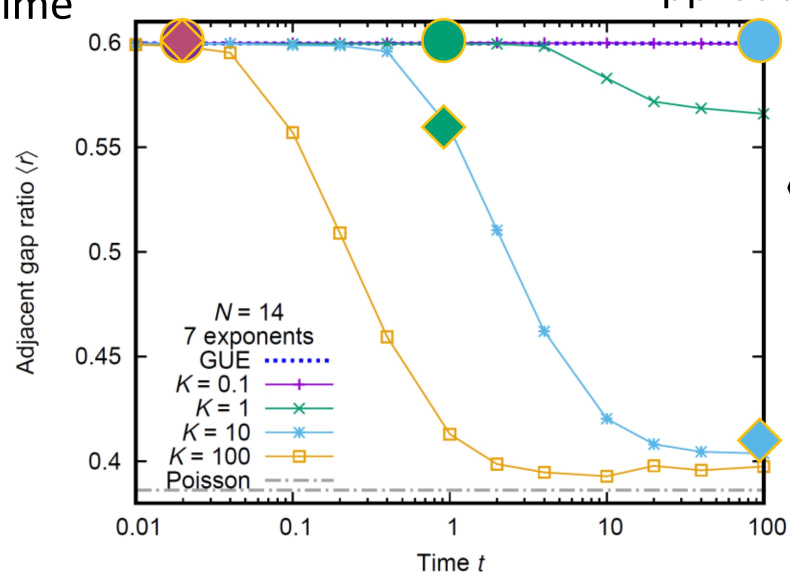


$K = 0.01$ (●):
Remains GUE for long time



$K = 10$ (◆):
Approaches Poisson

Energy eigenstates
$N/2$ larger exponents



$\langle r \rangle$: average of $\dfrac{\min(s_i, s_{i+1})}{\max(s_i, s_{i+1})}$

(fixed-i unfolding: unfold each gap $g_i = \lambda_{i+1} - \lambda_i$ using its average $\langle g_i \rangle_J$, $s_i = g_i / \langle g_i \rangle_J$)

QLS: The case of the random field XXZ model

$$\hat{H} = \sum_{i}^{N} \hat{\boldsymbol{S}}_i \cdot \hat{\boldsymbol{S}}_{i+1} + \sum_{i}^{N} h_i \widehat{S_i^z}$$

h_i: uniform distribution $[-W, W]$

Many-body localization (MBL) transition at $W = W_c \sim 3.5$

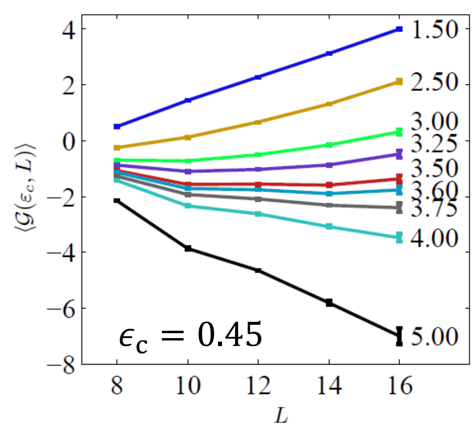
(though recently disputed; e.g. $W_c \geq 5$ proposed in E. V. H. Doggen et al., [1807.05051] using large systems with time-dependent variational principle & machine learning)

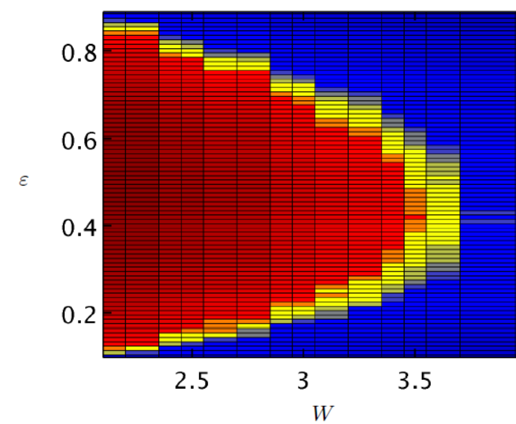
e.g. M. Serbyn, Z. Papic, and D. A. Abanin, Phys. Rev. X **5**, 041047 (2015) (arXiv:1507.01635)

Matrix element of local perturbation

$$\mathcal{G}(\varepsilon, L) = \ln \frac{|V_{n,n+1}|}{E'_{n+1} - E'_n}$$

Energy separation of neighboring energy eigenstates





cf. MBL in short-range SYK [García-García and MT, Phys. Rev. B **99**, 054202 (2019)]; Localization of fermions on quasiperiodic lattice with attractive on-site interaction [Phys. Rev. A **82**, 043613 (2010)]

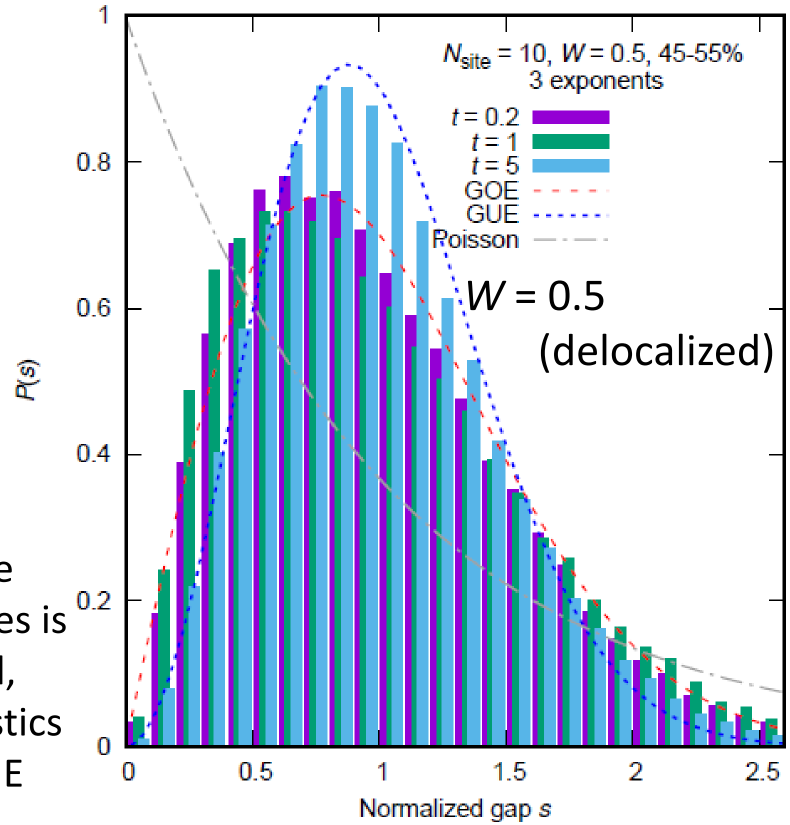
1809.01671

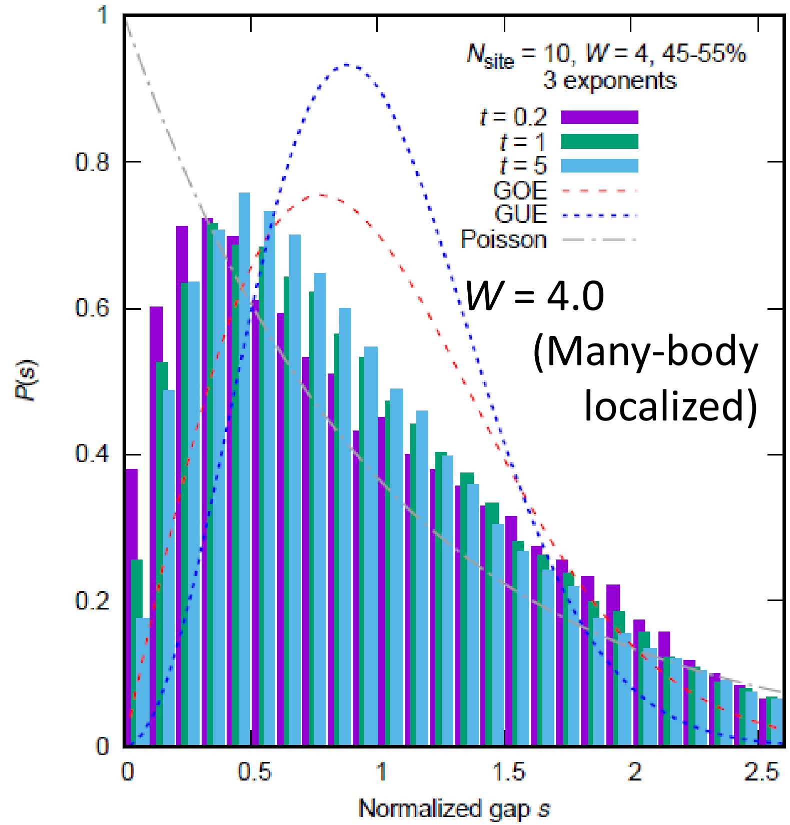
Spectral statistics of QLS for random field XXZ

$$\hat{H} = \sum_i^N \hat{\boldsymbol{S}}_i \cdot \hat{\boldsymbol{S}}_{i+1} + \sum_i^N h_i \widehat{S_i^z}$$

h_i: uniform distribution $[-W, W]$

$$\hat{M}_{ab}(t) = [\widehat{S_a^+}(t), \widehat{S_b^-}(0)]$$





- Exponential growth of the singular values is not observed, but the statistics approach GUE

Quantum Lyapunov spectrum distinguishes chaotic and non-chaotic phases

Singular value statistics of **two-point time correlators**

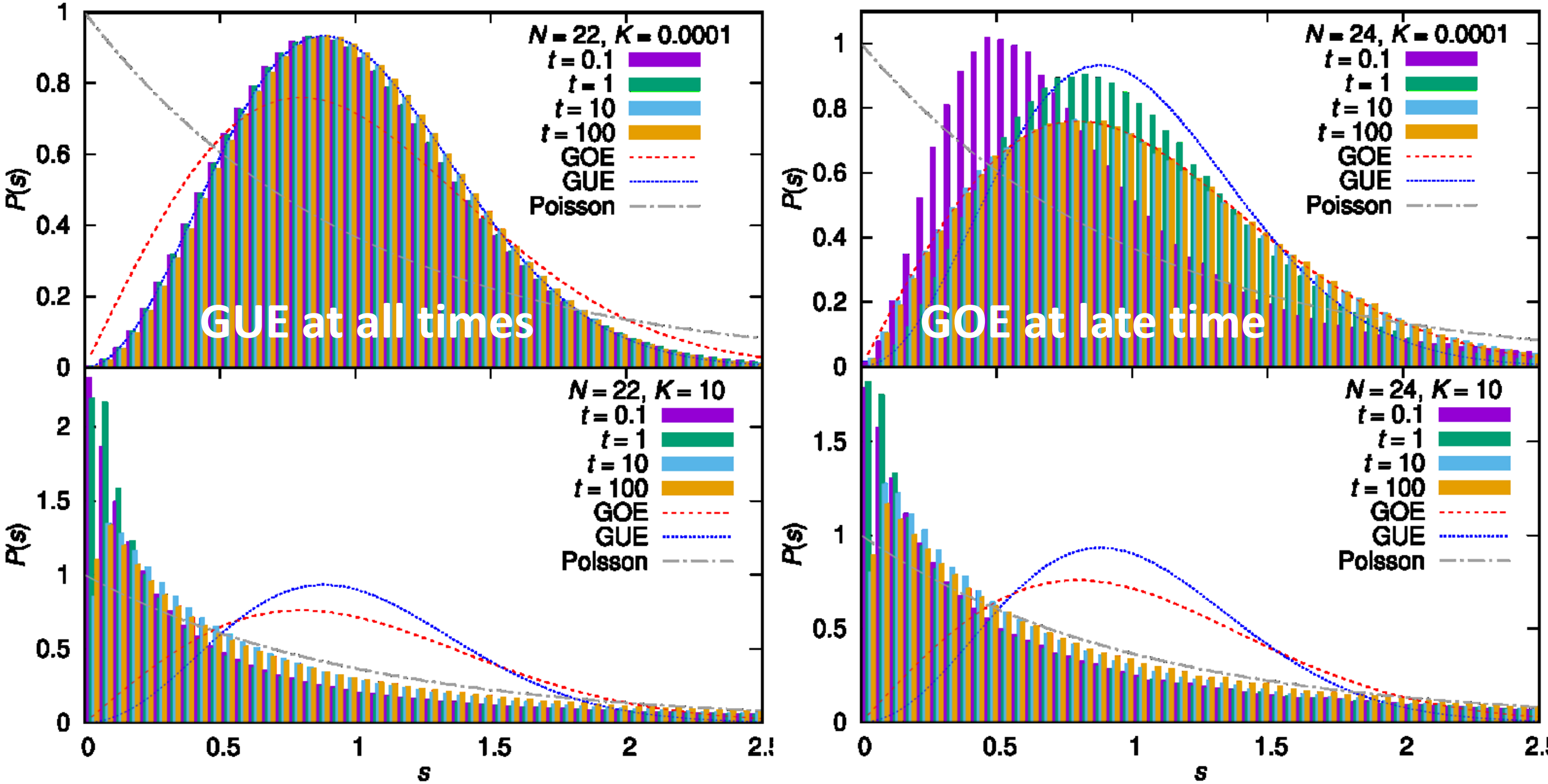
1902.11086

$$G_{ab}^{(\phi)}(t) = \langle \phi | \hat{\chi}_a(t) \hat{\chi}_b(0) | \phi \rangle \text{ as a matrix}$$

$$\lambda_j(t) = \log\left[\text{singular values of}\left(G_{ab}^{(\phi)}(t)\right)\right]$$

SYK$_4$

Strongly perturbed



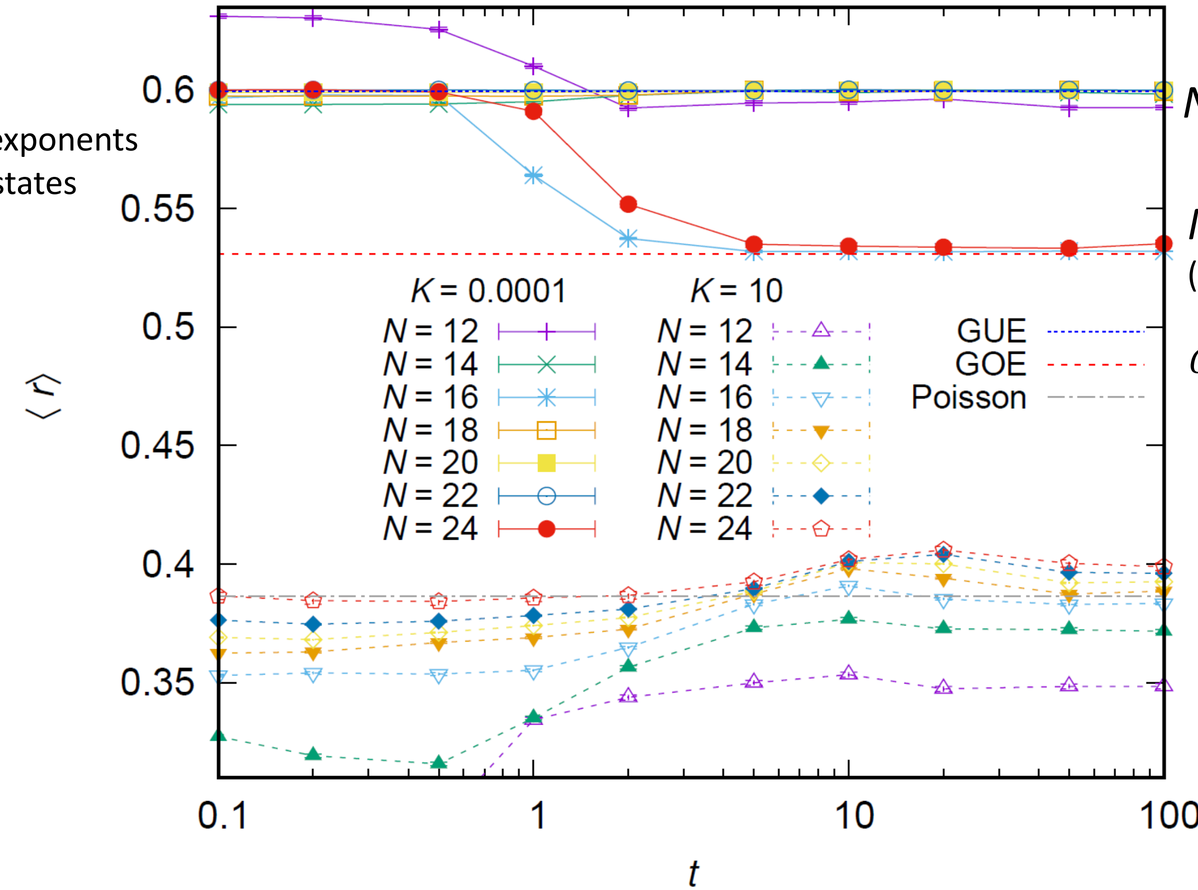
1902.11086

$\langle r \rangle$: average of the adjacent gap ratio $\frac{\min(\lambda_{i+1}-\lambda_i\,,\lambda_{i+2}-\lambda_{i+1})}{\max(\lambda_{i+1}-\lambda_i\,,\lambda_{i+2}-\lambda_{i+1})}$

Uncorrelated (Poisson): $2\log 2 - 1 \approx 0.386$

Correlated: larger (GOE: 0.5307, GUE: 0.5996 etc.) [Atas *et al.*, PRL 2013]

SYK, larger *N*/2 exponents
ϕ: energy eigenstates
fixed-*i* unfolded



N mod 8 = 2, 4, 6: GUE

N mod 8 = 0: GOE
(the matrix is symmetric)

$G^{(\phi)}_{ab} = \langle\phi|\hat{\chi}_a(t)\hat{\chi}_b(0)|\phi\rangle$

At late time, for two-point correlator singular values,
Random matrix behavior ⇔ chaotic

Summary

$$\hat{H}_{\mathrm{SYK}} = \frac{\sqrt{3!}}{N^{3/2}} \sum_{1 \le a < b < c < d \le N} J_{abcd}\hat{\chi}_a\hat{\chi}_b\hat{\chi}_c\hat{\chi}_d$$

black hole

holography

non-gravitational quantum system

cold atom realization

Danshita, Hanada and MT, PTEP 2017 [1606.02454]

maximally chaotic (chaos bound)

maximally chaotic (chaos bound)

numerical analysis; relation to other scrambling systems

Cotler et al., JHEP 2017 [1611.04650]
Gharibyan et al., JHEP 2018 [1803.08050]

modifications to study chaos / integrable transition, many-body localization

García-García et al., PRL 2018 [1707.02197]
García-García and MT, PRB 2019 [1801.03204]

$\lambda_{\mathrm{Lyapunov}}$

random matrix behavior of finite-time Lyapunov spectrum

[Classical] Hanada, Shimada and MT, PRE 2018 [1702.02197]

[Quantum] Gharibyan, Hanada, Swingle and MT, JHEP 2019 [1809.01671]

random matrix behavior of two-point correlators

Gharibyan, Hanada, Swingle and MT, 1902.11086

Lau, Ma, Murugan, and MT, Phys. Lett. B 2019 [1812.04770]

➔ Talk by Chris on Thursday

Summary

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  - Large-*N* solvability: conformal symmetry and maximal chaos
  - Experimental proposal 1606.02454 (and realization)
  - Random matrices (RM) and spectral form factor 1611.04650
  - $SYK_4 + SYK_2$: suppression of maximal chaos 1707.02197
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